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On the response of a system with bound states of particles to the perturbation by the external electromagnetic field

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The response of the system, consisting of two types of opposite-charged fermions and their bound states (hydrogen-like atoms), to the perturbation by the external electromagnetic field in low particle kinetic energies region is studied. Investigations are based on using a new formulation of the second quantization method that includes a capability of forming the particle bound states [1]. Expressions for Green functions that describe the system response to the external electromagnetic field and take into account the presence of particle bound states (atoms) are found. Macroscopic parameters of the system, such as conductivity, permittivity and magnetic permeability in terms of these Green functions are found. As an example, the perturbation of the ideal hydrogen-like plasma by the external electromagnetic field in low temperature region is considered. Expressions for the values are found that describe the ideal gas of hydrogen-like atoms Bose-condensate response to the external electromagnetic field.

Key words: *Green functions, bound states, response of systems, low-temperature hydrogen-like plasma, conductivity, magnetic permeability*

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1. Introduction

In the process of describing a behavior of many-particle systems a class of problems appear, that are concerned with the system response to the perturbing action of the external, in particular, electromagnetic field. Widespread approach to solving such kind of problems is based on using the Green functions formalism (see in that case e.g. [2]).

As well known, the most convenient method of describing physical processes in quantum many-particle theory is the second quantization method. Thus, within the framework of the second quantization it is the simplest to formulate an approach to a description of the system response to the perturbation by the external field, that is based on Green functions. However, if we try to realize such an approach, we can come across an essential difficulty, connected with the possible occurrence of the particle bound states.

Really, the key role of the second quantization method consists in the introduction of creation and annihilation operators of particles in a certain quantum state. The operators of physical quantities are constructed in terms of creation and annihilation operators. Such a description of quantum many-particle systems implies the particles to be elementary (not consisting of other particles). Moreover, it is absolutely accurate despite the possible existence of compound particles. Since the interactions between particles may lead to the formation of bound states, the standard second quantization method becomes too cumbersome. For this reason the construction of an approximate quantum-mechanical theory for many-particle systems consisting of elementary particles and their bound states represents a real problem. In this theory it is necessary to introduce the creation and annihilation operators of bound states as the operators of elementary objects (not compound). Moreover, it should preserve the required information concerning internal degrees of freedom for the bound states.

Such an approach has been realized in [1]. In this work the possibility of constructing such a theory is demonstrated for a system, that consists of two types of fermions, assuming that bound states (atoms or molecules) are formed by particles of two different types. The choice of such a model is not dictated by the principal difficulties but rather by the desire to simplify the calculations and obtain the visual results. Within the framework of this model a method of constructing the creation and annihilation operators of the bound state as a compound object is given. The substantiation of the conversion from the description of atoms as compound objects to elementary objects with the ordinary creation and annihilation Bose-operators is given. Such substantiation is considered in low-energy approximation in which the binding energy of a compound particle is much greater than its kinetic energy. In terms of the creation and annihilation operators of fermions and bosons (as elementary objects) a scheme for constructing the operators of physical quantities is formulated. Explicit expressions for the operators of principal physical quantities, such as density and charge density, momentum and current density, system Hamiltonian, are found. The Maxwell-Lorentz system of equations is found, describing the interaction between electromagnetic field and matter, that may also consist of neutral "atoms" (low-energy quantum electrodynamic equations).

In the present work we use this system of equations to study the response of the system with bound states to the perturbation by the external electromagnetic field within the framework of Green functions formalism. An essentially new issue in these considerations is the next circumstance. When we describe the system response to the perturbing action of the external electromagnetic field the approximate formulation of the second quantization method proposed in [1] makes it possible to take the neutral bound states into account in sufficiently simple way.

2. Quantum electrodynamic equations for the low-temperature hydrogen-like plasma

The quantum-electrodynamic system studied, consisting of fermions of two different types and their bound states, in low-energy region, in fact, can be considered as a low-temperature hydrogen-like plasma. Before we turn to the description of such plasma response to the external electromagnetic field, let us obtain the main equations that describe an evolution of this system. Taking into account the interaction between radiation and matter the system's Hamiltonian $\hat{\mathcal{H}}(t)$, according to [1], can be written as

$$\hat{\mathcal{H}}(t) = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{int}} + \hat{V}(t), \qquad \hat{\mathcal{H}}_0 = \hat{\mathcal{H}}_f + \hat{\mathcal{H}}_p\,, \tag{1}$$

where

$$\hat{\mathcal{H}}_f = \sum_{\mathbf{k},\lambda} \omega_k \hat{C}^{+}_{\mathbf{k}\lambda} \hat{C}_{\mathbf{k}\lambda} \tag{2}$$

is the Hamiltonian for free photons (ω_k is the frequency of photon with wave number k, $\hat{C}^{+}_{\mathbf{k}\lambda}$, $\hat{C}_{\mathbf{k}\lambda}$ are the creation and annihilation operators of photon with wave number k and polarization λ).

The value $\hat{\mathcal{H}}_p$ in the equation (1) is the Hamiltonian for free particles (free fermions and their bound states)

$$\hat{\mathcal{H}}_p = \sum\nolimits_{j=1}^{2} \tfrac{1}{2m_j} \int \mathrm{d}\mathbf{x} \tfrac{\partial \hat{\chi}^{+}_{j}(\mathbf{x})}{\partial \mathbf{x}} \tfrac{\partial \hat{\chi}_{j}(\mathbf{x})}{\partial \mathbf{x}} + \sum\nolimits_{\alpha} \int \mathrm{d}\mathbf{X} \left\{ \tfrac{1}{2M} \tfrac{\partial \hat{\eta}^{+}_{\alpha}(\mathbf{X})}{\partial \mathbf{X}} \tfrac{\partial \hat{\eta}_{\alpha}(\mathbf{X})}{\partial \mathbf{X}} + \varepsilon_\alpha \hat{\eta}^{+}_{\alpha}(\mathbf{X}) \hat{\eta}_{\alpha}(\mathbf{X}) \right\}, \tag{3}$$

$$M = m_1 + m_2,$$

where $\hat{\chi}^{+}_{j}(\mathbf{x})$, $\hat{\chi}_{j}(\mathbf{x})$ $(j = 1, 2)$ are the creation and annihilation operators of a free fermion of j type and mass m_j at the point $\mathbf{x}$; $\hat{\eta}^{+}_{\alpha}(\mathbf{X})$, $\hat{\eta}_{\alpha}(\mathbf{X})$ are the creation and annihilation operators of bound states of two different fermions ("hydrogen-like atoms") with the quantum numbers α at the point $\mathbf{X}$; ε_α is the energy of an atom at the level with the quantum numbers α.

The Hamiltonian $\hat{\mathcal{H}}_{\text{int}}$ in the equation (1) describes the interaction between particles

$$\hat{\mathcal{H}}_{\text{int}} = \hat{\mathcal{H}}^{1}_{\text{int}} + \hat{\mathcal{H}}^{2}_{\text{int}} + \hat{\mathcal{H}}^{3}_{\text{int}}, \tag{4}$$

where

$$
\begin{aligned}
\hat{\mathcal{H}}^1_{\rm int} &= \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{y} \hat{\varphi}^+(\mathbf{x}_2,\mathbf{y})\hat{\varphi}(\mathbf{x}_2,\mathbf{y})\{(\nu_{11}(\mathbf{x}_1-\mathbf{x}_2) \\
&+ \nu_{21}(\mathbf{x}_1-\mathbf{y}))\hat{\chi}^+_1(\mathbf{x}_1)\hat{\chi}_1(\mathbf{x}_1) + (\nu_{22}(\mathbf{x}_1-\mathbf{y}) + \nu_{12}(\mathbf{x}_1-\mathbf{x}_2))\hat{\chi}^+_2(\mathbf{x}_1)\hat{\chi}_2(\mathbf{x}_1)\}, \quad (5) \\
\hat{\mathcal{H}}^2_{\rm int} &= \frac{1}{2}\int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{y}_1 d\mathbf{y}_2 \hat{\varphi}^+(\mathbf{x}_1,\mathbf{y}_1)\hat{\varphi}^+(\mathbf{x}_2,\mathbf{y}_2)\hat{\varphi}(\mathbf{x}_2,\mathbf{y}_2)\hat{\varphi}(\mathbf{x}_1,\mathbf{y}_1)\{\nu_{11}(\mathbf{x}_1-\mathbf{x}_2) \\
&+ \nu_{22}(\mathbf{y}_1-\mathbf{y}_2) + \nu_{12}(\mathbf{x}_1-\mathbf{y}_2) + \nu_{21}(\mathbf{y}_1-\mathbf{x}_2)\}, \quad (6) \\
\hat{\mathcal{H}}^3_{\rm int} &= \frac{1}{2}\int d\mathbf{x}_1 d\mathbf{x}_2 \{\nu_{11}(\mathbf{x}_1-\mathbf{x}_2)\hat{\chi}^+_1(\mathbf{x}_1)\hat{\chi}^+_1(\mathbf{x}_2)\hat{\chi}_1(\mathbf{x}_2)\hat{\chi}_1(\mathbf{x}_1) \\
&+ \nu_{22}(\mathbf{x}_1-\mathbf{x}_2)\hat{\chi}^+_2(\mathbf{x}_1)\hat{\chi}^+_2(\mathbf{x}_2)\hat{\chi}_2(\mathbf{x}_2)\hat{\chi}_2(\mathbf{x}_1) \\
&+ 2\nu_{12}(\mathbf{x}_1-\mathbf{x}_2)\hat{\chi}^+_1(\mathbf{x}_1)\hat{\chi}^+_2(\mathbf{x}_2)\hat{\chi}_2(\mathbf{x}_2)\hat{\chi}_1(\mathbf{x}_1)\}. \quad (7)
\end{aligned}
$$

In equations (5)–(7) the operators $\hat{\varphi}^+(\mathbf{x}_1,\mathbf{x}_2)$, $\hat{\varphi}(\mathbf{x}_1,\mathbf{x}_2)$ are related to the creation $\hat{\eta}^+_\alpha(\mathbf{X})$ and annihilation operators $\hat{\eta}_\alpha(\mathbf{X})$ of atoms in quantum state α by expressions

$$
\begin{gathered}
\hat{\varphi}^+(\mathbf{x}_1,\mathbf{x}_2) = \textstyle\sum_\alpha \varphi^*_\alpha(\mathbf{x})\hat{\eta}^+_\alpha(\mathbf{X}), \qquad \hat{\varphi}(\mathbf{x}_1,\mathbf{x}_2) = \textstyle\sum_\alpha \varphi_\alpha(\mathbf{x})\hat{\eta}_\alpha(\mathbf{X}), \\
\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2, \qquad \mathbf{X} = \tfrac{m_1\mathbf{x}_1 + m_2\mathbf{x}_2}{m_1+m_2}\ , \quad (8)
\end{gathered}
$$

where $\varphi_\alpha(\mathbf{x})$ is the wave function of the bound state and $\nu_{ij}(\mathbf{x}-\mathbf{y}),\ i,j = 1,2$ is the potential energy of Coulomb interaction

$$\nu_{ij}(\mathbf{x}-\mathbf{y}) = \frac{e_i e_j}{|\mathbf{x}-\mathbf{y}|}, \quad (9)$$

(e_i is the electric charge of a fermion of i type).

In this way, the Hamiltonian $\hat{\mathcal{H}}^1_{\rm int}$ corresponds to scattering of particles of the first and second types by bound states, the Hamiltonian $\hat{\mathcal{H}}^2_{\rm int}$ corresponds to scattering of bound states by each other, the Hamiltonian $\hat{\mathcal{H}}^3_{\rm int}$ corresponds to scattering of particles of the first and second types by particles of the same types.

And, finally, the operator $\hat{V}(t)$ in (1) represents the Hamiltonian that describes the interaction of particles with the electromagnetic field

$$
\begin{gathered}
\hat{V}(t) = -\frac{1}{c}\int d\mathbf{x}\hat{\mathbf{A}}(\mathbf{x},t)\hat{\mathbf{J}}(\mathbf{x},t) - \frac{1}{2c^2}\int d\mathbf{x}\hat{\mathbf{A}}^2(\mathbf{x},t)\sum_{i=1}^{2}\frac{e_i}{m_i}\hat{\sigma}_i(\mathbf{x}) + \int d\mathbf{x}\varphi^{(e)}(\mathbf{x},t)\hat{\sigma}(\mathbf{x}), \\
\hat{\sigma}(\mathbf{x}) = \textstyle\sum_{i=1}^{2}\hat{\sigma}_i(\mathbf{x}). \quad (10)
\end{gathered}
$$

In this expression we have taken into account an interaction of particles with the external electromagnetic field $\mathbf{A}^{(e)}(\mathbf{x},t)$, $\varphi^{(e)}(\mathbf{x},t)$ ($\varphi^{(e)}(\mathbf{x},t)$ is the scalar potential of the external electromagnetic field) and the quantum electromagnetic field, that is described by the potential $\hat{\mathbf{a}}(\mathbf{x})$ (Coulomb's gauge):

$$\hat{\mathbf{A}}(\mathbf{x},t) = \hat{\mathbf{a}}(\mathbf{x}) + \mathbf{A}^{(e)}(\mathbf{x},t), \quad (11)$$

where $\mathbf{A}^{(e)}(\mathbf{x},t)$ is the vector potential of the external electromagnetic field and $\hat{\mathbf{a}}(\mathbf{x})$ is the quantum electromagnetic field operator, that is defined by the expression

$$\hat{\mathbf{a}}(\mathbf{x}) = \sum_{\mathbf{k}}\sum_{\lambda=1}^{2}\left(\frac{2\pi}{V\omega_{\mathbf{k}}}\right)^{1/2}\left(\mathbf{e}_{\mathbf{k}\lambda}\hat{C}_{\mathbf{k}\lambda}e^{i\mathbf{k}\mathbf{x}} + h.c.\right),$$

(V is the system volume, $\mathbf{e}_{\mathbf{k}\lambda}$ is the photon polarization vector).

The charge density operators $\hat{\sigma}_i(\mathbf{x})$ for particles of i type (see (10)) are connected with the density operators $\hat{\rho}_i(\mathbf{x})$ (see [1])

$$\begin{aligned}
\hat{\sigma}_i(\mathbf{x}) &= e_i\hat{\rho}_i(\mathbf{x}),\\
\hat{\rho}_1(\mathbf{x}) &= \hat{\chi}_1^+(\mathbf{x})\hat{\chi}_1(\mathbf{x}) + \int \mathrm{d}\mathbf{y}\int \mathrm{d}\mathbf{Y}\delta(\mathbf{x}-\mathbf{Y}-\frac{m_2}{M}\mathbf{y})\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\hat{\varphi}(\mathbf{y},\mathbf{Y}),\\
\hat{\rho}_2(\mathbf{x}) &= \hat{\chi}_2^+(\mathbf{x})\hat{\chi}_2(\mathbf{x}) + \int \mathrm{d}\mathbf{y}\int \mathrm{d}\mathbf{Y}\delta(\mathbf{x}-\mathbf{Y}-\frac{m_1}{M}\mathbf{y})\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\hat{\varphi}(\mathbf{y},\mathbf{Y}),
\end{aligned}\tag{12}$$

and, as easy to see, in the equation (12) we have also taken into account a contribution that had been made by charged particles, that are represented in the bound states (see (8)). The current density operator $\hat{\mathbf{J}}(\mathbf{x},t)$ in the formula (10) can be also expressed in terms of the creation and annihilation operators

$$\hat{\mathbf{J}}(\mathbf{x},t) = -\hat{\mathbf{A}}(\mathbf{x},t)\sum_{i=1}^{2}\frac{e_i}{m_i}\hat{\sigma}_i(\mathbf{x}) + \hat{\mathbf{j}}(\mathbf{x}), \qquad \hat{\mathbf{j}}(\mathbf{x}) = \sum_{i=1}^{2}\frac{e_i}{m_i}\hat{\boldsymbol{\pi}}_i(\mathbf{x}), \tag{13}$$

where the momentum density operators $\hat{\boldsymbol{\pi}}_i(\mathbf{x})$ are defined by expressions

$$\begin{aligned}
\hat{\boldsymbol{\pi}}_1(\mathbf{x}) &= -\frac{\mathrm{i}}{2}\left(\hat{\chi}_1^+(\mathbf{x})\frac{\partial\hat{\chi}_1(\mathbf{x})}{\partial\mathbf{x}} - \frac{\partial\hat{\chi}_1^+(\mathbf{x})}{\partial\mathbf{x}}\hat{\chi}_1(\mathbf{x})\right) - \frac{\mathrm{i}}{2}\int \mathrm{d}\mathbf{y}\int \mathrm{d}\mathbf{Y}\delta\left(\mathbf{x}-\mathbf{Y}-\frac{m_2}{M}\mathbf{y}\right)\\
&\times\left[\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\frac{\partial\hat{\varphi}(\mathbf{y},\mathbf{Y})}{\partial\mathbf{y}} - \frac{\partial\hat{\varphi}^+(\mathbf{y},\mathbf{Y})}{\partial\mathbf{y}}\hat{\varphi}(\mathbf{y},\mathbf{Y})\right.\\
&\left.+\frac{m_1}{M}\left(\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\frac{\partial\hat{\varphi}(\mathbf{y},\mathbf{Y})}{\partial\mathbf{Y}} - \frac{\partial\hat{\varphi}^+(\mathbf{y},\mathbf{Y})}{\partial\mathbf{Y}}\hat{\varphi}(\mathbf{y},\mathbf{Y})\right)\right],\\
\hat{\boldsymbol{\pi}}_2(\mathbf{x}) &= -\frac{\mathrm{i}}{2}\left(\hat{\chi}_2^+(\mathbf{x})\frac{\partial\hat{\chi}_2(\mathbf{x})}{\partial\mathbf{x}} - \frac{\partial\hat{\chi}_2^+(\mathbf{x})}{\partial\mathbf{x}}\hat{\chi}_2(\mathbf{x})\right) - \frac{\mathrm{i}}{2}\int \mathrm{d}\mathbf{y}\int \mathrm{d}\mathbf{Y}\delta\left(\mathbf{x}-\mathbf{Y}+\frac{m_1}{M}\mathbf{y}\right)\\
&\times\left[-\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\frac{\partial\hat{\varphi}(\mathbf{y},\mathbf{Y})}{\partial\mathbf{y}} + \frac{\partial\hat{\varphi}^+(\mathbf{y},\mathbf{Y})}{\partial\mathbf{y}}\hat{\varphi}(\mathbf{y},\mathbf{Y})\right.\\
&\left.+\frac{m_2}{M}\left(\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\frac{\partial\hat{\varphi}(\mathbf{y},\mathbf{Y})}{\partial\mathbf{Y}} - \frac{\partial\hat{\varphi}^+(\mathbf{y},\mathbf{Y})}{\partial\mathbf{Y}}\hat{\varphi}(\mathbf{y},\mathbf{Y})\right)\right].
\end{aligned}\tag{14}$$

Using formulas (12)–(14) we can write the expressions for the current and charge density operators in more suitable way:

$$\hat{\sigma}(\mathbf{x}) = \sum_a \hat{\sigma}_a(\mathbf{x}), \quad \hat{\mathbf{j}}(\mathbf{x}) = \sum_a \hat{\mathbf{j}}_a(\mathbf{x}), \quad a = 0,1,2, \tag{15}$$

where

$$\begin{aligned}
\hat{\sigma}_i(\mathbf{x}) &= e_i\hat{\chi}_i^+(\mathbf{x})\hat{\chi}_i(\mathbf{x}),\\
\hat{\mathbf{j}}_i(\mathbf{x}) &= -\frac{\mathrm{i}e_i}{2m_i}\left(\hat{\chi}_i^+(\mathbf{x})\frac{\partial\hat{\chi}_i(\mathbf{x})}{\partial\mathbf{x}} - \frac{\partial\hat{\chi}_i^+(\mathbf{x})}{\partial\mathbf{x}}\hat{\chi}_i(\mathbf{x})\right), \quad i = 1,2,\\
\hat{\sigma}_0(\mathbf{x}) &= \int \mathrm{d}\mathbf{y}\int \mathrm{d}\mathbf{Y}\left[e_1\delta(\mathbf{x}-\mathbf{Y}-\frac{m_2}{M}\mathbf{y}) + e_2\delta(\mathbf{x}-\mathbf{Y}-\frac{m_1}{M}\mathbf{y})\right]\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\hat{\varphi}(\mathbf{y},\mathbf{Y}),\\
\hat{\mathbf{j}}_0(\mathbf{x}) &= -\frac{\mathrm{i}}{2}\int \mathrm{d}\mathbf{y}\int \mathrm{d}\mathbf{Y}\left[\frac{e_1}{m_1}\delta(\mathbf{x}-\mathbf{Y}-\frac{m_2}{M}\mathbf{y}) - \frac{e_2}{m_2}\delta(\mathbf{x}-\mathbf{Y}-\frac{m_1}{M}\mathbf{y})\right]\\
&\times\left(\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\frac{\partial\hat{\varphi}(\mathbf{y},\mathbf{Y})}{\partial\mathbf{y}} - \frac{\partial\hat{\varphi}^+(\mathbf{y},\mathbf{Y})}{\partial\mathbf{y}}\hat{\varphi}(\mathbf{y},\mathbf{Y})\right)\\
&-\frac{\mathrm{i}}{2M}\int \mathrm{d}\mathbf{y}\int \mathrm{d}\mathbf{Y}\left[e_1\delta(\mathbf{x}-\mathbf{Y}-\frac{m_2}{M}\mathbf{y}) + e_2\delta(\mathbf{x}-\mathbf{Y}-\frac{m_1}{M}\mathbf{y})\right]\\
&\times\left(\hat{\varphi}^+(\mathbf{y},\mathbf{Y})\frac{\partial\hat{\varphi}(\mathbf{y},\mathbf{Y})}{\partial\mathbf{Y}} - \frac{\partial\hat{\varphi}^+(\mathbf{y},\mathbf{Y})}{\partial\mathbf{Y}}\hat{\varphi}(\mathbf{y},\mathbf{Y})\right).
\end{aligned}\tag{16}$$

As it is easy to see, the operators $\hat{\sigma}_0(\mathbf{x})$, $\hat{\mathbf{j}}_0(\mathbf{x})$ in these expressions define the bound states contribution to the charge and current densities.

In the momentum representation

$$\hat{\chi}_i(\mathbf{x}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{p}} \mathrm{e}^{\mathrm{i}\mathbf{p}\mathbf{x}}\hat{a}_{i\mathbf{p}}, \qquad \hat{\chi}_i^+(\mathbf{x}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{p}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\mathbf{x}}\hat{a}_{i\mathbf{p}}^+, \quad i=1,2,$$
$$\hat{\eta}_\alpha(\mathbf{x}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{p}} \mathrm{e}^{\mathrm{i}\mathbf{p}\mathbf{x}}\hat{\eta}_\alpha(\mathbf{p}), \qquad \hat{\eta}_\alpha^+(\mathbf{x}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{p}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\mathbf{x}}\hat{\eta}_\alpha^+(\mathbf{p})$$

expressions (16) according to (8) will have the next form:

$$\begin{aligned}
\hat{\sigma}_i(\mathbf{x}) &= \frac{e_i}{V}\sum_{\mathbf{p},\mathbf{p}'} \mathrm{e}^{\mathrm{i}\mathbf{x}(\mathbf{p}'-\mathbf{p})}\hat{a}_{i\mathbf{p}}^+\hat{a}_{i\mathbf{p}'},\\
\hat{\mathbf{j}}_i(\mathbf{x}) &= \frac{e_i}{2m_iV}\sum_{\mathbf{p},\mathbf{p}'} \mathrm{e}^{\mathrm{i}\mathbf{x}(\mathbf{p}'-\mathbf{p})}(\mathbf{p}+\mathbf{p}')\hat{a}_{i\mathbf{p}}^+\hat{a}_{i\mathbf{p}'},\\
\hat{\sigma}_0(\mathbf{x}) &= \frac{1}{V}\sum_{\mathbf{p},\mathbf{p}'}\sum_{\alpha,\beta} \mathrm{e}^{\mathrm{i}\mathbf{x}(\mathbf{p}'-\mathbf{p})}\sigma_{\alpha\beta}(\mathbf{p}-\mathbf{p}')\hat{\eta}_\alpha^+(\mathbf{p})\hat{\eta}_\beta(\mathbf{p}'),\\
\hat{\mathbf{j}}_0(\mathbf{x}) &= \frac{1}{V}\sum_{\mathbf{p},\mathbf{p}'}\sum_{\alpha,\beta} \mathrm{e}^{\mathrm{i}\mathbf{x}(\mathbf{p}'-\mathbf{p})}\left(\frac{(\mathbf{p}+\mathbf{p}')}{2M}\sigma_{\alpha\beta}(\mathbf{p}-\mathbf{p}') + \mathbf{I}_{\alpha\beta}(\mathbf{p}-\mathbf{p}')\right)\hat{\eta}_\alpha^+(\mathbf{p})\hat{\eta}_\beta(\mathbf{p}'),
\end{aligned} \tag{17}$$

where (see (8))

$$\begin{aligned}
\sigma_{\alpha\beta}(\mathbf{k}) &= \int \mathrm{d}\mathbf{y}\varphi_\alpha^*(\mathbf{y})\varphi_\beta(\mathbf{y})\left[e_1\exp\mathrm{i}\frac{m_2}{M}\mathbf{k}\mathbf{y} + e_2\exp\left(-\mathrm{i}\frac{m_1}{M}\mathbf{k}\mathbf{y}\right)\right],\\
\mathbf{I}_{\alpha\beta}(\mathbf{k}) &= -\frac{\mathrm{i}}{2}\int \mathrm{d}\mathbf{y}\left(\varphi_\alpha^*(\mathbf{y})\frac{\partial\varphi_\beta(\mathbf{y})}{\partial\mathbf{y})} - \frac{\partial\varphi_\alpha^*(\mathbf{y})}{\partial\mathbf{y})}\varphi_\beta(\mathbf{y})\right)\left[\frac{e_1}{m_1}\exp\mathrm{i}\frac{m_2}{M}\mathbf{k}\mathbf{y} - \frac{e_2}{m_2}\exp\left(-\mathrm{i}\frac{m_1}{M}\mathbf{k}\mathbf{y}\right)\right].
\end{aligned} \tag{18}$$

It is significant to note, that the Hamiltonian for free particles $\hat{\mathcal{H}}_p$ (see (1), (3)) in the momentum representation can be written as

$$\begin{aligned}
&\hat{\mathcal{H}}_p = \hat{\mathcal{H}}_{1p} + \hat{\mathcal{H}}_{2p} + \hat{\mathcal{H}}_{0p},\\
&\hat{\mathcal{H}}_{ip} = \sum_{\mathbf{p}} \varepsilon_i(\mathbf{p})\hat{a}_{i\mathbf{p}}^+\hat{a}_{i\mathbf{p}}, \qquad \varepsilon_i(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_i}, \qquad i=1,2,\\
&\hat{\mathcal{H}}_{0p} = \sum_{\alpha}\sum_{\mathbf{p}} \varepsilon_\alpha(\mathbf{p})\hat{\eta}_\alpha(\mathbf{p})^+\hat{\eta}_\alpha(\mathbf{p}), \qquad \varepsilon_\alpha(\mathbf{p}) = \varepsilon_\alpha + \frac{\mathbf{p}^2}{2M}, \qquad \varepsilon_\alpha < 0,
\end{aligned} \tag{19}$$

where ε_α is the energy of the atomic level with quantum numbers α, M is the bound state mass, $M = (m_1 + m_2)$.

The Maxwell equations for our system according to [1] can be written in the following form

$$\begin{aligned}
&\frac{\partial\hat{\mathbf{H}}}{\partial t} = -c\,\mathrm{rot}\hat{\mathbf{E}}, \qquad \mathrm{div}\hat{\mathbf{H}} = 0,\\
&\frac{\partial\hat{\mathbf{E}}}{\partial t} = -c\,\mathrm{rot}\hat{\mathbf{H}} - 4\pi(\hat{\mathbf{J}} + \mathbf{J}^{(e)}), \qquad \mathrm{div}\hat{\mathbf{E}} = 4\pi(\hat{\sigma} + \sigma^{(e)}),
\end{aligned} \tag{20}$$

where operators $\hat{\sigma}$, $\hat{\mathbf{J}}$ are still defined by the expressions (10), (12), (13) and values $\sigma^{(e)}$, $\mathbf{J}^{(e)}$ are the extrinsic current and charge densities. The electric $\hat{\mathbf{E}}$ and magnetic $\hat{\mathbf{H}}$ field intensity operators in terms of the scalar and vector potentials can be expressed as (see [2], [1] as well as equations (10), (11))

$$\hat{\mathbf{H}} = \mathrm{rot}\hat{\mathbf{A}}, \qquad \hat{\mathbf{E}} = -\frac{1}{c}\frac{\partial\hat{\mathbf{A}}}{\partial t} - \frac{\partial}{\partial\mathbf{x}}\left(\varphi^{(e)} + \int \mathrm{d}\mathbf{x}'\frac{\hat{\sigma}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\right). \tag{21}$$

Note that in deriving the electrodynamic equations we used the Coulomb's gauge.

3. The system response to the perturbation by the external electromagnetic field and Green functions

In this section in order to study the system response to the perturbing action of the external electromagnetic field we shall follow the principles that have been stated in [2]. Let us consider a system that at some moment of time t is characterized by statistical operator $\rho(t)$. Noting that the Hamiltonian of interaction $\hat{V}(t)$ is linear in respect to the external field and assuming that it is small in comparison with the Hamiltonian $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{int}}$ (see (1)), we can develop the perturbation theory over the week interaction. In accordance with [2] the mean value of an arbitrary quasilocal operator $\hat{a}(\mathbf{x})$ in linear approach for such system can be written as

$$\begin{aligned} \mathrm{Sp}\rho(t)\hat{a}(\mathbf{x}) &= \mathrm{Sp}w\hat{a}(0) + a^F(\mathbf{x},t), \\ a^F(\mathbf{x},t) &= \int_{-\infty}^{\infty} \mathrm{d}t' \int \mathrm{d}\mathbf{x}' G^{(+)}_{a\xi_i}(\mathbf{x}-\mathbf{x}', t-t') F_i(\mathbf{x}', t'), \end{aligned} \tag{22}$$

where w is the Hibbs distribution operator

$$w = \exp\left\{\Omega - \beta(\hat{\mathcal{H}} - \mu_1\hat{N}_1 - \mu_2\hat{N}_2)\right\}, \tag{23}$$

$\beta = 1/T$ is the reciprocal temperature, $\hat{N}_1$, $\hat{N}_2$ are the number operators of all fermions of the first and second type respectively (including fermions in bound states, see (12))

$$\hat{N}_1 = \int \mathrm{d}\mathbf{x}\hat{\rho}_1(\mathbf{x}), \qquad \hat{N}_2 = \int \mathrm{d}\mathbf{x}\hat{\rho}_2(\mathbf{x}) \tag{24}$$

and μ_1, μ_2 are the chemical potentials of fermions of the first and second type. The thermodynamic parameters β, μ_1, μ_2 can be found from the relations

$$\mathrm{Sp}w\hat{\mathcal{H}} = \mathcal{H}, \qquad \mathrm{Sp}w\hat{N}_1 = N_1, \qquad \mathrm{Sp}w\hat{N}_2 = N_2, \tag{25}$$

and the thermodynamic potential Ω dependence on thermodynamic parameters is defined by the expression

$$\mathrm{Sp}w = 1.$$

In the formula (22) $F_i(\mathbf{x},t)$ are the quantities, that define the external field and $\hat{\xi}_i(\mathbf{x})$ are quasilocal operators, related to the system (see also [2]); the summation convention is assumed for the repeated index i.

And, finally, the quantity $G^{(+)}_{a\xi_i}(\mathbf{x}-\mathbf{x}', t-t')$ in the expression (22) is the two-time retarded Green function (note, that "tilde" over operators means that they are taken in the Heisenberg representation)

$$G^{(+)}_{a\xi_i}(\mathbf{x}-\mathbf{x}', t-t') = -\mathrm{i}\theta(t-t')Spw[\tilde{\hat{a}}(\mathbf{x},t), \tilde{\hat{\xi}}_i(\mathbf{x}',t')], \tag{26}$$

where $\theta(t)$ is Heaviside function

$$\theta(t) = \begin{cases} 1, & t > 0, \\ 0, & t < 0. \end{cases}$$

Going over to Fourier transforms of values a^F, F_i

$$\begin{aligned} a^F(\mathbf{x},t) &= \frac{1}{(2\pi)^4}\int \mathrm{d}\mathbf{k}\mathrm{d}\omega e^{-i(t\omega-\mathbf{kx})} a^F(\mathbf{k},\omega), \\ F_i(\mathbf{x},t) &= \frac{1}{(2\pi)^4}\int \mathrm{d}\mathbf{k}\mathrm{d}\omega e^{-i(t\omega-\mathbf{kx})} F_i(\mathbf{k},\omega) \end{aligned}$$

one obtains

$$a^F(\mathbf{k},\omega) = G^{(+)}_{a\xi_i}(\mathbf{k},\omega) F_i(\mathbf{k},\omega), \tag{27}$$

where

$$G^{(+)}_{a\xi_i}(\mathbf{k},\omega) = \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}\mathbf{x} \mathrm{e}^{\mathrm{i}(t\omega - \mathbf{k}\mathbf{x})} G^{(+)}_{a\xi_i}(\mathbf{x},t). \tag{28}$$

It is significant, that in terms of the Fourier transforms of the introduced quantities we can express also the energy, transferred from field to matter. If we assume, that the field is acting only a limited period of time, the total energy Q, received by matter, is given by the expression [2]

$$Q = \frac{\mathrm{i}}{(2\pi)^4} \int_{-\infty}^{\infty} \mathrm{d}\omega \int \mathrm{d}\mathbf{k}\omega F_i(-\mathbf{k},-\omega) G^{(+)}_{\xi_i\xi_j}(\mathbf{k},\omega) F_j(\mathbf{k},\omega). \tag{29}$$

Now we can apply these expressions to the study of the response of the system, consisting of two types of oppositely charged fermions and their bound states. To make use of the Green function method, that was described above (see (22)–(28)), it is more convenient to represent the system Hamiltonian, that is defined by formulas (1)–(16), as

$$\hat{\mathcal{H}}(t) = \hat{\mathcal{H}} + \hat{V}^{(e)}(t), \qquad \hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\mathrm{int}} + \hat{V}, \tag{30}$$

where $\hat{\mathcal{H}}_0$ and $\hat{\mathcal{H}}_{\mathrm{int}}$ are given by the formulas (1)–(7), $\hat{V}$ is defined by the expression (see also (10)–(14))

$$\hat{V} = -\frac{1}{c} \int \mathrm{d}\mathbf{x} \hat{\mathbf{a}}(\mathbf{x},t) \hat{\mathbf{j}}(\mathbf{x},t) - \frac{1}{2c^2} \int \mathrm{d}\mathbf{x} \hat{\mathbf{a}}^2(\mathbf{x},t) \sum_{i=1}^{2} \frac{e_i}{m_i} \hat{\sigma}_i(\mathbf{x}), \tag{31}$$

and the Hamiltonian $\hat{V}^{(e)}(t)$ describes the system interaction with the external electromagnetic field

$$\hat{V}^{(e)}(t) = -\frac{1}{c} \int \mathrm{d}\mathbf{x} \mathbf{A}^{(e)}(\mathbf{x},t) \hat{\mathbf{j}}(\mathbf{x}) + \frac{1}{2c^2} \int \mathrm{d}\mathbf{x} \mathbf{A}^{(e)}(\mathbf{x},t)^2 \sum_{i=1}^{2} \frac{e_i}{m_i} \hat{\sigma}_i(\mathbf{x}) + \int \mathrm{d}\mathbf{x} \varphi^{(e)}(\mathbf{x},t) \hat{\sigma}(\mathbf{x}). \tag{32}$$

To get the Maxwell equations for the electromagnetic field in a medium, it is necessary to average the equations (20) with the system statistical operator containing the information both about medium and electromagnetic field. To this end we shall define the mean values of the electromagnetic fields $\mathbf{E}(\mathbf{x},t)$, $\mathbf{H}(\mathbf{x},t)$, acting in the matter

$$\mathbf{E}(\mathbf{x},t) = \mathrm{Sp}\rho(t)\hat{\mathbf{E}}(\mathbf{x},t), \qquad \mathbf{H}(\mathbf{x},t) = \mathrm{Sp}\rho(t)\hat{\mathbf{H}}(\mathbf{x},t), \tag{33}$$

as well as the induced charge and current averages (see (12)–(14))

$$\mathbf{J}(\mathbf{x},t) = \mathrm{Sp}\rho(t)\hat{\mathbf{J}}(\mathbf{x},t), \qquad \sigma(\mathbf{x},t) = \mathrm{Sp}\rho(t)\hat{\sigma}(\mathbf{x}). \tag{34}$$

The equations (20), averaged in accordance with the formulas (33)–(34), bring us to the Maxwell-Lorentz equations for the average fields in the matter

$$\begin{aligned} &\frac{\partial \mathbf{H}}{\partial t} = -c\,\mathrm{rot}\mathbf{E}, && \mathrm{div}\mathbf{H} = 0, \\ &\frac{\partial \mathbf{E}}{\partial t} = -c\,\mathrm{rot}\mathbf{H} - 4\pi(\mathbf{J} + \mathbf{J}^{(e)}), && \mathrm{div}\mathbf{E} = 4\pi(\sigma + \sigma^{(e)}), \end{aligned} \tag{35}$$

where quantities $\sigma^{(e)}$, $\mathbf{J}^{(e)}$ still represent the extrinsic charge and current densities.

The next problem is to find the charge $\sigma(\mathbf{x},t)$ and current $\mathbf{J}(\mathbf{x},t)$ densities induced by the external field. Calculating these quantities under assumption of week interaction between the system and the external field we shall use the equations (22)–(28), considering the potentials $\mathbf{A}^{(e)}(\mathbf{x},t)$, $\varphi^{(e)}(\mathbf{x},t)$ as $F_i(\mathbf{x},t)$, and $\hat{\sigma}(\mathbf{x})$ or $\hat{\mathbf{J}}(\mathbf{x},t)$ as a quasilocal operator $\hat{a}(\mathbf{x})$. As a result one gets

$$\begin{aligned} \widetilde{\sigma}(\mathbf{x},t) &= \sum_a \sigma_a + \int_{-\infty}^{\infty} \mathrm{d}t' \int \mathrm{d}^3x' \left[-\overline{G}^{(+)}_i(\mathbf{x}-\mathbf{x}',t-t') \frac{1}{c} A^{(e)}_i(\mathbf{x}',t') + G^{(+)}(\mathbf{x}-\mathbf{x}',t-t')\varphi^{(e)}(\mathbf{x}',t') \right], \\ \widetilde{J}_k(\mathbf{x},t) &= -\frac{1}{c} A^{(e)}_k(\mathbf{x},t) \sum_a \frac{e_a}{m_a}\sigma_a + \int_{-\infty}^{\infty} \mathrm{d}t' \int \mathrm{d}^3x' \left[-G^{(+)}_{kl}(\mathbf{x}-\mathbf{x}',t-t') \frac{1}{c} A^{(e)}_l(\mathbf{x}',t') \right. \\ &\quad \left. + G^{(+)}_k(\mathbf{x}-\mathbf{x}',t-t')\varphi^{(e)}(\mathbf{x}',t') \right], \end{aligned} \tag{36}$$

where $\sigma_a = \mathrm{Sp}w\hat{\sigma}_a(0)$, $a = 1,2,0$ (see (17)), w is the Hibbs statistical operator (23). Let us emphasize, that for quasineutral systems, where the number of fermions are equal ($N_1 = N_2$, see (24)) and their absolute charge values are also equal ($|e_1| = |e_2|$) $\sum\limits_a \sigma_a = 0$.

The retarded charge and current Green functions, that are included in the expression (36), are determined in accordance with the formula (26) (see also [2]):

$$\begin{aligned}
G^{(+)}(\mathbf{x},t) &= -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{\sigma}(\mathbf{x},t),\hat{\sigma}(0)], \qquad G_k^{(+)}(\mathbf{x},t) = -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{j}_k(\mathbf{x},t),\hat{\sigma}(0)],\\
\overline{G}_k^{(+)}(\mathbf{x},t) &= -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{\sigma}(\mathbf{x},t),\hat{j}_k(0)], \qquad G_{kl}^{(+)}(\mathbf{x},t) = -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{j}_k(\mathbf{x},t),\hat{j}_l(0)].
\end{aligned} \tag{37}$$

As the charge and current density operators of particles of different types (see (17)) commute with each other

$$[\hat{\sigma}_a,\hat{\sigma}_b] = [\hat{\sigma}_a,\hat{\mathbf{j}}_b] = [\hat{\mathbf{j}}_a,\hat{\mathbf{j}}_b] = 0, \quad a \neq b, \quad a,b = 1,2,0,$$

then, according to the equations (15), (17), (37), the contribution of different types of particles to Green functions will be additive

$$\begin{aligned}
G^{(+)}(\mathbf{x},t) &= \sum_a G_a^{(+)}(\mathbf{x},t), \quad G_a^{(+)}(\mathbf{x},t) = -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{\sigma}_a(\mathbf{x},t),\hat{\sigma}_a(0)],\\
\overline{G}_k^{(+)}(\mathbf{x},t) &= \sum_a \overline{G}_{ak}^{(+)}(\mathbf{x},t), \quad \overline{G}_{ak}^{(+)}(\mathbf{x},t) = -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{\sigma}_a(\mathbf{x},t),\hat{j}_{ak}(0)],\\
G_k^{(+)}(\mathbf{x},t) &= \sum_a G_{ak}^{(+)}(\mathbf{x},t), \quad G_{ak}^{(+)}(\mathbf{x},t) = -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{j}_{ak}(\mathbf{x},t),\hat{\sigma}_a(0)],\\
G_{kl}^{(+)}(\mathbf{x},t) &= \sum_a G_{akl}^{(+)}(\mathbf{x},t), \quad G_{akl}^{(+)}(\mathbf{x},t) = -\mathrm{i}\theta(t)\mathrm{Sp}w[\hat{j}_{ak}(\mathbf{x},t),\hat{j}_{al}(0)].
\end{aligned} \tag{38}$$

With the help of direct calculations, following the method [2], we can see, that also in the presence of particle bound states the following correspondence between Green functions (37) takes place

$$\begin{aligned}
&\overline{G}_k^{(+)}(\mathbf{x},t) = G_k^{(+)}(\mathbf{x},t), \qquad \frac{\partial G_i^{(+)}(\mathbf{x},t)}{\partial x_i} + \frac{\partial G^{(+)}(\mathbf{x},t)}{\partial t} = 0,\\
&\frac{\partial G_{ki}^{(+)}(\mathbf{x},t)}{\partial x_k} + \frac{\partial G_i^{(+)}(\mathbf{x},t)}{\partial t} + \sum_a \frac{e_a}{m_a}\sigma_a \delta(t)\frac{\partial}{\partial x_i}\delta(\mathbf{x}) = 0.
\end{aligned} \tag{39}$$

For the Green functions Fourier transforms (see (28)) these relations can be written as

$$\begin{aligned}
&\overline{G}_k^{(+)}(\mathbf{k},\omega) = G_k^{(+)}(\mathbf{k},\omega), \qquad G_i^{(+)}(\mathbf{k},\omega)k_i - \omega G^{(+)}(\mathbf{k},\omega) = 0,\\
&G_{ij}^{(+)}(\mathbf{k},\omega)k_j - \omega G_i^{(+)}(\mathbf{k},\omega) + k_i \sum_a \frac{e_a}{m_a}\sigma_a = 0.
\end{aligned} \tag{40}$$

4. Green functions and macroscopic characteristics of the ideal low-temperature hydrogen-like plasma

If we neglect all the interactions between particles in the investigated system, it can be considered as an ideal hydrogen-like low-temperature plasma (we note, that kinetic energy of particles should be small in comparison with the binding energy of compound particles). For an ideal hydrogen-like plasma the Green functions, that was introduced earlier, can be calculated exactly. To do this we should take into consideration, that neglecting the quantum fields presence, the Hamiltonian $\hat{\mathcal{H}}$ in the formula (30) should be interpreted as $\hat{\mathcal{H}}_p$, see (1), (3), (19). Taking into account this fact the Heisenberg representation of charge and current density operators, that appear in the equations (37) for the Green functions, is defined by expressions:

$$\begin{aligned}
\hat{\sigma}_i(\mathbf{x},t) &= \frac{e_i}{V}\sum_{\mathbf{p},\mathbf{p}'}\mathrm{e}^{-\mathrm{i}\mathbf{x}(\mathbf{p}-\mathbf{p}')}\mathrm{e}^{-\mathrm{i}t(\varepsilon_i(\mathbf{p})-\varepsilon_i(\mathbf{p}'))}\hat{a}^+_{i\mathbf{p}}\hat{a}_{i\mathbf{p}'},\\
\hat{\mathbf{j}}_i(\mathbf{x},t) &= \frac{e_i}{2m_iV}\sum_{\mathbf{p},\mathbf{p}'}\mathrm{e}^{-\mathrm{i}\mathbf{x}(\mathbf{p}-\mathbf{p}')}\mathrm{e}^{-\mathrm{i}t(\varepsilon_i(\mathbf{p})-\varepsilon_i(\mathbf{p}'))}\hat{a}^+_{i\mathbf{p}}\hat{a}_{i\mathbf{p}'},\quad i=1,2,\\
\hat{\sigma}_0(\mathbf{x},t) &= \frac{1}{V}\sum_{\mathbf{p},\mathbf{p}'}\sum_{\alpha,\beta}\mathrm{e}^{-\mathrm{i}\mathbf{x}(\mathbf{p}-\mathbf{p}')}\mathrm{e}^{-\mathrm{i}t(\varepsilon_\alpha(\mathbf{p})-\varepsilon_\beta(\mathbf{p}'))}\sigma_{\alpha\beta}(\mathbf{p}-\mathbf{p}')\hat{\eta}^+_\alpha(\mathbf{p})\hat{\eta}_\beta(\mathbf{p}'),\\
\hat{\mathbf{j}}_0(\mathbf{x},t) &= \frac{1}{V}\sum_{\mathbf{p},\mathbf{p}'}\sum_{\alpha,\beta}\mathrm{e}^{-\mathrm{i}\mathbf{x}(\mathbf{p}-\mathbf{p}')}\mathrm{e}^{-\mathrm{i}t(\varepsilon_\alpha(\mathbf{p})-\varepsilon_\beta(\mathbf{p}'))}\\
&\quad\times\left[\frac{(\mathbf{p}+\mathbf{p}')}{2M}\sigma_{\alpha\beta}(\mathbf{p}-\mathbf{p}')+\mathbf{I}_{\alpha\beta}(\mathbf{p}-\mathbf{p}')\right]\hat{\eta}^+_\alpha(\mathbf{p})\hat{\eta}_\beta(\mathbf{p}'),
\end{aligned}\tag{41}$$

where quantities $\sigma_{\alpha\beta}(\mathbf{k})$, $\mathbf{I}_{\alpha\beta}(\mathbf{k})$ are given by the formulas (18). If we substitute the operators (41) in (38) and do some calculations, we shall come to the following expressions for the Fourier transforms of scalar Green functions (see (28)):

$$\begin{aligned}
G_1^{(+)}(\mathbf{k},\omega) &= \frac{e_1^2}{V}\sum_{\mathbf{p}}\frac{f_1(\mathbf{p}-\mathbf{k})-f_1(\mathbf{p})}{\varepsilon_1(\mathbf{p})-\varepsilon_1(\mathbf{p}-\mathbf{k})+\omega+i0},\\
G_2^{(+)}(\mathbf{k},\omega) &= \frac{e_2^2}{V}\sum_{\mathbf{p}}\frac{f_2(\mathbf{p}-\mathbf{k})-f_2(\mathbf{p})}{\varepsilon_2(\mathbf{p})-\varepsilon_2(\mathbf{p}-\mathbf{k})+\omega+i0},\\
G_0^{(+)}(\mathbf{k},\omega) &= \frac{1}{V}\sum_{\mathbf{p}}\sum_{\alpha,\beta}\sigma_{\alpha\beta}(\mathbf{k})\sigma_{\beta\alpha}(-\mathbf{k})\frac{f_\alpha(\mathbf{p}-\mathbf{k})-f_\beta(\mathbf{p})}{\varepsilon_\alpha(\mathbf{p})-\varepsilon_\beta(\mathbf{p}-\mathbf{k})+\omega+i0}.
\end{aligned}\tag{42}$$

Similarly, for the vector Green functions we have

$$\begin{aligned}
G_{1l}^{(+)}(\mathbf{k},\omega) &= \frac{e_1^2}{2m_1V}\sum_{\mathbf{p}}(2\mathbf{p}-\mathbf{k})_l\frac{f_1(\mathbf{p}-\mathbf{k})-f_1(\mathbf{p})}{\varepsilon_1(\mathbf{p})-\varepsilon_1(\mathbf{p}-\mathbf{k})+\omega+i0},\\
G_{2l}^{(+)}(\mathbf{k},\omega) &= \frac{e_2^2}{2m_2V}\sum_{\mathbf{p}}(2\mathbf{p}-\mathbf{k})_l\frac{f_2(\mathbf{p}-\mathbf{k})-f_2(\mathbf{p})}{\varepsilon_2(\mathbf{p})-\varepsilon_2(\mathbf{p}-\mathbf{k})+\omega+i0},\\
G_{0l}^{(+)}(\mathbf{k},\omega) &= \frac{1}{V}\sum_{\mathbf{p}}\sum_{\alpha,\beta}\left[\frac{(2\mathbf{p}-\mathbf{k})}{2M}\sigma_{\alpha\beta}(\mathbf{k})+\mathbf{I}_{\alpha\beta}(\mathbf{k})\right]_l\frac{\sigma_{\beta\alpha}(-\mathbf{k})\left[f_\alpha(\mathbf{p}-\mathbf{k})-f_\beta(\mathbf{p})\right]}{\varepsilon_\alpha(\mathbf{p})-\varepsilon_\beta(\mathbf{p}-\mathbf{k})+\omega+i0}.
\end{aligned}\tag{43}$$

And, finally, the tensor Green functions for the investigated system is given by expressions:

$$\begin{aligned}
G_{1ls}^{(+)}(\mathbf{k},\omega) &= \frac{e_1^2}{4m_1^2V}\sum_{\mathbf{p}}(2\mathbf{p}-\mathbf{k})_l(2\mathbf{p}-\mathbf{k})_s\frac{f_1(\mathbf{p}-\mathbf{k})-f_1(\mathbf{p})}{\varepsilon_1(\mathbf{p})-\varepsilon_1(\mathbf{p}-\mathbf{k})+\omega+i0},\\
G_{2ls}^{(+)}(\mathbf{k},\omega) &= \frac{e_2^2}{4m_2^2V}\sum_{\mathbf{p}}(2\mathbf{p}-\mathbf{k})_l(2\mathbf{p}-\mathbf{k})_s\frac{f_2(\mathbf{p}-\mathbf{k})-f_2(\mathbf{p})}{\varepsilon_2(\mathbf{p})-\varepsilon_2(\mathbf{p}-\mathbf{k})+\omega+i0},\\
G_{0lj}^{(+)}(\mathbf{k},\omega) &= \frac{1}{V}\sum_{\mathbf{p}}\sum_{\alpha,\beta}\left[\frac{(2\mathbf{p}-\mathbf{k})}{2M}\sigma_{\alpha\beta}(\mathbf{k})+\mathbf{I}_{\alpha\beta}(\mathbf{k})\right]_l\\
&\quad\times\left[\frac{(2\mathbf{p}-\mathbf{k})}{2M}\sigma_{\beta\alpha}(-\mathbf{k})+\mathbf{I}_{\beta\alpha}(-\mathbf{k})\right]_j\frac{f_\alpha(\mathbf{p}-\mathbf{k})-f_\beta(\mathbf{p})}{\varepsilon_\alpha(\mathbf{p})-\varepsilon_\beta(\mathbf{p}-\mathbf{k})+\omega+i0}.
\end{aligned}\tag{44}$$

In the formulas (42)–(44) we have introduced the distribution functions for free fermions of the first $f_1(\mathbf{p})$ and second $f_2(\mathbf{p})$ type, as well as the distribution functions $f_\alpha(\mathbf{p})$ for hydrogen-like atoms (bound states) with the set of quantum numbers α

$$f_i(\mathbf{p})=\{\exp[(\varepsilon_i(\mathbf{p})-\mu_i)/T]+1\}^{-1},\quad f_\alpha(\mathbf{p})=\{\exp[(\varepsilon_\alpha(\mathbf{p})-\mu_\alpha)/T]-1\}^{-1}\tag{45}$$

in accordance with the relations

$$\mathrm{Sp}w\hat{a}^{+}_{i\mathbf{p}}\hat{a}_{i\mathbf{p}'} = \delta_{\mathbf{p},\mathbf{p}'}f_i(\mathbf{p}), \quad i=1,2, \qquad \mathrm{Sp}w\hat{\eta}^{+}_{\alpha}(\mathbf{p})\hat{\eta}_{\beta}(\mathbf{p}') = \delta_{\alpha,\beta}\delta_{\mathbf{p},\mathbf{p}'}f_{\alpha}(\mathbf{p}). \tag{46}$$

The particle energies $\varepsilon_{1,2}(\mathbf{p})$, $\varepsilon_{\alpha}(\mathbf{p})$ in formulas (42)–(45) are given by the expressions (19), and values $\delta_{\alpha,\beta}$, $\delta_{\mathbf{p},\mathbf{p}'}$ in (46) are the Kronecker symbols.

The particular feature of the obtained Green functions is that the contribution of bound states in the processes under consideration is now taken into account.

The expressions for Green functions that have been found enable us to get an expression for the matter macroscopic parameters, such as conductivity, permittivity and magnetic permeability. To this end we shall also use the method described in [2].

In accordance with the formulas (35), (36) the next relation between Fourier transforms takes place

$$\begin{aligned}
\widetilde{J}_i(\mathbf{k},\omega) &= \bar{\sigma}^l(\mathbf{k},\omega)k_i\frac{\mathbf{k}\mathbf{E}^{(e)}(\mathbf{k},\omega)}{k^2} + \bar{\sigma}^t(\mathbf{k},\omega)\frac{\Big[[\mathbf{k},\mathbf{E}^{(e)}(\mathbf{k},\omega)],\mathbf{k}\Big]}{k^2}\,, \\
\widetilde{\sigma}(\mathbf{k},\omega) &= \frac{1}{\omega}\bar{\sigma}^l(\mathbf{k},\omega)\mathbf{k}\mathbf{E}^{(e)}(\mathbf{k},\omega),
\end{aligned} \tag{47}$$

where

$$\begin{aligned}
\bar{\sigma}^l(\mathbf{k},\omega) &= \frac{i\omega}{k^2}G^{(+)}(\mathbf{k},\omega), \\
\bar{\sigma}^t(\mathbf{k},\omega) &= \frac{i}{\omega}\left[\sum_a \frac{e_a}{m_a}\sigma_a + \frac{1}{2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)G^{(+)}_{ij}(\mathbf{k},\omega)\right].
\end{aligned} \tag{48}$$

It is clear from equations (47) that the quantities $\bar{\sigma}^l$ and $\bar{\sigma}^t$, expressed in terms of Green functions according to formula (48), define the longitudinal and transversal current density components. They are usually interpreted as outer conductivity coefficients in contrast to inner (or true) longitudinal σ^l or transversal σ^t conductivity coefficients, that will be defined below. Note that according to (38) these coefficients are also additive quantities

$$\bar{\sigma}^{l,t}(\mathbf{k},\omega) = \sum_a \bar{\sigma}^{l,t}_a(\mathbf{k},\omega).$$

In terms of the introduced outer conductivity coefficients $\bar{\sigma}^l$ and $\bar{\sigma}^t$ we can express the energy, absorbed from the external field sources (see (29)):

$$Q_{\omega\mathbf{k}} = \frac{-2}{(2\pi)^4}\mathrm{Im}\frac{1}{\omega}\mathbf{E}^{*(e)}(\mathbf{k},\omega)G^{+}_{ij}(\mathbf{k},\omega)\mathbf{E}^{(e)}(\mathbf{k},\omega).$$

From this expression, in accordance with the formulas (47)–(48) one obtains

$$Q_{\omega\mathbf{k}} = \frac{2}{(2\pi)^4}\mathrm{Re}\left\{\bar{\sigma}^l(\mathbf{k},\omega)|\mathbf{E}^{(e)}_{\parallel}(\mathbf{k},\omega)|^2 + \bar{\sigma}^t(\mathbf{k},\omega)|\mathbf{E}^{(e)}_{\perp}(\mathbf{k},\omega)|^2\right\}. \tag{49}$$

In terms of these quantities ($\bar{\sigma}^l$ and $\bar{\sigma}^t$) the expressions for permittivity and magnetic permeability can be also defined. The relation between the permittivity and outer conductivity (see [2]) is given by the formula:

$$\epsilon = \left(1 + \frac{4\pi\bar{\sigma}^l}{i\omega}\right)^{-1}.$$

From this, according to the expression (48):

$$\epsilon^{-1}(\mathbf{k},\omega) = 1 + \frac{4\pi}{k^2}G^{(+)}(\mathbf{k},\omega). \tag{50}$$

It is more convenient to express the magnetic permeability in terms of inner conductivity coefficients σ^l and σ^t

$$\mu^{-1}(\mathbf{k},\omega) = 1 + \frac{4\pi\omega}{\mathrm{i}c^2k^2}(\sigma^l - \sigma^t), \tag{51}$$

that are connected with the outer conductivity coefficients $\bar{\sigma}^l$ and $\bar{\sigma}^t$ by the relations:

$$\sigma^l = \varepsilon\bar{\sigma}^l, \qquad \sigma^t = \frac{\bar{\sigma}^t}{1 + \dfrac{4\pi\bar{\sigma}^t}{i\omega}\left(1 - \dfrac{k^2c^2}{\omega^2}\right)^{-1}}\,.$$

In accordance with the expression (49), their relations with the Green functions (42), (44) take the form:

$$\sigma^l(\mathbf{k},\omega) = \frac{\mathrm{i}\omega G^{(+)}(\mathbf{k},\omega)}{k^2 + 4\pi G^{(+)}(\mathbf{k},\omega)}\,, \qquad \sigma^t(\mathbf{k},\omega) = \frac{k^2c^2 - \omega^2}{i\omega}\frac{A(\mathbf{k},\omega)}{(\omega^2 - k^2c^2) + 4\pi A(\mathbf{k},\omega)}\,,$$
$$A(\mathbf{k},\omega) \equiv \sum_a \frac{e_a}{m_a}\sigma_a + \frac{1}{2}\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)G^{(+)}_{ij}(\mathbf{k},\omega)\,. \tag{52}$$

So, we have defined the main macroscopic characteristics of the ideal hydrogen-like plasma in low temperature region. These characteristics allow us to solve a number of applied problems for our system. Let us demonstrate it on a few examples.

Using the developed theory it is not difficult to find the permittivity of an ideal gas of hydrogen-like (alkali) atoms at low temperatures. According to the expressions (42), (50) in neglect of free fermions contribution one gets

$$\epsilon^{-1}(\mathbf{k},\omega) = 1 + \frac{4\pi}{k^2}\frac{1}{V}\sum_{\mathbf{p}}\sum_{\alpha,\beta}\sigma_{\alpha\beta}(\mathbf{k})\sigma_{\beta\alpha}(-\mathbf{k})\frac{f_\alpha(\mathbf{p}-\mathbf{k}) - f_\beta(\mathbf{p})}{\varepsilon_\alpha(\mathbf{p}) - \varepsilon_\beta(\mathbf{p}-\mathbf{k}) + \omega + \mathrm{i}0}\,. \tag{53}$$

As is well known, at extremely low temperatures the Bose-Einstein condensate (BEC) of alkali atoms can be formed. At the temperatures much lower than the critical point temperature T_0 ($T \ll T_0$, see e.g. [2]), the bound states distribution functions $f_\alpha(\mathbf{p})$ are proportional to the Dirac delta-function $\delta(\mathbf{p})$. Therefore, according to the expressions (19), (53), after integration over momentum $\mathbf{p}$ the expression for permittivity of the studied gas in BEC state takes the form:

$$\begin{aligned}\epsilon^{-1}(\mathbf{k},\omega) \quad &\approx \quad 1 + \frac{(2s+1)}{2\pi^2k^2}\sum_{\alpha,\beta}\sigma_{\alpha\beta}(\mathbf{k})\sigma_{\beta\alpha}(-\mathbf{k}) \\ &\times\left[\frac{\nu_\alpha}{\Delta\varepsilon_{\alpha\beta} + k^2/2M + \omega + \mathrm{i}0} - \frac{\nu_\beta}{\Delta\varepsilon_{\alpha\beta} - k^2/2M + \omega + i0}\right], \qquad T \ll T_0, \end{aligned} \tag{54}$$

where s is the bound state spin, ν_α is the density of condensed atoms in the quantum state α and quantities $\sigma_{\alpha\beta}(\mathbf{k})$ are still defined by the formula (18). As it is easy to see, in the expression (54) at frequencies that are close to the energy interval $\Delta\varepsilon_{\alpha\beta}$ ($\Delta\varepsilon_{\alpha\beta} \equiv \varepsilon_\alpha - \varepsilon_\beta$) some peculiarities appear. In fact, such a behavior can strongly reflect on the dispersion characteristics of the gas studied. It appears to be a very interesting question, but we think this is the subject of a separate investigation.

Based on the developed theory we can also find the energy, that is dissipated by a charged particle when it passes through hydrogen-like plasma at low temperature (see in that case, e.g. [2]). In the case of a small dissipation the particle movement can be considered as uniform. Thus, the particle current density (the extrinsic current density in medium, see (20)) will be defined by the formula

$$\mathbf{J}^{(e)}(\mathbf{x},t) = ze\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t),$$

where ze is the particle charge and $\mathbf{v}$ is the particle velocity. It is easy to see, that the Fourier transform of the current density is given by the expression

$$\mathbf{J}^{(e)}(\mathbf{k},\omega) = 2\pi ze\mathbf{v}\delta(\omega - \mathbf{kv}). \tag{55}$$

Next we shall use the expression (49) for the energy, absorbed in the matter from external field sources. In this expression, using (47), the Fourier transforms of the longitudinal and transversal components of the external field can be expressed in terms of longitudinal and transversal components of the current particle density (55). If we do the necessary calculations, we shall come to the following expression for the energy $\mathrm{d}\mathcal{E}_{\mathbf{k}\omega}$, that was dissipated by the charged particle per unit time in the frequency $\mathrm{d}\omega$ and the wave vector $\mathrm{d}\mathbf{k}$ intervals, when it passes through the hydrogen-like plasma:

$$\mathrm{d}\mathcal{E}_{\mathbf{k}\omega} = -q_{\mathbf{k}\omega}\mathrm{d}\omega\mathrm{d}\mathbf{k}, \qquad q_{\mathbf{k}\omega} = \frac{Q_{\omega\mathbf{k}}}{T} = -\left(\frac{ze}{2\pi}\right)^2 \delta(\omega - \mathbf{kv})\omega \mathrm{Im}\left(\frac{v^2}{c^2} - \frac{1}{\epsilon\mu}\right)\left(\frac{\omega^2}{c^2}\epsilon - \frac{k^2}{\mu}\right)^{-1}, \quad (56)$$

where T is the particle time of flight. To get the expression (56) it is necessary to use the formula

$$\delta^2(\omega - \mathbf{kv}) = \frac{T}{2\pi}\delta(\omega - \mathbf{kv}).$$

The total dissipated particle energy $\mathcal{E}$ per unit length can be found by integrating the expression (56) over ω and $\mathbf{k}$

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}x} = -\frac{1}{v}\int \mathrm{d}\omega \mathrm{d}^3 k q_{\mathbf{k}\omega}\,. \qquad (57)$$

It is easy to see, that the main contribution in this integral comes from poles of the integrand (see (56))

$$\epsilon(\mathbf{k},\omega) = 0, \qquad \frac{\omega^2}{c^2}\epsilon(\mathbf{k},\omega)\mu(\mathbf{k},\omega) - k^2 = 0. \qquad (58)$$

The formulas (56)–(58) are similar to the expressions, that are given in [2], however, in microscopical approach they take account of particle bound states (atoms) to all processes that take place in our system (see (50)–(52), (42)–(46)). Note also that the expressions (58) represent the dispersion relations for free waves, that can spread in the system studied.

Thus, by using the microscopic approach, we have studied the linear response of the system with bound states of particles to disturbing effect of an external electromagnetic field. Our approach is based on novel formulation of the second quantization method in the presence of bound states of particles [1]. The use of such an approach has enabled us to obtain the expressions for the macroscopic characteristics of ideal hydrogen-like plasma at low temperatures taking into account not only the contribution of free charged fermions but also their bound states – the atoms of alkali metals. The expression for dielectric permittivity of an ideal gas of alkali atoms in the presence of Bose-Einstein condensation has been also obtained. The dispersion equation for the waves propagating in the system studied has been derived and the existence of resonance frequencies has been found.

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Про відгук системи зі зв'язаними станами частинок на збурення зовнішнім електромагнітним полем

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Досліджується відгук системи, у якій присутні два типи ферміонів з протилежними зарядами та їх зв'язані стани (атоми водневого типу), на збурення зовнішнім електромагнітним полем у області малих кінетичних енергій частинок. Дослідження базуються на використанні нового формулювання методу вторинного квантування, що включає можливість утворення зв'язаних станів частинок [1]. Знайдено вирази для функцій Гріна, що описують відгук системи на зовнішнє електромагнітне поле та враховують присутність зв'язаних станів частинок (атомів). За допомогою цих функцій Гріна знайдено макроскопічні параметри системи, такі як провідність, діелектрична та магнітна проникність. Як приклад розглянуто збурення ідеальної водневоподібної плазми зовнішнім електромагнітним полем у низькотемпературному діапазоні. Знайдено вирази для величин, що описують відгук бозе-конденсату ідеального газу водневоподібних атомів на зовнішнє електромагнітне поле.

Ключові слова: *функції Гріна, зв'язані стани, відгук системи, низькотемпературна водневоподібна плазма, провідність, магнітна проникність*

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