ELECTROMAGNETIC WAVE PROPAGATION THROUGH MAGNETOACTIVE PLASMA LAYERS

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We studied transmission of a *p*-polarized electromagnetic wave through two- and four-layer plasma structure immersed in an external magnetic field perpendicular to the incidence plane. The structure was composed of alternating layers of high and low density plasma. The layers had equal width. The transmission and reflection coefficients were derived using transfer matrix method. In this study we calculated spatial distribution of the tangential energy flux of the wave in the case of reflectionless transmission.

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INTRODUCTION

Interaction of the electromagnetic radiation with overdense plasma has been extensively studied in various contexts for many years. For the waves with frequencies beyond ω_{pe} plasma acts as a mirror, but in layered structures it could behave differently. For instance, a symmetric three-layer structure containing layer of overdense plasma sandwiched between two layers of less dense plasma has anomalously high transparency for certain frequencies and angles of incidence [1]. Similar effect is also observed in asymmetric two layer structure [2, 3]. Layered structures with arbitrary number of layers were considered in [4].

Recently, a great deal of interest is attracted to the structures called photonic crystals. Photonic crystals are artificial structures, which have a periodic dielectric structure with high index contrast, designed to control photons in the same way that crystals control electrons [5, 6].

The studies of the phenomena in these structures open the ways for building various kinds of tunable filters and spatial and spectral multiplexers or overcoming the blackout in radio communication with spacecrafts reentering the atmosphere.

1. MAIN EQUATIONS AND ASSUMPTIONS

We consider a periodic multilayer plasma structure surrounded by vacuum (Fig. 1). The structure is immersed in an external magnetic field \vec{H} directed along z-axis. It is assumed that the density of the layers P1 is small ($0 < \varepsilon_{I0} < 1$, where ε_{10} is the dielectric permittivity of layer at the absence of magnetic field), while the layers P2 are dense with $\varepsilon_{20} < 0$ (here ε_{20} is the dielectric permittivity of the layer P2 at H = 0). Consider propagation of a p-polarized (with field components E_x, E_y, H_z) electromagnetic wave with the wave vector $\vec{k} = k_x \vec{e}_x + k_y \vec{e}_y$ through the structure. The wave propagating from the half-infinite vacuum region V1 is obliquely incident at the plasma layer Pl1. In the vacuum region V1, there are the incident $(k_x > 0)$ and reflected ($k_x < 0$) waves. The transmitted wave propagates into the half-infinite vacuum region V2. The plasma regions P1 and P2 have equal widths $a_1 = a_2 = a$.

From Maxwell's equations we obtain the expressions for components of electromagnetic field of the wave

$$E_x(x) = -\frac{1}{k_0(\varepsilon^2 - g^2)} \left(k_y \varepsilon H_z + g \frac{dH_z}{dx} \right), \quad (1)$$

$$E_{y}(x) = -\frac{i}{k_{0}(\varepsilon^{2} - g^{2})} \left(k_{y}gH_{z} + \varepsilon \frac{dH_{z}}{dx}\right), \qquad (2)$$

$$\frac{d^2 H_z}{dx^2} + \kappa^2 H_z = 0 , \qquad (3)$$

where $\kappa^2 = k_y^2 - (\varepsilon^2 - g^2)k_0^2/\varepsilon$, $k_0 = \omega/c$, c is the

speed of light, ε and g are the components of the dielectric tensor for cold magnetoactive plasma neglecting ion motion and particle collisions.

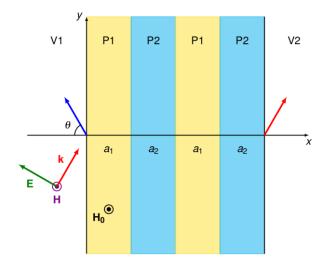


Fig. 1. Schematic representation of a p-polarized wave transmission through a symmetric layered plasma structure

Transmission of a p-polarized electromagnetic wave is studied using a transfer matrix method [7]. The layered plasma structure is characterized by a transfer matrix M, which relates wave amplitudes of the incident wave to the amplitudes of the transmitted wave. We derived transfer matrix for a two-layer plasma structure. The change in amplitude of the p-polarized wave after

propagation over distance a into the positive direction xis described by matrix equation

$$\begin{pmatrix} H_z(x+a) \\ E_y(x+a) \end{pmatrix} = M \begin{pmatrix} H_z(x) \\ E_y(x) \end{pmatrix}. \tag{4}$$

The wave tangential magnetic field component has the following form

$$H_z = [C_1 \exp(\kappa x) + C_2 \exp(-\kappa x)] \exp(ik_y y - i\omega t) . (5)$$

The solution does not change across the layer of magnetoactive plasma, and corresponding transfer ma-

$$M = \frac{1}{\xi} \begin{pmatrix} \xi \sinh(\kappa a) - \psi \cosh(\kappa a) & i \sinh(\kappa a) \\ i(\psi^2 - \xi^2) \sinh(\kappa a) & \xi \sinh(\kappa a) + \psi \cosh(\kappa a) \end{pmatrix},$$

where

$$\xi = \frac{\kappa \varepsilon}{k(\varepsilon^2 - g^2)}, \qquad \psi = \frac{k_y g}{k(\varepsilon^2 - g^2)}.$$

The transfer matrix of a multilayer structure composed of N layers is a product of transfer matrices M_n for each layer

$$M_S = M_N \cdot ... \cdot M_2 \cdot M_1$$
,

where M_1 – transfer matrix of the first layer facing the incident wave, M_2 – transfer matrix of the next adjacent

The electromagnetic field at the vacuum-plasma boundary at x=0 is a superposition of incident and reflected waves

$$H_{z}(0) = H_{0}(1+R),$$
 (6)

$$E_{v}(0) = k_{x} H_{0}(1-R),$$
 (7)

where H_0 – the amplitude of magnetic field of the wave, R – amplitude reflection coefficient, k_x is a wave number. The electromagnetic wave at the plasma-vacuum facing the transmitted wave at x=L is

$$H_z(L) = T \exp(ik_x L),$$
 (8)

$$E_{v}(L) = k_{x} T \exp(ik_{x} L), \qquad (9)$$

where T – amplitude transparency coefficient. The electromagnetic wave field at the structure interface with region V1 (6), (7) is related to electromagnetic field at the plasma boundary with region V2 (8), (9) by the equation (4), where $M=M_S$. The solution gives us transparency and reflection coefficients, correspondingly

$$T = 2k_x \exp(-ik_x x) \frac{M_{12}M_{21} - M_{11}M_{22}}{M_{12}k_x^2 + M_{21} - k_x(M_{11} + M_{22})} ,$$

$$R = \frac{M_{12}k_x^2 - M_{21} + k_x(M_{11} - M_{22})}{M_{12}k_x^2 + M_{21} - k_x(M_{11} + M_{22})},$$

where M_{ij} are components of the transfer matrix of the layered structure. The condition of the reflectionless transmission of the electromagnetic wave through the structure is determined by the equations

$$M_{11}$$
– M_{22} =0, (10)
 $M_{12}k_x^2$ – M_{21} =0. (11)

$$M_{12}k_{\rm r}^2 - M_{21} = 0.$$
 (11)

If wave frequency, angle of incidence and plasma parameters satisfy the conditions (10), (11), the absolute value of the transmission coefficient equals unity.

Meanwhile, the phase of the wave is shifted. For the two-layer structure the phase shift is determined by the following expression

$$\begin{split} & \operatorname{Arg}(T) = \\ & = -\frac{\xi_2}{\xi_1} \left(\frac{\psi_1 \sinh(\kappa_1 a_1) - \xi_1 \cosh(\kappa_1 a_1) - i k_x \sinh(\kappa_1 a_1)}{\psi_2 \sinh(\kappa_2 a_2) + \xi_2 \cosh(\kappa_2 a_2) - i k_x \sinh(\kappa_2 a_2)} \right). \end{split}$$

On Fig. 2 is shown the dependencies of absolute value and phase of the transparency coefficient on tangential component of the wave vector. The phase rapidly grows in the vicinity of the point corresponding to reflectionless transmission.

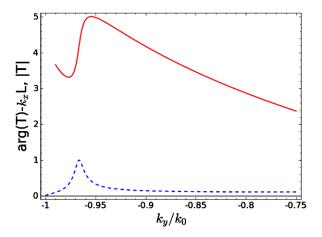


Fig. 2. The dependance of absolute value of transmission coefficient (dashed line) and phase of the transmitted wave (solid line) on wave number. The width of the layers is $a_1=a_2=\lambda/\pi$, the density ration is $n_2/n_1 = 2$, cyclotron frequency $\omega_c = 0.45\omega_{n2}$, wave frequency $\omega = 0.767 \omega_{n2}$

Using equation (4) one can find the value of tangential components of electromagnetic field of the wave at any boundary between layers of the structure. For instance, the tangential components of the electromagnetic wave at the boundary between first and second layers is found by multiplying the value of the wave field vector at vacuum-plasma interface by transfer matrix of the first layer. Due to continuity of the tangential components across the interfaces the tangential electromagnetic field vector at the next interface is obtained by multiplying field vector from the previous step by corresponding transfer matrix. In this way for each layer one gets the values of the field components at the right H_R and left H_L boundary. These values allow finding coefficients C_1 and C_2 in expression (5)

$$C_1 = \frac{H_R \exp(\kappa x_L) - H_L \exp(\kappa x_R)}{\exp(-\kappa a) - \exp(\kappa a)}, \quad (12)$$

$$C_2 = \frac{H_L \exp(-\kappa \alpha_R) - H_R \exp(-\kappa \alpha_R)}{\exp(-\kappa \alpha) - \exp(\kappa \alpha)},$$
 (13)

where x_R and x_L is the coordinates of the right and left boundaries, respectively, $a = x_R - x_L$ is the width of the layer. Equations (1)-(3) and coefficients (12), (13) give the spatial distribution of the electromagnetic field inside the layered structure.

2. THE ENERGY FLOW IN LAYERED PLASMA STRUCTURE

The energy flux carried by the electromagnetic wave is described by the Poynting vector

$$\mathbf{S} = \frac{c}{4\pi} \left[\mathbf{E} \mathbf{H} \right].$$

The Poynting vector has two components S_x and S_y . The time averaged normal to the interfaces component $\langle S_x \rangle$ is constant across the layers. We studied the spatial distribution of the tangential component of the energy flux

$$\langle S_y \rangle = \frac{c}{8\pi} \operatorname{Re} \left[E_x H_z^* \right].$$

On Figs. 3-6 we present the spatial distribution of the time averaged tangential energy flow normalized by the value $S_0=c|H_z(0)|^2/8\pi$, the coordinate x is measured in units of k^{-1} starting from the vacuum-plasma interface V1-P1.

The conditions for total transparency of the structure (10), (11) support two types of solutions. The first one called here evanescent resonance corresponds to the wave evanescent (κ^2 <0) in all the plasma regions. The second one corresponds to the wave propagating in the regions with positive dielectric permittivity (P1) and evanescent in regions with negative permittivity (P2) and thus called semi-evanescent resonance. The number of evanescent and semi-evanescent resonances is equal and increases with number of layers in the structure.

The flow of the energy of the electromagnetic wave has larger values in the layers with positive dielectric permittivity (P1).

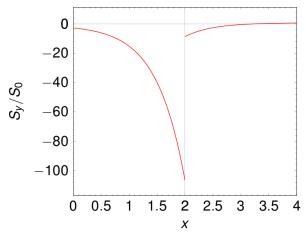


Fig. 3. The spatial distribution of the tangential energy flow in the two-layer structure in case of evanescent resonance. The wave number is $k_y/k_0=-0.967$; wave frequency is $\omega=0.767\omega_{p2}$

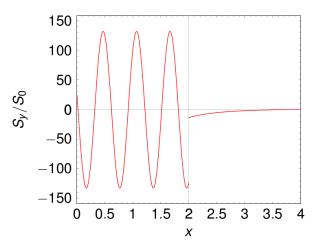


Fig. 4. The spatial distribution of the tangential energy flow in the two-layer structure in case of semievanescent resonance. The wave number is $k_v/k_0 = -0.457$; wave frequency is $\omega = 0.669\omega_{n^2}$

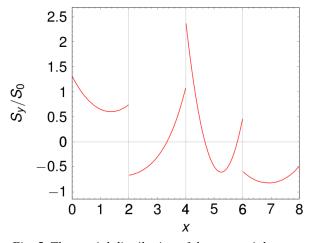


Fig. 5. The spatial distribution of the tangential energy flow in the four-layer structure in case of evanescent resonance. The wave number is $k_y/k_0=0.588$; wave frequency is $\omega=0.809\omega_{p2}$

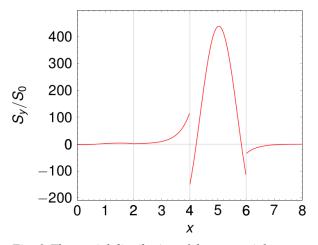


Fig. 6. The spatial distribution of the tangential energy flow in the four-layer structure in case of evanescent resonance. The wave number is $k_y/k_0=0.817$; wave frequency is $\omega=0.61\omega_{p2}$

CONCLUSIONS

Periodic multilayer plasma structure under consideration is resonantly transparent for p-polarized electromagnetic wave. It supports two types of resonances: evanescent (wave field is evanescent in all the layers) and semi-evanescent. If the structure composed of larger number of layers it has more resonances. Hence, transmission characteristics of the periodic structure can be tuned by varying number of layers.

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РАСПРОСТРАНЕНИЕ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ ЧЕРЕЗ СЛОИ МАГНИТОАКТИВНОЙ ПЛАЗМЫ

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Изучалось прохождение *р*-поляризованной электромагнитной волны через двух- и четырехслойную плазменную структуру во внешнем магнитном поле, перпендикулярном плоскости падения. Структура состояла из перемежающихся слоёв высокой и низкой плотности. Слои были равной толщины. Коэффициенты прохождения и отражения были получены при помощи метода матрицы распространения. Исследовалось пространственное распределение тангенциальной составляющей потока энергии волны в случае безотражательного прохождения.

ПОШИРЕННЯ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ КРІЗЬ ШАРИ МАГНІТОАКТИВНОЇ ПЛАЗМИ

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Вивчалось проходження *p*-поляризованої електромагнітної хвилі крізь дво- та чотиришарову плазмову структуру в зовнішньому магнітному полі, перпендикулярному до площини падіння. Структура складалася з шарів високої та низької густини, що перемежовувалися один з одним. Шари мали однакову товщину. Коефіцієнти проходження та відбиття було отримано методом матриці поширення. Досліджувався просторовий розподіл тангенціальної компоненти потоку енергії хвилі у випадку безвідбивного проходження.