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To the theory of spin–charge separation in one-dimensional correlated electron systems

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Spin—charge separation is considered to be one of the key properties that distinguish low-dimensional electron systems from others. Three-dimensional correlated electron systems are described by the Fermi liquid theory. There, low-energy excitations (quasiparticles) are reminiscent of noninteracting electrons: They carry charges $-e$ and spins 1/2. It is believed that for any one-dimensional correlated electron system, low-lying electron excitations carry either only spin and no charge, or only charge without spin. That is why recent experiments looked for such low-lying collective electron excitations, one of which carries only spin, and the other carries only charge. Here we show that despite the fact that for *exactly solvable* one-dimensional correlated electron models there exist excitations which carry only spin and only charge, in *all* these models with short-range interactions the low-energy physics is described by low-lying collective excitations, one of which carries *both spin and charge*.

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The spin—charge separation phenomenon arising from interactions between electrons in low-dimensional condensed matter models has attracted considerable interest since the discovery of high-T_c superconductivity in low-dimensional cuprates [1,2]. Other possible experimental realizations of spin—charge separation appear to exist in organic conductors [3,4] and in mesoscopic and nanoscale metallic systems [5], e.g., quantum wires and carbon nanotubes. Physicists believe that fundamental differences between low and higher dimensions can be traced back to the reduced phase space in the former, which results in the breakdown of the Fermi liquid description. The localization of the electrons with even a small amount of disorder, thermal fluctuations destroying long-range order at any nonzero temperature (if only short-range interactions are present), and quantum fluctuations tending to suppress a broken continuous symmetry are among other features of low-dimensional correlated electron systems. For «standard» three-dimensional electron systems an interaction between electrons is described in the framework of Landau's Fermi liquid theory [6]. A Fermi liquid consists of the Fermi sphere and a rarefied gas of weakly interacting «quasiparticles» defined via poles in one-particle Green's functions. Quasiparticles continuously evolve from free electrons when the interaction is adiabatically switched on and, hence, have the same quantum numbers and statistics as noninteracting electrons. In one space dimension the Fermi liquid quasiparticle pole disappears (its residue vanishes) and is replaced by incoherent structures, which follow from the global conformal invariance [7,8]. These structures involve nonuniversal power-law singularities. Although the Fermi surface is still properly defined, the discontinuity of the momentum distribution at the Fermi surface disappears as a consequence of the zero residue [8,9]. Systems displaying such breakdown of the Fermi liquid picture and exotic low-energy spectral properties are known as Luttinger liquids [7,8]. The theoretical understanding of two-dimensional electron systems is far from being complete. They frequently manifest the absence of ordering, and, hence, mean-field-like approximations fail. Many of normal state properties of two-dimensional high-T_c superconductors are very different from normal metals and cannot be reconciled with a standard Fermi liquid theory. A marginal Fermi or Luttinger liquid picture, similar to the one of one-dimensional conductors, which uses the spin–charge separation, has been proposed to explain some of these features [10].

An approximate description of one-dimensional correlated electron models in the framework of the bosonization approach [8] supports the Luttinger liquid picture. The latter is characterized, among other features, by a spin–charge separation: The charge and spin contents of wave functions move with different speeds and are related to different low-lying excitations of correlated electron systems. It is believed that for *any* one-dimensional correlated electron system the low-lying electron excitations carry either only spin and no charge, or only charge without spin [8,9]. However, the range of applicability of the Luttinger liquid bosonization theory is limited to the continuous models and weak enough interactions: The latter have to be smaller than the Fermi velocity (related to the bandwidth of electrons) by the consideration of only low-energy electrons close to Fermi points, for which one can linearize their dispersion law [7,9]. In recent experiment, which imply the observation of a spin–charge separation these conditions were often not satisfied [1–4]. Nonetheless, for many one-dimensional correlated electron models with strong interaction between electrons the powerful Bethe ansatz method can be applied, in the framework of which collective electron excitations also have different velocities [11]. It is often stated that a spin–charge separation persists in that description also (see, e.g., Refs. 1, 8, and 9), justifying in that way the applicability of the Luttinger liquid picture to any one-dimensional correlated electron systems with metallic behavior. In our work we exactly prove that it is *incorrect*: We show that in none of the known Bethe ansatz solvable models do the correlated electrons Dirac seas, and, hence, low-energy excitations, pertain to only spin-carrying and only charge-carrying quasiparticles: For those models necessarily one Dirac sea is formed by quasiparticles, which carry *both spin and charge*. Unfortunately, this relatively simple statement has never been clearly stated in published papers devoted to the Bethe ansatz solvable correlated electron models, and this has led to many misunderstandings.

The exact solution by means of Bethe's ansatz of numerous models of one-dimensional correlated electron systems provides deep insight into the ground state of systems, complete classification of states, thermodynamic properties, etc. The best known examples of models of strongly correlated electrons, solved by the Bethe ansatz are the one-dimensional Hubbard model and supersymmetric $t-J$ model [11]. With this method the Hamiltonian is diagonalized in terms of a set of parameters (quantum numbers) known as rapidities. A system with internal degrees of freedom (such as a spin) requires a sequence of nested generalized Bethe ansätze for wave functions. Each internal degree of freedom gives rise to one set of rapidities, i.e., in the case of electrons, which carry spin, one has two sets of rapidities. Rapidities parametrize the eigenfunctions and eigenvalues of a stationary Schrödinger equation. Independently of the symmetry of the wave function and spin, energy eigenstates are occupied according to the Fermi–Dirac statistics (hard-core particles). There are many other solutions of the Bethe ansatz equations for rapidities, which describe bound states of electrons with a complex structure. In the Bethe ansatz description in the ground state (and at low temperatures) each internal degree of freedom contributes with one Fermi sea. The Fermi velocities of these Fermi seas are in general different, giving rise to what is often also called the spin–charge separation. However, the question appears: Whether these low-lying excitations really carry only charge or only spin, as in the Luttinger liquid picture? To answer this question, let us consider the Bethe ansatz equations for two sets of rapidities $u_{1,j}$ ($j = 1,\ldots,M_1$, M_1 being the total number of electrons) and $u_{2,j}$ ($j = 1,\ldots,M_2$, M_2 is the number of electrons with down spins). The Bethe ansatz equations for correlated electron chains with periodic boundary conditions can be written in the form [11]:

$$2\pi J_{i,j} = Lp_i^0(u_{i,j}) - \sum_{m=1}^{2} \sum_{\substack{l=1,\\ l\neq j}}^{M_l} \varphi_{i,m}(u_{i,j}, u_{l,m}), \quad i = 1, 2, \tag{1}$$

with $p_1^0(u_{1,j}) = k_j$ (where k_j are quasimomenta), $p_2^0(x) = \varphi_{1,1}(x,y) = 0$, $\varphi_{i,j}(x,y) = -\varphi_{j,i}(y,x)$, where L is the number of sites, $J_{i,j}$ are (half)integer numbers, different from each other for each set. This solution is valid in the domain of parameters $0 \le M_2 \le M_{1/2}$ and $0 \le M_1 \le L$. For the Hubbard model one has $u_{1,j} = \sin k_j$, $\varphi_{1,2}(x,y) = 2\tan^{-1}[4(x-y)/U]$, $\varphi_{2,2}(x,y) = 2\tan^{-1}[2(x-y)/U]$ (U is the constant of the Hubbard local interaction between electrons situated at the same site of a ring but with different spin directions; the hopping integral between neighboring sites is equal to 1.) For the supersymmetric $t-J$ chain (for $J = 2t = 2$) these functions are

$$p_1^0(u_{1,j}) = 2\tan^{-1} 2u_{1,j},$$

$$\varphi_{1,2}(x,y) = 2\tan^{-1}[2(x-y)],$$

$$\varphi_{2,2}(x,y) = 2\tan^{-1}(x-y) \quad (k_j = 2\tan^{-1} 2p_j).$$

The energy of the state with M_1 electrons, M_2 of which having their spins down, is equal to

$$E = E_0 + \sum_{i=1}^{2}\sum_{j=1}^{M_i}\varepsilon_i^0(u_{i,j}), \qquad (2)$$

where E_0 is the energy for $M_i = 0$, $\varepsilon_1^0 = -\mu - H/2 - 2\cos k_j$, and for the repulsive Hubbard chain we have $\varepsilon_2^0(x) = H$, for the attractive Hubbard model

$$\varepsilon_2^0(x) = -2\mu - 4\,\mathrm{Re}\sqrt{1-[x+i(U/4)]^2},$$

and for the supersymmetric $t-J$ model one uses

$$\varepsilon_2^0(x) = -2 + 2\pi a_2(x) - 2\mu.$$

Here $a_n(x) = (n/2)/\pi[x^2 + (n/2)^2]$, μ is the chemical potential, and H is an external magnetic field. Solutions (each set of $u_{i,j}$ corresponds to only one eigenstate) of Eqs. (1) parametrize *all* eigenvalues and eigenfunctions in the domain [12,13] $0 \le M_2 \le M_1/2$, $0 \le M_1 \le L$. Actually, Eqs. (1) are quantization conditions for rapidities. This can be recognized, e.g., for the $U = 0$ case of the Hubbard chain, which pertains to the one-dimensional free lattice electron gas. «Separation» means that the Bethe ansatz eigenstate is determined by two sets of quantum numbers. However, Eqs. (1) imply that it is impossible to really separate their contributions: They are, obviously, coupled to each other.

Consider the case of the repulsive Hubbard chain. The ground state and low-energy excitations pertain to real rapidities $u_{i,j}$ [11]. What are the charges and spins related to those quantum numbers? Suppose one changes the number of rapidities $u_{2,j}$, keeping the number of $u_{1,j}$ fixed. Such a process yields the change of the total magnetic moment, while the total number of electrons is not changed. Hence, such excitations carry only spin, but not charge. [The redistribution of rapidities with their number fixed can produce only particle—hole excitations, which, by definition, carry no spin and charge (but can change the energy).] The simple way to see it without the connection to other kinds of spin and/or charge-carrying excitations, related to the change of the number of $u_{1,j}$, is to consider the state with $M_1 = L$ fixed (i.e., at half-filling, which belongs to the domain $0 \le M_2 \le M_1/2$, $0 \le M_1 \le L$), where the low-energy dynamics is known to be connected with only spin excitations [11].

Let us now change the number of rapidities $u_{1,j}$ with the number of $u_{2,j}$ being fixed. Such a process produces a change of *both* the total charge *and* the total magnetic moment of the system. Thus, this kind of excitation carries both spin and charge. Again, to avoid connection to the other set of rapidities, one can consider the state with $M_2 = 0$, which also belongs to the domain $0 \le M_2 \le M_1/2$, $0 \le M_1 \le L$. It is very difficult to believe that the fact that an excitation carries spin depends on the number of such excitations.

We emphasize that namely the above-mentioned two kinds of low-lying excitations are considered in the conformal limit of the Bethe ansatz solvable theories to compute correlation function exponents [14], and namely those results of the Bethe ansatz considerations are used to compare with the Luttinger liquid (bosonization) approach [8]. This is why, namely these two kinds of low-lying excitations are the most important ones for the repulsive Hubbard chain.

Hence, low-lying excitations of the repulsive Hubbard chain are related to eigenstates, ones of which carry only spin and others carry both charge and spin. These states are often called spinons and unbound electron excitations, respectively (sometimes, unbound electron excitations are called holons, which is, unfortunately, misleading). The SO(4) symmetry of the Hubbard Hamiltonian [15] implies the presence of excitations which carry only charge and no spin. They are spin-singlet bound states of electrons, e.g., local pairs. However, for the repulsive Hubbard chain these states have large energies and do not affect the low-energy thermodynamics [11]. These states (local spin-singlet pairs) are low-lying states for the attractive Hubbard chain, and together with unbound electron excitations determine low-energy properties of that model [11]. These low-energy excitations of the attractive Hubbard chain carry only charge and both spin and charge, respectively. Again, namely these two kinds of low-lying excitations for the attractive Hubbard chain are important, because they determine the correlation function exponents in the conformal limit [16].

The fact that excitations related to the change of the number of rapidities $u_{1,j}$ for the Hubbard model carry both spin and charge does not depend on the value of $U \ne 0$. It is often misunderstood [8,9] that the eigenfunction of the repulsive Hubbard chain with $U \to \infty$ is reminiscent of the one of spinless fermions multiplied by the wave function of the Heisenberg spin 1/2 chain [17]. In fact, this limit $U \to \infty$ of the repulsive Hubbard chain has been the only argument used to prove the spin—charge separation for Bethe ansatz solvable models, cf. Ref. 8. However, the careful inspection of the expressions for energies of charge-carrying excitations [11] in this limit shows that they carry also spin-1/2 (one can see it by taking the derivative of the energy with respect to H, and then putting $H \to 0$). It turns out that charge-carrying excitations (the wave function of which is reminiscent of spinless fermions) for the $U \to \infty$ repulsive

Hubbard model pertain to the spin-polarized phase (the critical field of the quantum phase transition to the spin-polarized phase for the $0 < U \to \infty$ case is zero [11]). But namely in the spin-polarized case the fact that charge-carrying excitations carry also spin is especially clear, see above. Hence, the claim that in the $U \to \infty$ repulsive Hubbard chain one has factorization of the wave function into only charge-carrying and only spin-carrying parts is incorrect.

The same conclusions follow from the study of low-energy properties of the supersymmetric $t-J$ chain. Here also one kind of low-energy excitations carries both spin and charge. Again, namely these excitations determine the correlation function exponents in the conformal limit [18].

Hence, the statement that for any one-dimensional correlated electron system the low-energy excitations carry either only spin or only charge is invalid.

Why, then, does the Luttinger liquid description manifest a spin—charge separation? [Note that Luttinger liquid bosons formally do not carry either spin or charge, because they are particle—hole excitations of related correlated electron systems and, by definition, cannot carry spin and charge but can only change energies and momenta.] One can see that the Luttinger liquid approach and the Bethe ansatz approach use *different* sets of quantum numbers. As we have shown above, the Bethe ansatz eigenstates and eigenvalues are determined by two sets of quantum numbers, one of which is the total number of electrons, and the other is the total number of spins down. Contrary, in the Luttinger liquid approach one classifies states with two sets of quantum numbers, one of which is again the total number of electrons, but the other is the total magnetic moment of electrons (not the number of spins down!) [7]. Then, naturally, the Luttinger liquid approach manifests the spin-charge separation, unlike the low-energy behavior of the Bethe ansatz solvable models of correlated electrons; see above.

Then one is faced with an obvious contradiction. The Bethe ansatz is the exact solution, and it does not manifest the spin—charge separation, as we proved above. The Luttinger liquid approach (an approximate one) manifests the spin—charge separation *for the same models* of correlated electrons, e.g., for the Hubbard model. Where, then, can the Luttinger liquid approach be used? This can be seen, e.g., from the conformal limit of the Bethe ansatz solution of the Hubbard chain, where one linearizes the energy of low-lying excitations about Fermi points. In this limit for small U it is possible to rewrite the Bethe ansatz answer [14,19] in such a way that states will be classified not by the total number of electrons and number of spin-down electrons, but by the former and the total magnetic moment. This answer, naturally, coincides with the Luttinger liquid one [8,9]. However, this approach is only correct in the limit of *weak* electron—electron interactions and linearized dispersion laws of low-energy excitations. Simply by using the corrections of higher order in U, one can see that it is impossible to reformulate the Bethe ansatz conformal limit of the Hubbard chain in terms of *separated* contributions, which pertain to changes of the total number of electrons and the total magnetic moment. Hence, one can conclude that the approximate bosonization (Luttinger liquid) procedure can be correctly applied to exactly (Bethe ansatz) solvable correlated electron models for only *weak* electron—electron correlations; otherwise the results of the Luttinger liquid approach contradict known exact Bethe ansatz results.

Experiments [1–5] have actually observed low-lying excitations, characterized by two different energy scales. However, nothing permits one to conclude from those experiments that one of these excitations carries only spin while the other carries only charge. Also, for the main issues of the Luttinger liquid-like theory for the two-dimensional correlated electron systems [10] it is not necessary that real spin—charge separation occurs. One needs only the presence of two low-lying excitations (with one of them carrying only spin) instead of Fermi liquid quasiparticles. It seems interesting to study the behavior of charge and spin persistent currents in correlated electron rings with a strong electron—electron interaction for nonzero H. In such a case different (but nonzero) spins (charges) of two kinds of low-energy excitations can yield an interference of oscillations of persistent currents with two different periods [20].

Summarizing, in this work we have shown that for *all* known Bethe ansatz solvable one-dimensional models of correlated electrons with short-range interactions between electrons one of low-lying excitations carries *both spin and charge*. This is very different from what is often believed when considering the spin—charge separation in low-dimensional correlated electron models. This is why the applicability of the conclusions of the Luttinger liquid approach to the spin—charge separation for one-dimensional lattice models with short-range electron—electron interactions that are much stronger than the bandwidth of the electrons is under question.

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