Radio Physics and Radio Astronomy, 2002, v. 7, No. 4, pp. 415-419

RADAR RANGE EQUATION FOR ULTRA WIDEBAND SIGNALS AND ULTRA SHORT PULSES

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The range equation for radiolocation of an object and its elements with ultrawideband signals (UWBS) and ultra short pulses (USP) are resulted. The criticism of other published equations is given.

1. Introduction

Key ratio in the theory of a radiolocation of objects by using the UWBS and USP is the range equation. The examples of such equation are given in [1-3]. Unfortunately, the examples in [2-3] – are erroneous, and the area of application of the equation given in [1], is limited to a rare case, when the pulse, reflected from object, has the form of a rectangular. The purpose of this work is to offer a universal variant of the equations based on using the auto correlation characteristics of a radio-line.

2. The Range Equation for Radiolocation of Object by Using Narrow-Band Signals

For simplicity we shall assume, that the transmitting and receiving antennas (TA and RA) and motionless object are in free space. In this case the classical range equation, in which monochromatic signal with frequency ω is used, has a form:

$$\bar{P}_R = \bar{P}_T H(\omega)/16\pi^2 r_T^2 r_R^2 \geq \bar{P}_{R\,\min}, \qquad (1)$$

where r_T and r_R – distances from TA to object and from object to RA; $\bar{P}_T$ – average (for the period $2\pi/\omega$) capacity of a signal, leaded to TA; $\bar{P}_R$ and $\bar{P}_{R\,\min}$ – average capacity of a signal and, accordingly, given minimally necessary average capacity of a signal on an output of RA;

$$H(\omega) \equiv G_T(\omega)\sigma(\omega)A_R(\omega)\times [1-\eta_T(\omega)][1-\eta_R(\omega)]\gamma(\omega) - \qquad (2)$$

transfer function of a radio-line; $G_T(\omega)$ and $A_R(\omega)$ – TA gain and effective area of aperture of RA in the direction to the object; $\sigma(\omega)$ – bistatic radar cross section (RCS); $\eta_{T,R}(\omega)$ – factors of reflection of a transmitted signal from an input of TA and received signal from an output of RA; $\gamma(\omega)$ – factor of polarisation losses.

In a case of a quasi-monochromatic signal the average capacities in (1) need to be replaced with energy:

$$W_R = W_T H(\omega)/16\pi^2 r_T^2 r_R^2 \geq W_{R\,\min}, \qquad (3)$$

where W_T – the energy of the signal, leaded to TA; W_R and $W_{R\,\min}$ – the energy of the signal and, accordingly, the minimal necessary energy on RA output.

However, in practice a ratio (3) is used not only for quasi-monochromatic, but also for real signals occupying a frequency bands distinguished from a zero. Usually the width $\Delta\omega_T$ of an active band of a power spectrum of a signal, leaded to TA, is so small, that the relative change of value $H(\omega)$ within the limits of such a band is insignificant:

$$\Delta H/H \equiv |H(\omega_{T2}) - H(\omega_{T1})|/H(\omega_{T0}) \approx |H'_\omega(\omega_{T0})|\Delta\omega_T/H(\omega_{T0}) \ll 1. \qquad (4)$$

Here $\Delta\omega_T = \omega_{T2} - \omega_{T1}$; $\omega_{T0} = (\omega_{T1} + \omega_{T2})/2$; $\omega_{T1,2}$ and ω_{T0} – extreme and average frequency of active band, within the limits of which is overwhelming part (for example, 95%) of the energy of the signal. Then

$$W_R \approx W_T H(\omega_{T0})/16\pi^2 r_T^2 r_R^2 \geq W_{R\,\min}. \qquad (5)$$

From the physical point of view in the considered situation width $\Delta\omega_H$ of active band of transfer function $H(\omega)$ of radio-line is greater than $\Delta\omega_T$: $\Delta\omega_H \gg \Delta\omega_T$ (here $\Delta\omega_H = \omega_{H2} - \omega_{H1}$; $\omega_{H1,2}$ – extreme frequencies of an active band). Therefore, energetic spectrums of transmitted and received signals are approximately similar, and, hence, the width $\Delta\omega_{T,R}$ of active bands of energetic spectrums of these signals, and also their times $\tau_{c\;T,R}$ of auto correlation are about equal: $\Delta\omega_T \approx \Delta\omega_R$, $\tau_{c\,T} \approx \tau_{c\,R}$.

For pulses with finite duration, it is possible to proceed from energies to average capacities, thus averaging to carry out during auto correlation. In result, we shall receive:

$$\tilde{P}_R \approx \tilde{P}_T H(\omega_{T0}) / 16\pi^2 r_T^2 r_R^2 \geq \tilde{P}_{R\,\min}, \quad (6)$$

where the values, averaged during auto correlation, are marked with tildes: $\tilde{P}_{T,R} = W_{T,R} / \tau_{c\,T,R}$, $\tilde{P}_{R\,\min} = W_{R\,\min} / \tau_{c\,R}$.

If the phase characteristic of complex transfer function (the square of which module is equal to $H(\omega)$) is rather linear within the limits of a band, transfer-reception of pulses will occur practically without distortions of their form and consequently for them not only the times of auto correlation, but the duration will be equal also: $\tau_T \approx \tau_R$. In this case averaged in time auto correlation $\tau_{c\,T,R}$ can be replaced with averaged in time pulse duration, and such image to pass to pulse capacities. Then (6) accepts a form close to (1):

$$\bar{P}_R \approx \bar{P}_T H(\omega_{T\,0}) / 16\pi^2 r_T^2 r_R^2 \geq \bar{P}_{R\,\min}, \quad (7)$$

where $\bar{P}_{T,R} = W_{T,R} / \tau_{T,R}$, $\bar{P}_{R\,\min} = W_{R\,\min} / \tau_R$.

3. The Range Equation for Radiolocation of Object by Using UWBS and USP

However, for many transmitted signals the condition (4) is not carried out and for this reason the ratio (5)–(7) become inapplicable. For such signals $\Delta\omega_T \sim \Delta\omega_H$ or $\Delta\omega_T \gg \Delta\omega_H$. As a rule, we can relate to them UWBS, for which $\Delta\omega_T \sim \omega_{T0}$, and also USP with duration $\tau_T \sim \tau_H \equiv 2\pi / \Delta\omega_T$ and $\tau_T \ll \tau_H$.

It is specified in [4,5], that it is expedient of this situation to proceed to the integrated description of the characteristics of a radio-line. In the given case – to integration the partial (for frequency ω) range equation which has been written down with use bilateral (in view of negative frequencies) of spectral density $w_T(\omega)$ of energy of a transmitted signal and bilateral transfer function:

$$W_R = \frac{\int\limits_0^\infty w_T(\omega) H(\omega) d\omega}{8\pi^2 r_T^2 r_R^2} \geq W_{R\,\min}. \quad (8)$$

Thus $w_T(\omega)$ should satisfy the power condition:

$$W_T = \int\limits_{-\infty}^{\infty} w_T(\omega) d\omega = 2\int\limits_0^{\infty} w_T(\omega) d\omega. \quad (9)$$

The equation (8) is a generalisation of the equations (3), (5)-(7) and can be used for any transmitted signals. For quasi-monochromatic signal, when $w_T(\omega) = W_T[\delta(\omega - \omega') + \delta(\omega + \omega')]/2$, it coincides with (3). For a narrow-band signal, when by virtue of a condition (4) it is possible to replace integrals $2\int_0^\infty w_T(\omega) H(\omega) d\omega$ with value $H(\omega_{T0}) W_T$ equal on a condition (9), it coincides with (5), and in more special cases – with (6) and (7).

The equation (8) can be copied in other, sometimes more convenient form with use of temporary parameters of process of transfer-reception. For this purpose, we shall spread out even functions $H(\omega)$ and $w_T(\omega)$ in Fourier-integrals. For function $w_T(\omega)$ this operation is quite applicable, as integral on a condition (9) has finite value. Concerning function $H(\omega)$ we shall notice, that it is regular and in a limit $\omega \to \infty$ it is equal to zero because of vanishing the values $G_T(\omega)$, $A_R(\omega)$ or $\eta_{T,R}(\omega)$. It can occur for the different reasons. For example, for mirror antennas functions $G(\omega)$ or $A(\omega)$ vanish because of increasing at $\omega \to \infty$ the influence of asynchronous of excitation of the aperture, which is caused by the finite sizes of both focal stain, and radiator, and roughness of the surface of the mirror. Not pressing further in these details, we shall believe, that at $\omega \to \infty$ speed of decrease of function $H(\omega)$ rather large so, that integral $\int_0^\infty H(\omega) d\omega$ converges. Then, using cosine-decomposition

$$\begin{aligned} \begin{Bmatrix} B_H(\tau) \\ B_{w_T}(\tau) \end{Bmatrix} &= 2\int\limits_0^\infty \begin{Bmatrix} H(\omega) \\ w_T(\omega) \end{Bmatrix} \cos(\omega\tau) d\omega, \\ \begin{Bmatrix} H(\omega) \\ w_T(\omega) \end{Bmatrix} &= \frac{1}{\pi}\int\limits_0^\infty \begin{Bmatrix} B_H(\tau) \\ B_{w_T}(\tau) \end{Bmatrix} \cos(\omega\tau) d\tau, \end{aligned} \quad (10)$$

and known integral equality

$$\int\limits_0^\infty w_T(\omega) H(\omega) d\omega = \frac{1}{2\pi}\int\limits_0^\infty B_{w_T}(\tau) B_H(\tau) d\tau,$$

we shall rewrite (8) as:

$$W_R = \frac{\int\limits_0^\infty B_{w_T}(\tau) B_H(\tau) d\tau}{16\pi^3 r_T^2 r_R^2} \geq W_{R\,\min}, \quad (11)$$

where $B_{w_T}(\tau)$ and $B_H(\tau)$ – the auto-correlation functions (ACF) of a transmitted signal and radio-line.

The ratio (8) and (11) are equivalent, however depending on a situation one of them can be more acceptable because of simplicity of calculation of integral. How already it was marked above, for rather narrow-band signals, when $\Delta\omega_T \ll \Delta\omega_H$ and condition (4) is satisfied, the equation (8) becomes simpler and is reduced to (5). Otherwise, for rather broadband signals, when $\Delta\omega_T \gg \Delta\omega_H$ and the condition of rather small varying of function $w_T(\omega)$ in an active band of frequencies $\omega_{H1} \div \omega_{H2}$ is satisfied calculation of integral in (8) with taking into account (10) results in a ratio:

$$W_R \approx w_T(\omega_{H0})B_H(0)/16\pi^2 r_T^2 r_R^2 \geq W_{R\,\min}. \quad (12)$$

Let us consider a case, when factor of auto correlation (FAC) $k_{w_T}(\tau) \equiv B_{w_T}(\tau)/B_{w_T}(0)$ is of a push-button kind. Let's take an approximation $k_{w_T}(\tau) = \exp[-\chi p(\tau)]g(\tau)$, where $\chi \gg 1$ is a large dimensionless parameter, $p(\tau)$ and $g(\tau)$ – any smooth functions, and $p(0) = 0$, $g(0) = 1$, $p'_\tau(0) = 0$, $p''_{\tau\tau}(0) > 0$. Using Taylor-decomposition $p(\tau) \approx p''_{\tau\tau}(0)\tau^2/2$ in a vicinity of the point $\tau = 0$, it is easy to show, that the value $\sqrt{2/[\chi p''_{\tau\tau}(0)]}$ can be identified as $\tau_{c\,T}$ – time of auto correlation of a transmitted signal determined on a level e^{-1} of fall of function $k_{w_T}(\tau)$. At $\tau_{c\,T} \to 0$ FAC accepts a push-button form: $k_{w_T}(\tau) \to 1$ at $\tau = 0$ and $k_{w_T}(\tau) \to 0$ at $\tau \neq 0$. By applying a method of asymptotic calculation of integral in (11) (at $\tau_{c\,T} \to 0$) and taking into account following from (9)-(10) equality

$$B_{w_T}(0) = W_T, \quad (13)$$

we receive:

$$W_R \approx \sqrt{\pi}B_H(0)W_T\tau_{c\,T}/16\pi^3 r_T^2 r_R^2 \geq W_{R\,\min}. \quad (14)$$

For monostatic radar, when $r_T = r_R \equiv r$, from (14) it follows: $r_{\max} \sim \sqrt[4]{W_T\tau_{c\,T}}$. The analysis of this dependence shows, that at reduction of time $\tau_{c\,T}$ the maximal range $r_{\max}$ decreases. To prevent it one need to increase proportionally the energy W_T of the transmitted signal.

The given feature is quite stacked in frameworks of known physical law, according to which the best power opportunity on transfer-reception of signals is realising not in broadband but in quasi-monochromatic radio-lines. Because as in this case by choosing frequency of a transmitted signal equal to frequency of a global maximum of the function $H(\omega)$, it is possible to achieve a global maximum of the received energy W_R. At the same time, the broadband radio-lines in comparison with narrow-band have new quality consisting in small time of process of transfer-reception, that in radiolocation is using for the identification of objects and their elements. However, "a power payment" for purchase of this new quality at preservation of former range of action of a radio-line is the appropriate increase of energy W_T of signals.

4. The Range Equation for Radiolocation of Elements of Object with UWBS and USP

We modify (8), written down for detection of objects wholly, with reference to detection of separate parts of object. For this purpose, let us take the ACF $B_{w_R}(\tau)$ of received signal:

$$B_{w_R}(\tau) = \frac{\int_0^\infty w_T(\omega)H(\omega)\cos\omega\tau\, d\omega}{8\pi^2 r_T^2 r_R^2} \geq B_{w_R\,\min}(\tau), \quad (15)$$

where $B_{w_R\,\min}(\tau)$ is a given threshold function. At excess of a threshold, the presence of some (but still not identified) elements of object responsible for formation of ACF at the moment τ is considered found out. The meaning $B_{w_R}(0)$ characterises influence of all elements of object, therefore at $\tau = 0$ (15) passes in (8). For detection and maintenance of identification of elements of object most approach ACF with narrow peaks. Such ACF are necessary to formation for applying special kinds of transmitted signals, including UWBS and USP.

Let us consider last case in more detail, thus for simplicity we shall be limited to monostatic radar. Let us present the object as a set of N bright points carried in a radial direction. Let's assume, that RCS σ_i of each bright point does not depend on frequency ω. Then (15) accepts a form:

$$B_{w_R}(\tau) = B_1(\tau)\sum_{i=1}^{N}\sigma_i + \sum_{j=i+1}^{N}\sum_{i=1}^{N}\sqrt{\sigma_i\sigma_j} \times \left[B_1\left(\tau + \frac{2l_{ij}}{c}\right) + B_1\left(\tau - \frac{2l_{ij}}{c}\right)\right] \geq B_{w_R\,\min}(\tau), \quad (16)$$

where l_{ij} – radial distance between bright points,

$$B_1(\tau) = \frac{\int\limits_0^\infty w_T(\omega) H(\omega,\sigma)|_{\sigma=1} \cos\omega\tau \, d\omega}{8\pi^2 r^4} \quad - \quad (17)$$

specific (on unit of RCS) ACF of received signal connected to ACF by ratio: $B_{w_R}(\tau) = \sigma B_1(\tau)$. If specific ACF has a form close to push-button – with one peak at $\tau = 0$, then, how it follows from (16), the ACF $B_{w_R}(\tau)$ besides the basic peak at $\tau = 0$ with amplitude equal to $B_1(0)\sum_{i=1}^{N}\sigma_i$, will have $N(N-1)$ additional peaks at $\tau = \pm 2l_{ij}/c$ with amplitudes equal to $B_1(0)\sqrt{\sigma_i\sigma_j}$. By applying the inequality $2\sum_{j=i+1}^{N}\sum_{i=1}^{N}\sqrt{\sigma_i\sigma_j} \le \sum_{i=1}^{N}\sigma_i$, it is easy to be convinced that the sum of amplitudes of all additional peaks does not surpass amplitude of the basic peak. From the analysis (16) also follows, that for detection of a bright point with the least value of RCS $(\sigma_i)_{\min}$ the threshold $B_{w_R \min}(\tau)$ should not surpass $B_1(0)(\sigma_i)_{\min}$. If $B_{w_R \min}(\tau) = \text{const}$, then for detection of a bright point of object with least value of RCS in comparison with a case of detection of object as a whole minimally necessary value $B_1(0)$ should be increased in $\frac{\sum_{i=1}^{N}\sigma_i}{(\sigma_i)_{\min}}$ times. For identical bright points the increase must be of N times. In this case equation (15) for one element of object can be presented as (8) for object as a whole, but with factor of a stock on energy equal N:

$$\frac{N\int\limits_0^\infty w_T(\omega) H(\omega) d\omega}{8\pi^2 r^4} \ge W_{R \min} \equiv B_{w_R \min}. \quad (18)$$

At increase of number of bright points (that usually happens at complication of a configuration of object) requirement to value of factor of a stock accordingly grow. If these requirements do not hold and energy of a transmitted signal do not increase in N times, then, how it follows from (18), the maximal range $r_{\max}$ of detection of one bright point will decrease in $\sqrt[4]{N}$ times. The maximal range of detection of object as a whole, certainly, remains former. Other, less power-intensive way of increase of value $B_1(0)$ is the rational redistribution of energy density $w_T(\omega)$ on spectrum, which is equivalent to change of the form of a transmitted signal. One of radical ways of change of the form is transition to the coded sequence of transmitted pulses.

5. Criticism of Other Range Equations for UWBS and USP

In light stated we should return to criticism of the equations appearing in [2-3]. The equation [2] at $\eta_{T,R}(\omega) = 0$ and $\gamma(\omega) = 1$ for bistatic case has a form:

$$\frac{[P_T(t)]_{\max} G_T(\omega_{T0}) \sigma_i(\omega_{T0}) A_R(\omega_{T0})}{16\pi^2 r_T^2 r_R^2} \ge P_{R \min}, \quad (19)$$

where $[P_T(t)]_{\max}$ – maximal instant meaning of capacity of the transmitter; $P_{R \min}$ – minimally allowable meaning of capacity on an input of the receiver.

The ratio (19) is received in a heuristic way and combines time and spectral approaches, which, at the end, results in mistakes. To confirm it, we shall consider the elementary case, when the object consists of one reflecting element, and the signal generated by the transmitter is a pulse with finite duration and rather narrow-band spectrum. Then the ratio (19) should coincide with (7). However it does not occur because of basic distinction between $[P_T(t)]_{\max}$ and $\overline{P}_T$. The presence in (19) G, σ and A, taken only for the average frequency ω_{T0} of the spectrum of the transmitted signal, also is a mistake, because dependence on other part of a spectrum and, hence, on the form of a transmitted signal is ignored. In (8) as against (18) it is taken into account by integration on all spectrums of a signal.

The range equation given in [3], at $\eta_{T,R}(\omega) = 0$, $\gamma(\omega) = 1$, in view of generalisation on bistatic case has a form:

$$\frac{W_T G_T(S_T, \tau_T) \sigma(t) A_R(S_T, \tau_T)}{16\pi^2 r_T^2 r_R^2} \ge W_{R \min}. \quad (20)$$

Here symbol S_T is implied to be the form of the transmitted signal. In (8) the RCS is the function of frequency ω, while in (20) – is a function of time t. In opinion of the author [3], basis for this purpose is dependence on time of intensity $E_s(t)$ of scattered field, which square, ostensibly, appears in definition of RCS: $\sigma \sim E_s^2$. However, actually in classical definition of RCS appears average (for the period $2\pi/\omega$) density of capacity of scattered signal, which is calculated as the integral on time t: $\sigma \sim \frac{\omega}{2\pi}\int_0^{2\pi/\omega} E_s^2(t)\,dt$. Therefore, the RCS is the function of frequency, instead of the time. From the physical point of view, RCS plays a role of transfer function of a radio-line and to mix it with time characteristics is inadmissible. Unfortunately, the similar mistake is repeated later in [6-8]. Consequently, the authors of [6] came to essentially incorrect final conclusions, about what, in particular, was already informed in [4]. Thus, coming back to the equation (19) it is necessary to ascertain, that it is wrong.

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ОСНОВНОЕ УРАВНЕНИЕ РАДИОЛОКАЦИИ ДЛЯ СВЕРХШИРОКОПОЛОСНЫХ СИГНАЛОВ И УЛЬТРАКОРОТКИХ ИМПУЛЬСОВ

В.Б. Авдеев

Получено основное уравнение радиолокации для локации объекта и его элементов сверхширокополосными сигналами и ультракороткими импульсами. Приведен критический анализ других опубликованных вариантов этого уравнения.

ОСНОВНЕ РІВНЯННЯ РАДІОЛОКАЦІЇ ДЛЯ НАДШИРОКОСМУГОВИХ СИГНАЛІВ ТА УЛЬТРАКОРОТКИХ ІМПУЛЬСІВ

В.Б. Авдєєв

Отримано основне рівняння радіолокації для локації об’єкта та його елементів надширокосмуговими сигналами та ультракороткими імпульсами. Наведено критичний аналіз інших опублікованих варіантів цього рівняння.