PLASMA EXPANSION INTO GAS

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An initial problem of the low-temperature plasma expansion in an unbounded gaseous medium is considered. It is assumed that the plasma had been created locally in the background gas by a source of ionization which acts during the small enough time. The ions have a temperature of the background gas, the ionization degree is small and the macroscopic gas dynamics is neglectable. We assume the electrons are Maxwellian. The mathematical model and the numerical method for the problem treatment are elaborated. The problem can be interesting in the plasma afterglow studies and for the tasks of laser pumping by a short pulse.

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1. PROBLEM STATEMENT

Investigations of the low temperature plasma, which freely expands into the gaseous medium or a vacuum, have a long history (see, e.g, [1-3] and references therein). There are a wide set of astrophysical, spacecraft, technological problems, which lead to the necessity of such studies.

The system under consideration is shown on the Fig. 1. The plasma density spatial distribution is radially symmetrical. The cases of the flat layer, the cylinder and the sphere are considered. The fast electrons run away of the plasma and the radial ambipolar electric field arises. The last leads to the ion drift through the background gas and to the plasma expanding. So, our model consists of the ambipolar field equation, the ion movement equation, and the balance conditions for the power and the energy

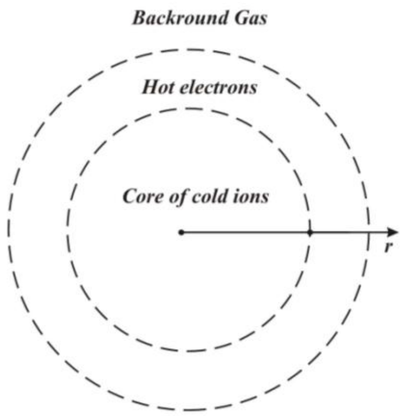

Fig. 1. The configuration of the problem

2. AMBIPOLAR FIELD EQUATION

We start from the equation for the divergence of the ambipolar electric field $\vec{E}$,

$$\nabla\vec{E} = 4\pi e(n - n_e)\,, \quad (1)$$

where n_e is the electrons density, n is the ions density, summarized by components with the charge of the ion Z_α (in elementary charge e units) and density n_α:

$$n = \sum\nolimits_\alpha Z_\alpha n_\alpha. \quad (2)$$

The characteristic time of the changing of the electron density is significantly less compared with the characteristic time of the ions, so it is helpful for the robust numerical calculations to exclude n_e from (1). For this purpose let us consider the equation of the movement of the electron fluid [4]:

$$m_e n_e \frac{\partial \vec{v}_e}{\partial t} = -T\nabla n_e - e n_e. \quad (3)$$

Here, m_e, n_e, $\vec{v}_e$, T are the electron mass, the density, the velocity and the temperature of the electron fluid, correspondingly. We confine ourselves to the case when the inertial term is neglectable compared with other two terms in (3). Let us to introduce the dimensionless variables r', n', E' as follows:

$$r = r' r_{De}\,, \quad (4)$$

$$n = n' n_0\,, \quad (5)$$

$$E = E' \frac{T_0}{e r_{De}}\,, \quad (6)$$

$$r_{De}^2 = \frac{T_0}{4\pi e^2 n_0}. \quad (7)$$

Here n_0 and T_0 are the electron density and the electron temperature in the center of the plasma in the initial moment of time. Under the above assumption the equations (1), (3) pass to

$$\nabla' n_e' + n_e' \vec{E}' = 0\,, \quad (8)$$

$$\nabla' \vec{E}' = n' - n_e'\,, \quad (9)$$

and we obtain the equation for the ambipolar electric field with the spatial distribution density n' of ions given:

$$\nabla'(\nabla'\vec{E}' - n') + \vec{E}'(\nabla'\vec{E}' - n') = 0. \quad (10)$$

In the case of the radial symmetry we have

$$\vec{E}' = E' \vec{e}_r\,, \quad (11)$$

$$\nabla' n' = \frac{\partial n'}{\partial r'} \vec{e}_r , \qquad (12)$$

$$\nabla' \vec{E}' = \frac{1}{r'^{\alpha}} \frac{\partial}{\partial r'} r'^{\alpha} E', \qquad (13)$$

where parameter $\alpha = 0,1,2$ denotes the case of the flat layer, the cylinder and the sphere, correspondingly. Then, the equation (10) transforms to

$$\frac{\partial}{\partial r'}\left(\frac{\partial E'}{\partial r'} + WE'\right) + AE' = \frac{\partial n'}{\partial r'} , \qquad (14)$$

$$W = \frac{\alpha}{r'} + \frac{1}{2} E' , \qquad (15)$$

$$A = \frac{\alpha}{r'} E' - n' . \qquad (16)$$

The obvious boundary condition is

$$\vec{E}'\Big|_{r' \to \infty} = 0 . \qquad (17)$$

3. DRIFT OF THE IONS

The movement equation of the ion of the kind α reads [4]:

$$m_\alpha n_\alpha \frac{\partial \vec{v}_\alpha}{\partial t} + \nu_\alpha \vec{v}_\alpha = \frac{eZ_\alpha}{m_\alpha} \vec{E}. \qquad (18)$$

Here m_α, $\vec{v}_\alpha$, ν_α are the ion mass, the ion velocity and the ion-neutral collision frequency, correspondingly. The dimensionless values t', $\vec{v}_\alpha$, ν_α are determined as follows:

$$t = t'/\omega_0, \qquad (19)$$

$$\nu_\alpha = \nu_\alpha' \omega_0, \qquad (20)$$

$$v_\alpha = v_\alpha' v_0, \qquad (21)$$

where

$$\omega_0 = \sqrt{\frac{4\pi e^2 n_0}{m_0}} \qquad (22)$$

is the plasma frequency of the ion component with the smallest ion mass m_0. The corresponding Bohm velocity plays the role of the velocity scaling coefficient:

$$v_0 = r_{De}\omega_0 = v_\alpha' \sqrt{T_0 / m_0}. \qquad (23)$$

Generally, the law of the ion movement (18) supports multistream solutions. We avoid them for simplicity, and suppose the inertia term small enough:

$$m_\alpha = 0 . \qquad (24)$$

So, the movement of the ion of the kind α is determined only by the ion mobility μ_α. In dimensionless variables we have

$$v_\alpha' = \mu_\alpha' E' . \qquad (25)$$

The use of the variable mobility model [4] gives:

$$\mu_\alpha' = \frac{2\lambda_\alpha' Z_\alpha}{\pi m_\alpha' \sqrt{v_\alpha'^2 + T_\alpha'}} , \qquad (26)$$

where the dimensionless mass m_α', the temperature T_α', and the free pass length λ_α' of the ions of the kind α are

$$m_\alpha' = m_\alpha / m_0 , \qquad (27)$$

$$T_\alpha' = T_\alpha / T_0 , \qquad (28)$$

$$\lambda_\alpha' = \lambda_\alpha / r_{De} . \qquad (29)$$

4. PARTICLES AND ENERGY BALANCE

For the ion density we have the continuity equation:

$$\frac{\partial n_\alpha'}{\partial t'} + \nabla' n_\alpha' \vec{v}_\alpha' = \nu_{i\alpha}' n_e' , \qquad (30)$$

where $\nu_{i\alpha}'$ is the normalized ionization frequency. The density of the plasma internal energy consists of the electric field energy and the electron gas internal energy:

$$u = \frac{\vec{E}^2}{8\pi} + \frac{3}{2} n_e T . \qquad (31)$$

The plasma expansion is accompanied with the transformation of the total internal energy u to the charged particles kinetic energy, the ohmic heating, and the inelastic processes such as excitation and ionization:

$$\frac{\partial U'}{\partial t'} = -\int_V (P'_{Ohm} + P'_{inel}) dV' , \qquad (32)$$

$$U' = \int_V u' dV' , \qquad (33)$$

$$P'_{inel} = n' \sum_m \nu_m' \varepsilon_m' , \qquad (34)$$

$$P'_{Ohm} = E' \sum_m Z_\alpha' n_\alpha' v_\alpha' , \qquad (35)$$

$$u' = \frac{\vec{E}'^2}{2} + \frac{3}{2} n_e' T' , \qquad (36)$$

where ν_m' and ε_m' are the frequency and the energy loss of the inelastic process m. In the balance equation (32) we neglect the ions kinetic energy due to the assumption (24). The dimensionless energies are introduced by

$$u = u' n_0 T . \qquad (37)$$

5. NUMERICAL SOLUTION

The calculation procedure is organized in the form of sequential solving of the equations (14), (30), (32) at the each time step. Globally, the evolution of the system can be represented as follows:

$$\frac{\partial}{\partial t} x(t) = f(x(t)) , \qquad (38)$$

$$x = \begin{pmatrix} n_\alpha' \\ U' \end{pmatrix} . \qquad (39)$$

We approximate the derivation in (38) by the finite difference

$$\frac{\partial}{\partial t} x(t) \to \frac{x_k - x_{k-1}}{\tau} , \qquad (40)$$

where x_k is the value of the x at the time $t_k = k\tau$, and τ is the time step. The x_k is achieved by the iterations according to the relation:

$$x_k^i = x_{k-1} + \tau(f(x_k^{i-1})(1-p) + f(x_{k-1})p), \quad (41)$$

where i is the iteration number, p is the convergence parameter. For the calculation of x_k^{i-1} we must find E', v_α' and T' with x_k^{i-1} (i.e, n_α' and U') given. The ambipolar field only depends on the total ion density n' and should be obtained from (10), the ions velocities only depends on the ambipolar field (25), and the temperature is found from (32), (36), (9):

$$T' \int_V \frac{3}{2} n_e' dV' = U' - \int_V \frac{E'^2}{3} dV'. \quad (42)$$

The ambipolar electric field equation (14) is nonlinear and it is also solved iteratively:

$$\frac{\partial}{\partial r'}\left(\frac{\partial E'^i}{\partial r'} + W^{i-1}E'^i\right) + A^{i-1}E'^i = \frac{\partial n'}{\partial r'}. \quad (43)$$

The iterations converge quickly, so the nonlinearity doesn't bring any troubles and allows us to avoid a headache associated with the calculation of the electron density. Partial derivative equation (43) belongs to the convection-diffusion type and is solved by the Scharfetter-Gummel scheme [5].

As for continuity equation (30), it can be solved by a great variety of ways, but we do it in the spirit of Euler-Lagrange scheme described in [6]. The calculation domain is divided into set of cells with the impenetrable walls which are occupied by the ions of the certain kind. Between an adjacent moments of time (say, t_{k-1} and t_k), the walls move with the ion velocity (25). Before the next time step the domain is covered by the new set of cells and the ion density is redistributed. This scheme is totally conservative, and has a moderate numerical diffusion. But most importantly, it is helpful in the numerical treatment of the shock wave near the ion core boundary.

CONCLUSIONS

In this paper we consider the free diffusion of the plasma into the background gas. The equation set of plasma time-space evolution is formulated. The robust and cost effective numerical method for the equations solving is elaborated.

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РАСШИРЕНИЕ ПЛАЗМЫ В ГАЗ

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Рассмотрена начальная задача расширения низкотемпературной плазмы в неограниченную газовую среду. Предполагается, что плазма создана локально источником ионизации, действующим достаточно малое время. Ионы имеют температуру фонового газа, степень ионизации плазмы невелика и макроскопические течения газа отсутствуют. Электроны имеют максвелловское распределение. Разработаны математическая модель и численный метод её решения. Задача может быть полезна в исследованиях послесвечения и лазерной накачки коротким импульсом.

РОЗШИРЕННЯ ПЛАЗМИ В ГАЗ

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Розглянуто початкова задача розширення низькотемпературної плазми в необмежене газове середовище. Передбачається, що плазма створена локально джерелом іонізації, що діє досить короткий час. Йони мають температуру фонового газу, ступінь іонізації плазми невелика і макроскопічні течії газу відсутні. Електрони мають максвелівський розподіл. Розроблено математичну модель та її числовий розв’язок. Задача може бути корисна в дослідженнях післясвітіння і лазерного накачування коротким імпульсом.