STOCHASTIC DIFFERENTIAL EQUATIONS OF CHARGED PARTICLE MOTION IN TOROIDAL PLASMAS

A.A. Gurin, V.O. Yavorskij

Institute for Nuclear Research, Kyiv, Ukraine

Stochastic equations of charged particle motion in toroidal plasma are derived using the Ito theory of stochastic processes. Expressions for stochastic differentials of the full set of drift variables associated with the kinetic theory of charged particles in plasma with Coulomb collisions are obtained. Equations obtained may be used for the modelling of fast charged particle motion in toroidal plasmas, namely for Monte-Carlo simulation the dynamics of charged fusion products and beam ions in tokamaks.

PACS: 52.55.Hc, 52.55.Pi, 55.25.Fi, 52.35.Bj, 05.45.-a

INTRODUCTION

Description of the charged particle behaviour in toroidal plasmas usually is based on the kinetic equations with the Fokker-Plank collisional term accounting for the effect of Coulomb collisions. Alternatively the detailed microscopic depiction of single particle motion may be achieved on the base of the Markov theory in terms of the Ito approach of stochastic differential equations [1, 2].

This paper aims the derivation of stochastic equations of single particle motion in plasma using the Ito theory of stochastic processes. We obtain the expressions for the stochastic differentials of the full set of drift variables in exact correspondence with the kinetic theory of charged particles in plasma with Coulomb collisions [3]. Equations derived can be used for the Monte-Carlo simulation of the dynamics of charged fusion products and beam ions in tokamaks. Notice that such kind of modelling is usually based on the use of supporting Monte-Carlo models [4].

1. BASIC FORMULAS OF STOCHASTIC ANALYSIS

Ito stochastic differential equations [1, 2] of multi-dimensional diffusive process $X(t) = (X_1(t),..., X_n(t))$,

$$dX_i = a_i\left(t, X\right)dt + \sum_{j=1}^{m} b_{ij}\left(t, X\right)b_{ij}\left(t, X\right)dW_j(t)\,, \quad (1)$$

as well as the Kolmogorov equation for the transition probability of Markov process $P_{t_0 X_0}(t, X)$ from the state X_0 in an arbitrary time t_0 into the state X at a time $t>t_0$

$$\partial_t P_{t_0X_0}\left(t, X\right) = -\sum_{i=1}^{n} \partial_{x_i}\left[a_i\left(t, X\right)P_{t_0X_0}\left(t, X\right)\right] + \frac{1}{2}\sum_{i,j=1}^{n}\partial_{x_i}\partial_{x_j}\left[D_{ij}\left(t, X\right)P_{t_0X_0}\left(t, X\right)\right] \quad (2)$$

represent the alternative approaches of the complete description of the process. According to Ito approach the coefficients *a*, *b* in expression for stochastic differential are determined by the left edge of time interval $(t, t+dt)$ and supposing the values $X(t)$ to be known the stochasticity of differentials dX is delivered by the independent increments of the components $dW_j = W_j(t+dt) - W_j(t)$ of Wiener process. Namely the above structure of Ito stochastic differential results in the Markoviety of process $X(t)$ (alternative definitions of stochastic differential, e.g. Stratonovich meaning, are not considered here). The components of Wiener process are independent and represent the elementary Markovian Gaussian processes with the independent growths and transition probabilities as follows

$$P_{t,X_j}(t+\tau, X_j') = \exp\left[-\left(X_j - X_j'\right)^2 \big/ \left(2\tau\right)\right] \Big/ \sqrt{2\pi\tau}\,, \quad (3)$$

where τ is an arbitrary value. It follows from Eq. (3) that the random part of stochastic differential (1) is the dominant as the Wiener differential is of the order of $\sqrt{dt}$. Respectively the random part may be represented as $dW_j = \sqrt{dt}\gamma_j$ with γ_j – the Gaussian random numbers with a dispersion that equals 1. This circumstance is crucial for the process calculation. Nevertheless the quadratic terms $dW_j dW_k$ are of the order of dt. From Eq. (3) it follows that for infinitely small $\tau = dt$ the square of infinitely small growth $dW_j=W_j(t+dt)-W_j(t)$ is determined by the process dispersion with a probability one and can be considered as a non-random value, i.e. $dW_j^2 = dt$, with

$$dW_j dW_k = \delta_{jk} dt \quad (4)$$

for multidimensional processes. From above equation it follows also the subsequent equality

$$dx_i dx_j = \sum_{k=1}^{m} b_{ik} b_{jk} dt\,. \quad (5)$$

These relations determine the rule of correspondence of the diffusion coefficients $D_{ij} = \sum_{k=1}^{m} b_{ik} b_{jk}$ in stochastic equations (1) and in Kolmogorov equation (2). They represent the basis of Ito formula for the stochastic differential of arbitrary function $F(X)$ –

$$dF(X) = \sum_{i=1}^{n} dx_i \partial_{x_i} F + \frac{dt}{2}\sum_{i,j=1}^{n} D_{ij}\partial^2_{x_i x_j} F \quad (6)$$

as well. Eqs. (1-5) represent the basis of Ito analysis that essentially extends the classical mathematical analysis.

The number of Wiener components m in stochastic differential equation (1) can be less than the number n of the components of process X and what is more – not every of Eqs. (1) may contain a fluctuating part. Namely such a situation is realised in plasma theory formulated in terms of the kinetic equations for the distribution functions $f_a(\mathbf{r,v})$ of plasma components a in the phase space of spatial coordinates and velocities. The most complete form of these equations is as follows [4]

$$\begin{aligned}
&\frac{\partial}{\partial t} f_a + \mathbf{v}\cdot\nabla f_a + \mathbf{a}\cdot\nabla_{\mathrm{v}} f_a = \sum_b C^{a/b},\\
&\mathbf{a} = \frac{e_a}{m_a}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right),\ C^{a/b} = \nabla_v \cdot \left(-\mathbf{A} f_a + \frac{1}{2}\nabla_v \cdot \overset{\leftrightarrow}{\mathbf{D}} f_a\right),\\
&\mathbf{A} = L^{a/b}\left(1 + \frac{m_a}{m_b}\right)\nabla_{\mathrm{v}}\psi(\mathbf{r},\mathbf{v}), \qquad (7)\\
&\overset{\leftrightarrow}{\mathbf{D}}(t, X) = L^{a/b}\nabla_{\mathbf{v}}\nabla_{\mathbf{v}}\varphi, \quad L^{a/b} = 4\pi\Lambda\frac{e_a^2 e_b^2}{m_a^2},\\
&\psi = \int \frac{d\mathbf{v}_b}{|\mathbf{v}-\mathbf{v}_b|} f_b(t,\mathbf{r},\mathbf{v}_b), \varphi = \int |\mathbf{v}-\mathbf{v}_b| f_b(t,\mathbf{r},\mathbf{v}_b)\, d\mathbf{v}_b
\end{aligned}$$

where $C^{a/b}$ is the collisional term, Λ – the Coulomb logarithm. Kinetic equations (6) can be treated as the direct Kolmogorov equations for the unconditional simultaneous single-particle probability renormalized in accordance with the definition of distribution function f_a as the density of particles a in the phase space X=($\mathbf{r}$,$\mathbf{v}$). Though the kinetic theory does not use the approach of Markov processes the unambiguous correspondence of kinetic and Markov approaches is established by the equality $f(t, X) = \int (dX_0)^n f(t_0, X_0) P_{t_0, X_0}(t, X)$ that serves as the basis for the Monte-Carlo modelling of a macro-canonical ensemble. Therefore the stochastic differential equations of particle motion corresponding to the kinetic theory have a form:

$$d\mathbf{r} = \mathbf{v}dt,\ \ d\mathbf{v} = (\mathbf{a}+\mathbf{A})dt + \overset{\leftrightarrow}{\mathbf{b}}\cdot d\mathbf{W}, \quad \overset{\leftrightarrow}{\mathbf{D}} = \overset{\leftrightarrow}{\mathbf{b}}\cdot\overset{\leftrightarrow}{\mathbf{b}}. \qquad (8)$$

Here $\mathbf{W}$ is a 3D Wiener process.

2. VELOCITY DIFFUSION IN GYRO-TROPIC PLASMA

Stochastic equations (8) are rather compact and striking for the numerical modelling. Though, even if the problems of the numerical modelling of random changes are solved, in general case of 3D velocity diffusion there is a complex problem of the expansion of the matrix of diffusion coefficients $\overset{\leftrightarrow}{\mathbf{D}}$ of kinetic equation as a product of two matrixes of diffusion coefficients $\overset{\leftrightarrow}{\mathbf{b}}$ in equation of stochastic motion. In practice this problem is avoided in modelling of charged particle motion in strong magnetic field. In this case the fast gyration can be excluded from the analysis using the drift theory of motion for longitudinal u=$\mathbf{v}\cdot\mathbf{h}$ and transverse $w = |\mathbf{v}-u\mathbf{h}|$ velocity components with $\mathbf{h} = \mathbf{B}/B$ [5, 6]. Drift theory is developed for determinable motion of charged particle and is based on the averaging over the fast gyration. Similar approach is applicable for stochastic equation of motion as well. The basic formulae of drift theory are written in local orthogonal coordinate system ($\mathbf{h}$, $\mathbf{e}_1$, $\mathbf{e}_2$) associated with the inhomogeneous magnetic field:

$$\begin{aligned}
&\mathbf{v} = u\mathbf{h} + w\mathbf{e}_w, \quad \mathbf{e}_w = \mathbf{e}_1\cos\zeta + \mathbf{e}_2\sin\zeta,\\
&\nabla_{\mathbf{v}} = \mathbf{h}\partial_u + \mathbf{e}_w\partial_w + \mathbf{e}_\rho w^{-1}\partial_\zeta \equiv\\
&\mathbf{h}\partial_u + \mathbf{e}_w w^{-1}\partial_w w + w^{-1}\partial_\zeta \mathbf{e}_\rho,\\
&\text{because } \mathbf{e}_\rho = \mathbf{h}\times\mathbf{e}_w = \partial_\zeta \mathbf{e}_w,\ \ \mathbf{e}_w = -\partial_\zeta \mathbf{e}_\rho. \qquad (9)\\
&\mathbf{D} = L^{a/b}\left[\bar{\nabla}_{\mathbf{v}}\bar{\nabla}_{\mathbf{v}} + \mathbf{e}_\rho\mathbf{e}_\rho w^{-1}\partial_w\right]\varphi(u,w) =\\
&\begin{pmatrix} D_{uu} & D_{wu} & 0\\ D_{uw} & D_{ww} & 0\\ 0 & 0 & D_{\rho\rho}\end{pmatrix}, \quad \bar{\nabla}_{\mathbf{v}} = \mathbf{h}\partial_u + \mathbf{e}_w\partial_w,\\
&\mathbf{A} = L^{a/b}\left(1+\frac{m_a}{m_b}\right)\bar{\nabla}_{\mathbf{v}}\psi(u,w).
\end{aligned}$$

Thus if the potential φ of kinetic theory is independent on gyro phase ζ the problem of the expansion of diffusive matrix is simplified as the 2x2 block of the matrix can be represented as two matrixes of same dimensionality (corresponding solution will be provided separately). It should be pointed out that 3D Wiener process determined in Eq. (7) in arbitrary (not related with the magnetic field) coordinate system is characterised by the components (W_h, W_u, W_ρ), which are the standard independent Wiener processes. In fact, using relationships (4), it can be shown that

$$dW_h dW_w = (\mathbf{h}\cdot d\mathbf{W})(\mathbf{e}_w\cdot d\mathbf{W}) = (\mathbf{h}\cdot\mathbf{e}_w)dt = 0\,.$$

Derivation of the stochastic equations for u, w is based on the Ito formula (5) as the differential dr does not contain the Wiener fluctuations, variable u=$\mathbf{h}\cdot\mathbf{v}$ is linear with respect of $\mathbf{v}$ and differentiation of u is equivalent to a standard differentiation:

$$\begin{aligned}
&du = d(\mathbf{h}\cdot\mathbf{v}) = \left[\mathbf{v}\cdot(\mathbf{v}\cdot\nabla)\mathbf{h} + (\mathbf{a}+\mathbf{A})\cdot\mathbf{h}\right]dt +\\
&b_{hh}dW_h + b_{hw}dW_w.
\end{aligned}$$

Variable w is linear regarding $\mathbf{v}$ with

$$\nabla_{\mathbf{v}} w = \mathbf{e}_w, \qquad \nabla_{\mathbf{v}}\nabla_{\mathbf{v}} w = \frac{1}{w}\mathbf{e}_\rho\,\mathbf{e}_\rho.$$

Correspondingly the equation for dw has a following form:

$$\begin{aligned}
&dw = d(\mathbf{e}_w\cdot\mathbf{v}) =\\
&\left[\mathbf{v}\cdot(\mathbf{v}\cdot\nabla)\mathbf{e}_w + (\mathbf{a}+\mathbf{A})\cdot\mathbf{e}_w + \frac{1}{2w}D_{\rho\rho}\right]dt +\\
&\quad + b_{wh}dW_h + b_{ww}dW_w.
\end{aligned}$$

Finally last two stochastic equations should be averaged over the gyro phase. Correct averaging procedure supposes the usage of stochastic integrals for growths

$$\delta u = \int_t^{t+\delta} du,\ \delta w = \int_t^{t+\delta} dw,$$

where time interval δt includes a lot of gyro periods, however, is small over the time scales of the characteristic variation of values u,w. This procedure leads to the well-known (in drift theory) regular terms proportional to δt, while the stochastic contributions are introduced using the independent Wiener components δW_h, δW_w. In drift stochastic differential equations the values δt, δW_h, δW_w are supposed to be infinitely small. Finally we get:

$$du = \left(\frac{e}{m}\mathbf{h}\cdot\mathbf{E} + \frac{w^2}{2}\,\mathrm{div}\mathbf{h} + A_h \right)dt + b_{hh}dW_h + b_{hw}dW_w,$$
$$dw = \left(-\frac{uw}{2}\,\mathrm{div}\mathbf{h} + A_w + \frac{1}{2w}D_{\rho\rho} \right)dt + b_{wh}dW_h + b_{ww}dW_w.$$

Using Ito formulae

$$d\left(u^2\right) = 2udu + D_{uu}dt,\; d\left(w^2\right) = 2wdw + D_{ww}dt,$$

we get equations for well-known constants of motion

$$\mu = \frac{B_0 w^2}{2B}, \quad \varepsilon = \frac{1}{2}\left(\mathrm{u}^2 + \mathrm{w}^2\right)$$

as follows:

$$d\mu = \frac{B_0}{B}\left\{ \left[wA_w + \frac{1}{2}\left(D_{ww} + D_{\rho\rho}\right) \right]dt + wdW \right\},$$
$$d\varepsilon = \left[\frac{e}{m}u\mathbf{h}\cdot\mathbf{E} + uA_u + wA_w + \frac{1}{2}\left(D_{uu} + D_{ww} + D_{\rho\rho}\right) \right]dt +$$
$$\left(u+w\right)dW, \quad dW = b_{wu}dW_u + b_{ww}dW_w.$$

Knowledge of the stochastic equation of motion allows deriving a corresponding direct Kolmogorov equation or Fokker-Planck equation. Below is the relevant derivation.

3. VELOCITY DIFFUSION IN ISOTROPIC PLASMA

In case of isotropic plasma, when the distribution function of charged particles f_b in collisional term $C^{a/b}$ depends on the absolute value of the velocity $v=(u^2+w^2)^{1/2}$, the potentials φ, ψ depend only on v and diffusion coefficients are as follows:

$$\ddot{\mathbf{D}} = L^{a/b}N_b\left(\frac{1}{v}\varphi'\ddot{\mathbf{U}}_\perp + \varphi''\ddot{\mathbf{U}}_\parallel \right),\; \ddot{\mathbf{U}}_\perp = \ddot{\mathbf{1}} - \frac{\mathbf{vv}}{v^2},\; \ddot{\mathbf{U}}_\parallel = \frac{\mathbf{vv}}{v^2}\;. \quad (9')$$

Important is the orthogonality of $\ddot{\mathbf{U}}_\perp$ and $\ddot{\mathbf{U}}_\parallel$, $\ddot{\mathbf{U}}_\perp\cdot\ddot{\mathbf{U}}_\parallel = 0$ and following relationships $\ddot{\mathbf{U}}_\perp\cdot\ddot{\mathbf{U}}_\perp = \ddot{\mathbf{U}}_\perp$, $\ddot{\mathbf{U}}_\parallel\cdot\ddot{\mathbf{U}}_\parallel = \ddot{\mathbf{U}}_\parallel$, $\ddot{\mathbf{U}}_\parallel\cdot\mathbf{e}_\rho = 0$. Consequently the problem of the expansion of the matrix of diffusion coefficients is easily resolved. The stochastic equations of the velocity diffusion are of the following form (drifts are not included):

$$d\mathbf{v} = (\mathbf{a}+\mathbf{A})dt + \left(\sqrt{D_\perp}\ddot{\mathbf{U}}_\perp + \sqrt{D_\parallel}\ddot{\mathbf{U}}_\parallel \right)\cdot d\mathbf{W}_t.$$
$$\mathbf{A} = L^{a/b}N_b\left(1 + m_a/m_b\right)v^{-1}\mathbf{v}\psi' dt, \qquad (10)$$
$$D_\perp = L^{a/b}N_b\; v^{-1}\varphi',\; D_\parallel = L^{a/b}N_b\varphi''.$$

In variable u, w the equations can be rewritten as

$$du = (a_u + A_u)dt + b_{uu}dW_u + b_{uw}dW_w,$$
$$dw = \left(a_w + A_w + \frac{1}{2w}D_{\rho\rho} \right)dt + b_{wu}dW_u + b_{ww}dW_w,$$
$$\begin{pmatrix} b_{uu} & b_{uw} \\ b_{wu} & b_{ww} \end{pmatrix} = \qquad (10')$$
$$v^{-2}\begin{pmatrix} w^2\sqrt{D_\perp} + u^2\sqrt{D_\parallel} & uw\left(\sqrt{D_\parallel} - \sqrt{D_\perp}\right) \\ uw\left(\sqrt{D_\parallel} - \sqrt{D_\perp}\right) & u^2\sqrt{D_\perp} + w^2\sqrt{D_\parallel} \end{pmatrix}.$$

However, usage of the asymmetric matrix of diffusion coefficients allows the more compact matrix expansion. With new independent Wiener differentials dU_t, dW_t the basic equations of stochastic dynamics can be essentially simplified

$$du = (a_u + A_u)dt + \left(w\sqrt{D_\perp}dU_t + u\sqrt{D_\parallel}dW_t \right)v^{-1}$$
$$dw = \left(a_w + A_w + D_{\rho\rho}/(2w) \right)dt - \left(u\sqrt{D_\perp}dU_t - w\sqrt{D_\parallel}dW_t \right)v^{-1}$$

Ito stochastic differential equations for energy $\varepsilon = (u^2+w^2)/2$, pitch-parameter $\lambda = \mu_0/\varepsilon$; and transverse energy $\mu_0 = w^2/2$

$$d\mu_0 = \left[wa_w + \frac{1}{2}\left(D_{ww} + D_{\rho\rho}\right) \right]dt +$$
$$+ w\left(b^*_{wu}dU_t + b^*_{ww}dV_t \right),$$
$$d\varepsilon = \left(uA_u + wA_w + \mathrm{Sp}\{\mathbf{D}\}/2 \right)dt + v\sqrt{D_\parallel}dV_t,$$
$$d\lambda = \frac{\mu_0}{\varepsilon}\left(1 + \frac{d\mu_0}{\mu_0} \right)\left[1 - \frac{d\varepsilon}{\varepsilon} + \left(\frac{d\varepsilon}{\varepsilon}\right)^2 - \ldots \right] =$$
$$\approx \frac{1}{\varepsilon^2}\left(\varepsilon d\mu_0 - \mu_0 d\varepsilon\right)\left(1 - \frac{d\varepsilon}{\varepsilon} \right), \qquad (11)$$
$$\varepsilon d\mu_0 - \mu_0 d\varepsilon = \frac{1}{2}\left(u^2 d\mu_0 - \mu_0 du^2 \right) =$$
$$\frac{1}{4}\left[u^2\left(D_{ww} + D_{\rho\rho}\right) - w^2 D_{uu} \right]dt +$$
$$\frac{u^2 w}{2}\left(b^*_{wu}dU_t + b^*_{ww}dW_t \right) - \frac{uw^2}{2}\left(b^*_{uu}dU_t + b^*_{uw}dW_t \right).$$

In Eq. (12) terms with dW_t annihilate and term in Ito differential $d\lambda$ is reduced to $dU_t\, dW_t$. Finally

$$d\lambda = v^{-4}\left[u^2\left(D_{ww} + D_{\rho\rho}\right) - w^2 D_{uu} \right]dt +$$
$$+ 2v^{-4}\left(u^2 w b^*_{wu} - uw^2 b^*_{uu} \right)dU_t = \qquad (12)$$
$$= v^{-4}\left[\left(2u^2 - w^2\right)D_\perp dt - 2uvw\sqrt{D_\perp}dU_t \right].$$

4. SPATIAL DIFFUSION

In guiding centre coordinates $\mathbf{R} = \mathbf{r} - \boldsymbol{\rho}$, $\boldsymbol{\rho} = \mathbf{g}\times\mathbf{v}$, $\mathbf{g} = \mathbf{h}/\Omega$ the Ito differential of $\mathbf{R}$ looks like

$$d\mathbf{R} = d\mathbf{r} - (d\mathbf{r}\cdot\nabla)\mathbf{g}\times\mathbf{v} - \mathbf{g}\times d\mathbf{v}. \qquad (13)$$

For fluctuating Wiener part of d$\mathbf{R}$ we get

$$\tilde{d}\mathbf{R} = -\frac{1}{\Omega}[\mathbf{e}_\rho (b^*_{wu} dW^*_u + b^*_{ww} dW^*_w) - \mathbf{e}_w b_{\rho\rho} dV^*_\rho]. \quad (14)$$

Considering stochastic Ito integral within the interval $(t, t+\Delta t)$, $\Omega\Delta t \gg 1$ we account for the Gaussian nature of stochastic integrals which can be approximated by Wiener processes with dispersion and correlation determined by the integrals

$$\Delta\mathbf{R}\Delta\mathbf{R} = \int_t^{t+\Delta t}\int_t^{t+\Delta t} d\mathbf{R}(t_1) d\mathbf{R}(t_2). \quad (15)$$

Gyro averaging of above expression results in

$$\Delta\mathbf{R}\Delta\mathbf{R} = \frac{\Delta t}{2\Omega^2}(\vec{\mathbf{1}} - \mathbf{hh})(b^{*2}_{wu} + b^{*2}_{ww} + b^{*2}_{\rho\rho}). \quad (16)$$

In axisymmetric tokamak-like configuration

$$\tilde{d}\mathbf{R} = \sqrt{D_s}(\mathbf{n} dW_n + \mathbf{n}^* dW^*_n),$$

$$D_s = \frac{1}{2\Omega^2}(b^{*2}_{wu} + b^{*2}_{ww} + b^{*2}_{\rho\rho}) = \frac{1}{2\Omega^2}(D_{ww} + D_{\rho\rho}), \quad (17)$$

where D_s the 2D spatial diffusion. Finally we get

$$d\mathbf{R} = \left[u\mathbf{h}(\mathbf{R}) + \mathbf{v}_d(u, w, \mathbf{R})\right]dt +$$

$$\sqrt{D_\mathbf{R}}\left(\mathbf{n} dW_n + \mathbf{n}^* dW^*_n\right),$$

$$D_\mathbf{R} = \frac{L^{a/b} N_b(\mathbf{R})}{2\Omega^2(\mathbf{R}) v^2}\left(\frac{u^2}{v}\varphi'_v + w^2\varphi''_{vv} + v^2\varphi''_{vv}\right). \quad (18)$$

CONCLUSIONS

Correct stochastic equations of charged particle motion which correspond to the drift kinetic approach are derived in terms of Ito theory of stochastic processes. They represent the set of four equations containing the four independent Wiener components.

Obtained equations are consistent with the theory of Coulomb collisions and are not more complex as compared to those used in the conventional approaches [4]. They can be used for the Monte-Carlo simulation of the dynamics of charged fusion products and beam ions in tokamaks.

ACKNOWLEDGEMENTS

This work was supported in part by the Project #0112U002797 of the National Academy of Sciences of Ukraine.

REFERENCES

1. K. Ito. Stochastic integral // *Proc. Imp. Acad*. Tokyo, 1944, v. 20.
2. I.I. Gikhman, A.V. Skorokhod. *Stochastic differential equations*. Kyiv: „Naukova Dumka“, 1968 (in Russian).
3. B.A. Trubnikov. Particle Interactions in a Fully Ionized Plasma // *Reviews of Plasma Physics*. Bureau, New York, 1965, v. 1, p. 105.
4. A.H. Boozer. Monte Carlo collision operators for use with exact trajectory integrators // *Physics of Plasmas*. 2002, v. 9, p. 4389.
5. R.D. Hazeltine. Recursive derivation of drift-kinetic equation // *Plasma Phys*. 1973, v. 15, p. 77-80.
6. N.N. Bogolyubov, Yu.A. Mitropolsky. *Asymptotic methods in the theory of nonlinear oscillations*. Moscow: “State Publishing House technical and theoretical literature”, 1955.

Article received 29.09.2016

СТОХАСТИЧЕСКИЕ УРАВНЕНИЯ ДВИЖЕНИЯ ЗАРЯЖЕННЫХ ЧАСТИЦ В ТОРОИДАЛЬНОЙ ПЛАЗМЕ

А.А. Гурин, В.А. Яворский

В терминах теории стохастических процессов Ито получены выражения для стохастических дифференциалов полного набора дрейфовых переменных, соответствующих кинетической теории заряженных частиц в плазме с кулоновскими столкновениями. Полученные стохастические уравнения движения являются последовательными с точки зрения учёта эффектов кулоновских столкновений и не являются более сложными по сравнению с теми, что обычно используются в традиционных модельных подходах.

СТОХАСТИЧНІ РІВНЯННЯ РУХУ ЗАРЯДЖЕНИХ ЧАСТИНОК У ТОРОЇДАЛЬНІЙ ПЛАЗМІ

А.А. Гурин, В.О. Яворський

В термінах теорії стохастичних процесів Іто отримано вирази для стохастичних диференціалів повного набору дрейфових змінних, що відповідають кінетичній теорії заряджених частинок у плазмі з кулонівськими зіткненнями. Отримані стохастичні рівняння руху є послідовними щодо врахування ефектів кулонівських зіткнень, та не є складнішими у порівнянні із тими, що використовуються зазвичай в поширених модельних підходах.