OPTICAL PROPERTIES OF METAL-COMPOSITE-BASED THIN FILMS

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Abstract

In the first part of this work we study the effect of a semi-infinite matrix dispersed system (MDS) on the external electromagnetic radiation in the electrostatic approximation. With the help of our previous technique, we obtain general expressions for the multipole expansion coefficients of the electric potential for a sphere accounting for the interaction between ambient particles and the substrate. The polarizability tensor and resonant frequencies of a single sphere show anisotropy due to the influence of a substrate. In the second part electrodynamical properties of thin percolating layers manufactured on the basis of the MDS are considered. Transition from 3-D to 2-D behavior, which is observed near the percolation threshold and shows itself as changing of some parameters (in comparison with those for 3-D percolating system) like the values of percolation threshold, critical indices of conductivity and permittivity, were studied.

Introduction

Interest in matrix dispersed systems is stimulated, first of all, by the possibility of manufacturing materials with predicted optical properties. At the same time, the properties of MDS may strongly differ from those of the materials used for the formation of MDS [1]. In the theoretical studies, MDS are usually considered as infinite systems.

In the first part of this work, we take into consideration the effects of an MDS interface. Namely, the MDS is considered as a half space dielectric matrix with a plane interface separating it from another half space of homogeneous dielectric. The matrix is filled with spherical inclusions of different diameters that are randomly located. The results [2] obtained for the system of spheres on a dielectric substrate can be obtained from our model as a particular case. Basically, this part is a generalization of [3, 4].

In the second part we consider percolating layer of sizes $L \times L \times H$ ($L<<H$), which is a useful model of disordered composite film, particularly, of metal-dielectric layer deposited on a substrate. In such a system near the percolation threshold a transition from 3-D to 2-D behavior is observed, that manifests itself in changing (in comparison with 3-D system) the values of percolation threshold, critical indices of conductivity and permittivity, frequency dependence of dielectric response and other parameters. Below scaling expressions for electrodynamics parameters of percolating layer near the threshold are obtained by method of percolation renormalization group.

Spheres near a substrate

Basic equations.

We consider a semi-infinite MDS consisting of dielectric spheres of different diameters embedded in a homogeneous dielectric (ambient) as shown in Fig. 1. Another half space is filled with another homogeneous dielectric (substrate). The system is placed in the electric field proportional to $e^{i\omega t}$. Let $\varepsilon_a(\omega), \varepsilon_s(\omega)$ and $\varepsilon_i(\omega)$ be the dielectric functions of the ambient, substrate and the i^{th} sphere, respectively, and R_i be the radius of the i^{th} sphere.

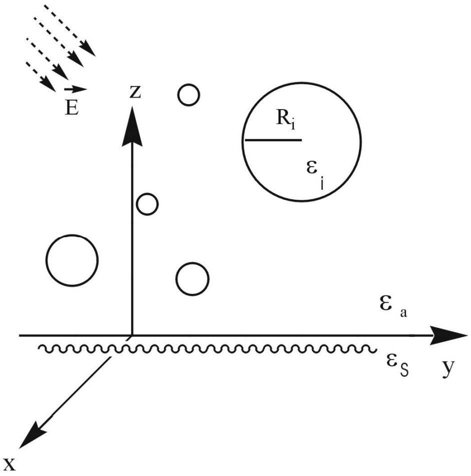


Fig. 1. Geometry of the semi-infinite matrix dispersed system.

Let the wavelength of the external electromagnetic field be much larger than radii of the spheres and the distances between them. In other words, we use the electrostatic approximation. In such a case resulting electric field is caused by the interaction of the external field with the MDS and the substrate and its potential satisfies the Laplace equation

$$\Delta\psi(\vec{r}) = 0 \qquad (1)$$

in the regions I - inside MDS (out of spheres), II - inside the spheres, III - inside the substrate, and does the standard boundary conditions

$$(\psi_i = \psi_j)_{\sigma_{ij}}, \quad \left(\varepsilon_i \frac{\partial\psi_i}{\partial n_i} = \varepsilon_j \frac{\partial\psi_j}{\partial n_j}\right)_{\sigma_{ij}}, \qquad (2)$$

where ε_i is dielectric function of the matter filling out the i^{th} region (*i=I, II, III*),

ψ_i is the resulting field potential in the i^{th} region,

σ_{ij} denotes the common bound surface of the regions *i* and *j*.

Using ideas of the image and multipole expansion methods of solving of electrostatic problems we seek a solution of the problem (1, 2) in the following form:

$$\psi^I = \psi^I_{ext} + \sum_i \psi^I_{i-th\,spere} + \psi^I_{substrate} = -\vec{E}_o\vec{r} + \sum_{ilm} A_{ilm}F_{lm}(\vec{\rho}_i) + \sum_{ilm} A'_{ilm}F_{lm}(\vec{\rho}\,'_i) \qquad (3)$$

$$\psi^{II}_i = \sum_{lm} B_{ilm}G_{lm}(\vec{\rho}_i); \qquad (4)$$

$$\psi^{III} = \psi^{III}_{ext} + \psi^{III}_0 + \sum_{ilm} C_{ilm}F_{lm}(\vec{\rho}\,'_i); \qquad (5)$$

$$\begin{aligned} \psi^I_{ext} &= -\vec{E}_0\vec{r} = -(E_{ox}x + E_{oy}y + E_{oz}z) \\ \psi^{III}_{ext} &= -\vec{E}'_0\vec{r} = -(aE_{ox}x + bE_{oy}y + cE_{oz}z) \end{aligned} \qquad (6)$$

where $F_{lm}(\vec{r}) \equiv r^{-l-1}Y_{lm}(\vec{r})$; $G_{lm}(\vec{r}) \equiv r^l Y_{lm}(\vec{r})$; $\vec{\rho}_i \equiv \vec{r} - \vec{r}_i$; $\vec{\rho}\,'_i \equiv \vec{r} - \vec{r}_i\,'$;
$\vec{r}_i$ is a radius-vector of the center of the i^{th} sphere ; $\vec{r}_i\,'$ is a radius-vector of the i^{th} sphere center image and ψ^{III}_0 is a constant contribution to the potential ψ^{III} related with a choice of radius-vector origin point. Note, that all the individual terms in (3, 4, 5) automatically satisfy

equation (1), and (6) expresses the idea of force lines refraction on the boundary of different media.

The unknown coefficients $A_{lmi}, A'_{lmi}, B_{lmi}, C_{lmi}, a, b, c$ are obtained after applying the boundary conditions (2) to the expansions (3, 4, 5).

Boundary conditions on the substrate surface.

1. Potential continuity condition on the surface σ_{I-III} takes the form

$$(\vec{E}'_0 - \vec{E}_0)\vec{r} - \psi_0^{III} + \sum_{ilm}\{A_{ilm}F_{lm}(\vec{\rho}_i) + A'_{ilm}F_{lm}(\vec{\rho}'_i) - C_{ilm}F_{lm}(\vec{\rho}_i)\}\underset{\sigma_{I-III}}{=} 0 .$$

Different terms here have different arguments. It proves to be more convenient to reduce all the terms to a common argument, e.g. to $\vec{\rho}_i$. Using the fact, that for any point at the boundary surface σ_{I-III}

$$\vec{\rho}_i = (\rho_i, \theta_i, \varphi_i)$$

$$\vec{\rho}'_i = (\rho'_i, \theta'_i, \varphi'_i) = (\rho_i, \pi - \theta_i, \varphi_i)$$

and using the relation [5]

$$Y_{lm}(\pi - \theta, \varphi) = (-1)^{l+m} Y_{lm}(\theta, \varphi)$$

we obtain

$$\Delta\vec{E}_0^{//}\vec{r}^{//} + \Delta\vec{E}_0^{\perp}\vec{r}^{\perp} - \psi_0^{III} + \sum_{ilm}\{A_{ilm} + (-1)^{l+m} A'_{ilm} - C_{ilm}\}F_{lm}(\vec{\rho}_i)\underset{\sigma_{I-III}}{=} 0 ,$$

where we have used decomposition $\vec{r} = \vec{r}^{//} + \vec{r}^{\perp}$ and analogous to it for $\Delta\vec{E} \equiv \vec{E}'_0 - \vec{E}_0$. Obtained equation is equivalent to the set

$$\begin{cases} \Delta\vec{E}_0^{//} \cdot \vec{r}^{//} = 0 \\ \Delta\vec{E}_0^{\perp} \cdot \vec{r}^{\perp} - \psi_0^{III} = 0 \\ A_{ilm} + (-1)^{l+m} A'_{ilm} - C_{ilm} = 0 \end{cases} \quad (7)$$

2. Potential derivative continuity condition on the surface σ_{I-III} in view of $\frac{\partial}{\partial n} = \frac{\partial}{\partial z}$ takes the form

$$(c\varepsilon_s - \varepsilon_a)E_{oz} + \varepsilon_a \sum_{ilm} A_{ilm}\frac{\partial}{\partial z}F_{lm}(\vec{\rho}_i) + \varepsilon_a \sum_{ilm} A'_{ilm}\frac{\partial}{\partial z}F_{lm}(\vec{\rho}'_i) - \varepsilon_s \sum_{ilm} C_{ilm}\frac{\partial}{\partial z}F_{lm}(\vec{\rho}_i)\underset{\sigma_{I-III}}{=} 0 .$$

Again, reducing all the terms to argument $\vec{\rho}_i$ and using relation

$$\frac{\partial}{\partial z}F_{lm}(\pi - \theta, \varphi) = (-1)^{l+m-1}\frac{\partial}{\partial z}F_{lm}(\theta, \varphi) ,$$

which can be seen from [5]

$$\frac{\partial}{\partial z}[f(r)Y_{lm}(\theta,\varphi)] = \left[\frac{(l+1)^2 - m^2}{(2l+1)(2l+3)}\right]^{\frac{1}{2}}\left(\frac{\partial f}{\partial r} - \frac{l}{r}f\right)Y_{l+1,m}(\theta,\varphi) +$$

$$+\left[\frac{l^2 - m^2}{(2l-1)(2l+1)}\right]^{\frac{1}{2}}\left(\frac{\partial f}{\partial r} + \frac{l+1}{r}f\right)Y_{l-1,m}(\theta,\varphi) ,$$

we obtain equation

$$(c\varepsilon_s - \varepsilon_a)E_{oz} + \sum_{ilm}[\varepsilon_a A_{ilm} + \varepsilon_a (-1)^{l+m-1} A'_{ilm} - \varepsilon_s C_{ilm}]\frac{\partial}{\partial z}F_{lm}(\vec{\rho}_i)\underset{\sigma_{I-III}}{=} 0$$

or equivalent set

$$\begin{cases} c\varepsilon_s - \varepsilon_a = 0 \\ \varepsilon_a A_{ilm} + \varepsilon_a (-1)^{l+m-1} A'_{ilm} - \varepsilon_s C_{ilm} = 0 \end{cases} \tag{8}$$

3. The solution of eq. (7, 8) is

$$\begin{cases} a = 1 \\ b = 1 \\ c = \dfrac{\varepsilon_a}{\varepsilon_s} \\ \psi_0^{III} = \left(\dfrac{\varepsilon_a}{\varepsilon_s} - 1 \right) E_{0z} h_0 \\ A'_{ilm} = (-1)^{l+m} \dfrac{\varepsilon_a - \varepsilon_s}{\varepsilon_a + \varepsilon_s} A_{ilm} \\ C_{ilm} = \dfrac{2\varepsilon_a}{\varepsilon_a + \varepsilon_s} A_{ilm} \end{cases} \tag{9}$$

where h_0 is the height of the global origin over the substrate.

Boundary conditions on the sphere surface and equation for A_{ilm}

1. On the surface of j^{th} sphere the potential continuity condition takes the form

$$-\vec{E}_0 \cdot \vec{r} + \sum_{ilm} A_{ilm} F_{lm}(\vec{\rho}_i) + \sum_{ilm} A'_{ilm} F_{lm}(\vec{\rho}\,'_i) - \sum_{lm} B_{jlm} G_{lm}(\vec{\rho}_j) \underset{\sigma_{I-II\,j}}{=} 0 .$$

Applying representations $\vec{r} = \vec{r}_j + \vec{\rho}_j$ and $\vec{\rho}_i = \vec{\rho}_j - (\vec{r}_i - \vec{r}_j)$, well-known addition theorem [6] for spherical harmonics

$$F_{lm}(\vec{r} - \vec{R}) = \sum_{l_1 m_1} T_{lm}^{l_1 m_1} F_{LM}(\vec{R}) G_{l_1 m_1}(\vec{r}) , \qquad (r < R)$$

where $$T_{lm}^{l_1 m_1} \equiv (-1)^{l+m_1} \left[4\pi \frac{(2l+1)}{(2l_1+1)(2L+1)} \cdot \frac{(L+M)!(L-M)!}{(l+m)!(l-m)!(l_1+m_1)!(l_1-m_1)!} \right]^{\frac{1}{2}} ,$$

$$L \equiv l + l_1 , \qquad M \equiv m - m_1 ,$$

and taking into account that $\vec{\rho}_j \big|_{\sigma_{I-II\,j}} = (R_j, \theta_j, \varphi_j)$, we obtain equation

$$\sum_{l_1 m_1} Y_{l_1 m_1}(\Omega_j) R_j^{l_1} \left\{ A_{j l_1 m_1} R_j^{-2l_1-1} + \sum_{ilm} T_{lm}^{l_1 m_1} \left[A_{ilm} F'_{LM}(\vec{r}_i - \vec{r}_j) + A'_{ilm} F_{LM}(\vec{r}\,'_i - \vec{r}_j) \right] - B_{j l_1 m_1} \right\} = \vec{E}_0 \cdot \vec{r}_j + (\vec{E}_0 \cdot \vec{\rho}_j)_{\sigma_{I-II\,j}}$$

where $F'_{lm}(\vec{r}_i - \vec{r}_j) \equiv \begin{cases} F_{lm}(\vec{r}_i - \vec{r}_j), i \neq j \\ 0, i = j \end{cases}$

Interpreting this equation as multipole expansion, we can obtain expression for the coefficients $\{\ldots\}$ by using standard procedure $\int d\Omega \cdot Y_{lm}^* \cdot \ldots$, that leads to

$$\left\{ A_{j l_1 m_1} R_j^{-2l_1-1} + \sum_{ilm} T_{lm}^{l_1 m_1} \left[A_{ilm} F'_{LM}(\vec{r}_i - \vec{r}_j) + A'_{ilm} F_{LM}(\vec{r}\,'_i - \vec{r}_j) \right] - B_{j l_1 m_1} \right\} R_j^{l_1} = \sqrt{4\pi} \vec{E}_0 \vec{r}_j \delta_{00}^{l_1 m_1} + \frac{4}{3} \pi E_0 R_j \sum_{m=-1}^{1} \left[Y_{1m}^*(\vec{E}_0) \delta_{1m}^{l_1 m_1} \right]$$

While deriving last expression we have used relations [5]

$$\int Y_{lm}(\Omega) Y_{l'm'}^*(\Omega) d\Omega = \delta_{ll'} \delta_{mm'} \equiv \delta_{lm}^{l'm'} ,$$

$$\vec{a} \cdot \vec{b} = ab \cdot \cos\left(\widehat{\vec{a} \cdot \vec{b}} \right) = \frac{4}{3} \pi ab \sum_{m=-1}^{1} Y_{1m}(\vec{a}) Y_{1m}^*(\vec{b}) .$$

2. Potential derivative continuity condition on the surface of j^{th} sphere in view of $\left.\frac{\partial}{\partial n}\right|_{\sigma_{I-II_j}} = \frac{\partial}{\partial \rho_j}$ and $\left.\rho_j\right|_{\sigma_{I-II_j}} = R_j$ takes the form

$$\sum_{l_1 m_1}\left\{\varepsilon_a \frac{l_1+1}{l_1} R_j^{-1} A_{jl_1m_1} + \varepsilon_j B_{jl_1m_1} - \varepsilon_a \sum_{ilm} T_{lm}^{l_1m_1}\left[A_{ilm} F'_{LM}\left(\vec{r}_i - \vec{r}_j\right) + A'_{ilm} F_{LM}\left(\vec{r}_i{}' - \vec{r}_j\right)\right]\right\} l_1 R_j^{l_1-1} Y_{l_1m_1}\left(\Omega_j\right) = -\varepsilon_a (\vec{E}_0 \cdot \hat{\rho}_j)$$

Applying to this expression the same procedure as earlier, we obtain relation

$$\left\{\varepsilon_a \frac{\left(l_1+1\right)}{l_1} R_j^{-2l_1-1} A_{jl_1m_1} + \varepsilon_j B_{jl_1m_1} - \varepsilon_a \sum_{ilm} T_{lm}^{l_1m_1}\left[A_{ilm} F'_{LM}\left(\vec{r}_i - \vec{r}_j\right) + A'_{ilm} F_{LM}\left(\vec{r}_i{}' - \vec{r}_j\right)\right]\right\} l_1 R_j^{l_1-1} = -\frac{4}{3}\pi \varepsilon_a E_0 \sum_{m=-1}^{1}\left[Y_{1m}^{*}\left(\hat{E}_0\right)\delta_{l_1 m_1}^{1m}\right].$$

Two equations obtained from the boundary conditions on the surface of j^{th} sphere form the full set defining unknown coefficients A_{ilm} and B_{ilm} (note, that explicit form of A'_{ilm} as function of A_{ilm} was found earlier, see eq. (9)). After some transformations it can be reduced to the form

$$\begin{cases} B_{ilm} = f\left(A_{ilm}\right) \\ \sum\limits_{\substack{ilm \\ l\neq 0}}\left\{\delta_{jl_1m_1}^{ilm} + K_{jl_1m_1}^{ilm}\right\} A_{ilm} = V_{jl_1m_1} \end{cases}, \qquad (10)$$

where $K_{jl_1m_1}^{ilm} \equiv \alpha_{jl_1} T_{l_1m_1}^{lm}\left\{F'_{LM}\left(\vec{r}_i - \vec{r}_j\right) + \left(-1\right)^{l+m}\frac{\varepsilon_a - \varepsilon_s}{\varepsilon_a + \varepsilon_s} F_{LM}\left(\vec{r}_i{}' - \vec{r}_j\right)\right\}$,

$$\alpha_{jl_1} \equiv \frac{l_1\left(\varepsilon_j - \varepsilon_a\right)}{l_1\varepsilon_j + (l_1+1)\varepsilon_a} R_j^{2l_1+1},$$

$$V_{jl_1m_1} = \frac{4}{3}\pi \alpha_{jl_1}\delta_{l_1}^{1} E_0 \sum_{m=-1}^{1} Y_{1m}^{*}\left(\hat{E}_0\right)\delta_{m_1}^{m},$$

$$\vec{E}_0 = \left(E_{0x}, E_{0y}, E_{0z}\right) = E_0 \cdot \hat{E}_0.$$

The explicit form of the function f in (10) is not needed for further consideration.

Second equation of (10) can be written in the matrix form $\left[\hat{1} + \hat{K}\right]\hat{A} = \hat{V}$ or $\hat{A} = \left[\hat{1} + \hat{K}\right]^{-1}\hat{V}$, that allows us to interpret the matrix $\hat{M} \equiv \left[\hat{1} + \hat{K}\right]^{-1}$, which connects external potential matrix V_{ilm} and multipole coefficients A_{ilm}, as the multipole polarizability matrix of the MDS spheres.

A single sphere near a substrate. The resonant frequencies.

For a single sphere near a substrate, we can obtain the polarizability tensor in the dipole-dipole approximation by using (10):

$$\hat{\alpha} = \frac{4}{3}\pi R^3 \varepsilon_a\left(\varepsilon - \varepsilon_a\right)\begin{pmatrix}\alpha_{II} & 0 & 0 \\ 0 & \alpha_{II} & 0 \\ 0 & 0 & \alpha_{\perp}\end{pmatrix}, \qquad (11)$$

where $\alpha_i = \left[\varepsilon_a + L_i\left(\varepsilon - \varepsilon_a\right)\right]^{-1}$; $(i = II, \perp)$; $L_i = \frac{1}{3}\left(1 + l_i \frac{\varepsilon_a - \varepsilon_s}{\varepsilon_a + \varepsilon_s}\right)$;

$$l_i = \frac{R}{h}\cdot\begin{cases}\frac{1}{8}, (i = //) \\ \frac{1}{4}, (i = \perp)\end{cases}$$

h is the distance between the sphere's center and the substrate.

Let us consider the case of Lorentz's dielectric functions and $\varepsilon_a = 1$ (vacuum):

$$\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega} \quad ; \quad \varepsilon_s(\omega) = 1 + \frac{\omega_{ps}^2}{\omega_{0s}^2 - \omega^2 - i\gamma_s\omega} .$$

The resonant frequency is obtained by using the condition $\alpha_i(\omega_{res}) = \infty$. In our case it reduces to the following algebraic equation with respect to the frequency:

$$\omega^4 + a_3\omega^3 + a_2\omega^2 + a_1\omega + a_0 = 0, \tag{12}$$

where $a_3 = i(\gamma + \gamma_s)$

$$a_2 = -\left(\omega_0^2 + \omega_{0s}^2 + \frac{1}{3}\omega_p^2 + \frac{1}{2}\omega_{ps}^2 + \gamma\gamma_s \right)$$

$$a_1 = -i\left(\gamma_s\omega_0^2 + \gamma\omega_{0s}^2 + \frac{1}{3}\gamma_s\omega_p^2 + \frac{1}{2}\gamma\omega_{ps}^2 \right)$$

$$a_0 = \omega_0^2\omega_{0s}^2 + \frac{1}{3}\omega_{0s}^2\omega_p^2 + \frac{1}{2}\omega_0^2\omega_{ps}^2 + \frac{1}{6}(1 - l_i)\omega_p^2\omega_{ps}^2$$

A solution to (12) neglecting damping $(\gamma = \gamma_s = 0)$ is

$$(\omega_{1,2}^i)^2 = \frac{1}{2}\left\{ y_1 + y_2 \pm \sqrt{(y_1 - y_2)^2 + 4l_i y_3} \right\} \tag{13}$$

where $y_1 = \varpi_0^2 + \dfrac{\varpi_p^2}{3}$; $y_2 = \varpi_{os}^2 + \dfrac{\varpi_{ps}^2}{2}$; $y_3 = \dfrac{\varpi_p^2}{3} \cdot \dfrac{\varpi_{ps}^2}{2}$.

Particularly, for a metallic sphere on the dielectric substrate from (13), using the inequality $\omega_{ps}/\omega_p << 1$, we obtain the following approximate expressions

$$\begin{cases} (\omega_{res}^{(1)})^2 = \dfrac{\varpi_p^2}{3} + l_i \dfrac{\varpi_{ps}^2}{2} \\ (\omega_{res}^{(2)})^2 = \omega_{0s}^2 + (1 - l_i)\dfrac{\varpi_{ps}^2}{2} \end{cases} \tag{14}$$

for the four (i=//,⊥) resonant frequencies. Note that $\varpi_p/\sqrt{3}$ is well-known surface plasmon frequency of a sphere and $\varpi_{ps}/\sqrt{2}$ is one of a substrate.

As we see, substrate changes the dipole moment of a sphere in such a way, that the four resonant frequencies arise in the absorption spectrum of a sphere. What causes arising of such a number of the resonant frequencies? First, one pair of the frequencies is observed when the field direction is parallel to the substrate, while another one – when perpendicular, and these two pairs don't coincide in addition. In general case field has both the components and absorption spectrum has the four resonant frequencies respectively.

Second, under certain field direction (// or ⊥ to the substrate) the pair of frequencies arises due to an interaction between surface plasmons of the sphere and of the substrate. Under increasing the distance between sphere and substrate this interaction vanishes and we obtain well-known result: a single sphere and a single half-infinite substrate absorb radiation at the frequencies $\varpi_p/\sqrt{3}$ and $\varpi_{ps}/\sqrt{2}$ respectively.

Electrodynamical properties of percolating layers based on metal-dielectric composites

In this part we consider electrophysical and optical properties of percolating system like a layer consisting of conducting and non-conducting inclusions of typical size *a*, at that its volume fractions are *p* and *1-p* respectively. The layer size is supposed to be infinite in the longitudinal directions, and of thickness *H* in the transversal direction. The layer is bound by the planes *z=0* and *z=H* and has cubic packing consisted of randomly arranged conducting (black) and non-conducting (white) cubes. Such a system structure leads to no loss of generality of further obtained results, because inclusion shape is insufficient near metal-dielectric percolation phase transition. The system has anisotropical electrodynamics properties from the very beginning, because it always has finite conducting cluster that connects *z=0* and *z=H* planes, when $H << \infty$ and *p* is close to a critical value [7]. It is clear, that percolation threshold for longitudinal direction $p_c^{//}$ is dependent from *H* and $p_c^{//} \to p_c^{(3)}$ when $H \to \infty$, where $p_c^{(3)}$ is percolation threshold for 3-D packing. In the case *H=a* we have $p_c^{//} = p_c^{(2)}$, where $p_c^{(2)}$ is percolation threshold for 2-D packing. In our case, as it follows from [7], $p_c^{(3)} \approx 0.3117$ (percolation threshold of site problem for cubic lattice) and $p_c^{(2)} \approx 0.59275$ (percolation threshold of site problem for square lattice).

At finite layer thickness $a << H < \infty$ the system properties are defined by the relation between *H* and the correlation length of 3-D percolating system

$$\xi_3 \approx a|p - p_c|^{-\nu_3}, \tag{15}$$

where $\nu_3 \approx 0.9$ is the critical index of the correlation length [7]. If $\xi_3 << H$, then 3-D system is isotropic and its electrodynamics parameters are independent from *H*. In this case evaluation of the parameters should be performed by using the mean field approximation [9] at $\xi_3 \approx a$ and the scaling relations at $\xi_3 >> a$. Particularly, the layer conductivity σ at $p > p_c^{(3)}$ is given by

$$\sigma_\perp = \sigma_{//} = \sigma_1 (p - p_c^{(3)})^{t_3}, \tag{16}$$

where $t_3 \approx 1.6 \div 2$ [7], and σ_1 is the specific conductivity of respective (black) cubes. At $\xi > H$ the layer behaves itself like a 2-D system consisting of effective $H \times H \times H$ blocks.

Characteristics of the blocks can be calculated by the method of percolation renormalization group transformation (PRGT) [13, 14]. This transformation fulfils transition from percolating system consisting of elements of size *a* to the system, which is equivalent to that on macroscopical properties but consists of effective elements (blocks) of magnified *n* times sizes. The effective element involves n^d elements of original system (*d* is the space dimension; in our case *d*=3).

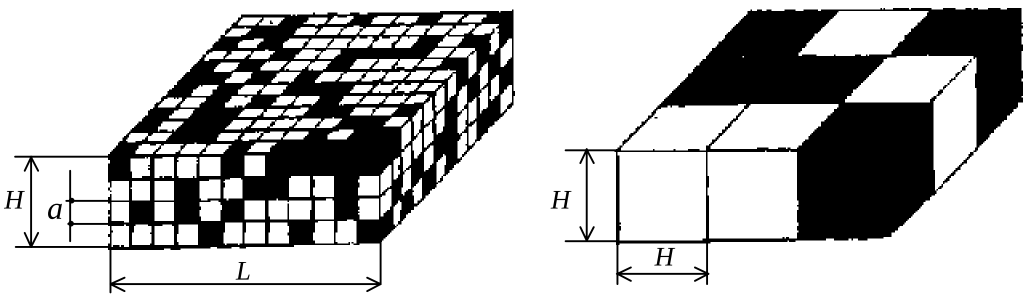
Applying of PRGT is able under condition $an << \xi_3$. In the transformed system the fraction of effective conductors *p'* and its specific conductivity σ_1^* are equal to [13]

$$p^* = p_c^{(3)} + n^{1/\nu_3}(p - p_c^{(3)}), \tag{17}$$

$$\sigma_1^* = \sigma_1 n^{-t_3/\nu_3}. \tag{18}$$

Note, that relations (17, 18) reflect supposition that percolation threshold $p_c^{(3)}$ is the fixed point of PRGT and that correlation length ξ as well effective conductivity σ (16) are conserving under PRGT.

Let's now apply PRGT to the system and put $n = H/a$. Then we transit from percolating layer of thickness $H=na$ to 2-D mosaic formed by effective blocks of size $H \times H \times H$ (Fig. 2, *b*). The mosaic properties are defined by the relations (17, 18).



a b

Fig. 2. (*a*) - Percolating layer; (*b*) - 2-D percolating system obtained as result of PRGT.

Because the percolation threshold $p_c^{(2)}$ for 2-D mosaic is larger, than that for respective 3-D packing, percolation breakdown in the longitudinal direction occurs when the effective conductor fraction p^* becomes equal to $p_c^{(2)}$. Therefore the percolation threshold of percolating layer is defined by the condition

$$p_{c,2} = p_{c,3} + (H/a)^{1/\nu_3}(p_{c,\|} - p_{c,3}) \tag{19}$$

and is equal to

$$p_{c,\|} = p_{c,3} + (p_{c,2} - p_{c,3})(H/a)^{-1/\nu_3} \tag{20}$$

Dependence (20) of percolation threshold of layer on its thickness for cubic packing is shown on Fig. 2. It consents qualitatively with experimental results [14] obtained by studying conductivity of a layer of conducting and nonconducting spheres and with results [13] obtained in studying of metal-ceramic films $Au\text{-}Al_2O_3$.

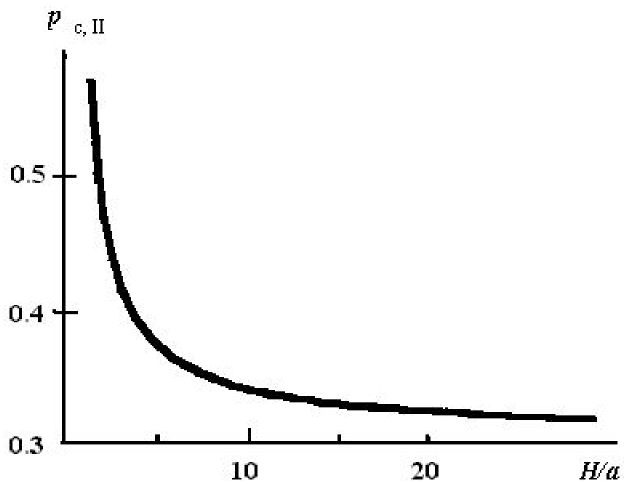


Fig. 3. Percolation threshold for longitudinal direction as a function of film thickness.

Near the percolation threshold at $\xi_3 > H$ transversal conductivity of the layer is defined by the effective conductivity σ^* of the conducting blocks accounting for its fraction p^*:

$$\sigma_\perp(H) = p^*\sigma_1^* \approx p_c^{(3)}\sigma_1(H/a)^{-t_3/\nu_3} \qquad (21)$$

This formula corresponds to fractal law of conductivity of isotropic percolating system of finite size.

Longitudinal characteristics of the layer at $\xi_3 > H$ are defined by the critical indices of 2-D systems. In particular, the correlation length is

$$\xi_2 = H\left|p - p_c^{(2)}\right|^{-\nu_2} = a(H/a)^{1-\nu_2/\nu_3}\left|p - p_c^{//}\right|^{-\nu_2} \qquad (22)$$

where the critical index $\nu_2 = 4/3$ [14] and longitudinal conductivity at $p > p_c^{//}$ is

$$\sigma_\| = \sigma_1^*(p^* - p_{c,2}) = \sigma_1(H/a)^{(t_2-t_3)/\nu_3}(p^* - p_\|)^{t_2} \qquad (23)$$

where critical index $t_2 \approx 1{,}3$ [7].

Above used PRGT method allows us, in our opinion, to consider more complicated problems too.

Evaluation of effective permittivity of the layer is performed in the low frequency limit. In this case, starting with expression for permittivity of conducting conclusions [1]

$$\varepsilon_1(\omega) = \varepsilon_{1\infty} - \frac{\omega_p^2}{\omega(\omega + i\nu)}\ , \qquad (24)$$

that at $\nu > \omega$ takes the form

$$\varepsilon_1(\omega) \approx \varepsilon_{1\infty} + i\frac{4\pi\sigma_1(\omega)}{\omega}\ , \qquad (25)$$

where $\sigma_1(\omega) \equiv \dfrac{\omega_p^2}{4\pi\nu}$ is conductivity of conclusions near the percolation transition,

we obtain for effective values of $\varepsilon_1(\omega)$ the same relations as (25), but only with $\tilde{\varepsilon}_{1\infty}$ instead of $\varepsilon_{1\infty}$, and $\tilde{\sigma}_1(\omega) \equiv \sigma(\omega)$ instead of $\sigma_1(\omega)$. The form of dependency $\sigma(\omega)$ is evaluated earlier, and that of $\tilde{\varepsilon}_{1\infty}$ can be easily evaluated from the well-known relations [1].

Conclusions

We obtained the general expression for the resonant frequency of the model system, which is a dielectric sphere in vacuum on a dielectric substrate. The latter results in splitting and shifting of the resonant frequency depending on a direction of the external field according to (13). This allows one to suggest that layers of small particles on a substrate possess anisotropic electrodynamical properties. Using PRGT method, we have developed the theory on calculation of conductivity and permittivity of metal-dielectric films of any thickness near the percolation transition. The percolation threshold was found as well as its dependency on film thickness and scaling dependencies (at $\xi_3 > H$) of both the longitudinal and transversal conductivities on the thickness. Obtained theoretical results consent qualitatively with experimental data [12, 13].

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