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Distribution of areas of radiation generation at different frequencies in pulsar magnetospheres

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We analyse homogeneous samples of pulsars at 10 cm and 20 cm and propose to use n, the ratio of the half-width of the radiation cone θ to the minimum angular sight distance from the centre of this cone, to determine the levels of radiation generation at different frequencies in pulsar magnetospheres.

Key words: pulsars; magnetosphere; plasma; instabilities

INTRODUCTION

Earlier [2, 3] we proposed some methods for calculating the angles between the rotation axis of the pulsar, the magnetic moment of the neutron star (β) and the line of sight of the observer (ζ), based on the solution of the following system:

$$\sin\beta = C\sin(\zeta-\beta)\,,\quad \cos\theta = \cos\zeta\cos\beta + D\sin\beta\sin\zeta,\quad \theta = n(\zeta-\beta)\,. \qquad (1)$$

Here C is the maximum value of the derivative of the position angle ψ of linear polarization, the parameter $D=\cos(W_{10}/2)$ is determined from the observed pulse width W_{10} by the 10%-level. The value of n indicates how many times the half-width of the radiation cone θ is greater than the minimum angular distance $(\zeta-\beta)$ of line of sight from the centre of the cone. The value of n can be estimated from the profile shape or the depth of the minimum in the centre of the profile.

DETERMINATION OF LEVELS OF RADIATION GENERATION

If the magnetic field has a dipole structure, then the minimum angular distance $(\zeta-\beta)$ for a concrete pulsar is the same at all frequencies, and the angular radius θ of the cone increases with distance from the neutron star surface (at lower frequencies). Therefore, decreasing frequency should increase the value of n. The values $(\zeta-\beta)$ and β are fixed and the derivative C must be the same at all levels of the magnetosphere. For the dipole field the third system of equations (1) gives:

$$\frac{\theta_{20}}{\theta_{10}} = \frac{n_{20}}{n_{10}}. \qquad (2)$$

The excess of θ_{20} over θ_{10} obtained from direct observations confirms the validity of the basic assumptions of all radio pulsar models about the generation of lower frequencies at larger distances from the neutron star.

In the case of the usually applied equation, if the field is dipole:

$$\theta \approx \sqrt{\frac{r}{r_{LC}}}, \qquad (3)$$

then for an individual pulsar we have

$$\frac{r_{20}}{r_{10}} = \frac{n_{20}^2}{n_{10}^2}, \qquad (4)$$

that allows to calculate the relative positions of levels of radiation generation at wavelengths of 10 and 20 cm. Statistical dependence given in [2, 3] for 10 cm is the following:

$$\lg\theta_{10} = (1.12\pm0.05) + (-0.25\pm0.09)\lg P, \qquad (5)$$

and for 20 cm:

$$\lg\theta_{20} = (1.22\pm0.03) + (-0.24\pm0.05)\lg P. \qquad (6)$$

It allows to calculate the average ratio for all pulsars of the sample as

$$\frac{\theta_{20}}{\theta_{10}} = 1.26. \qquad (7)$$

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Our calculations show that for some objects the ratio r_{20}/r_{10} can reach values close to 2. For a number of pulsars $r_{20}/r_{10} \approx 1$. However, these objects have rather short periods, and it is quite possible that their radiation is generated at all frequencies near the light cylinder with almost equal pulse widths of 20 and 10 cm, and it is not possible to use this method for estimation of n for them. Equation (3) is written for the aligned rotator. In the case of arbitrary inclination of the dipole axis to the rotation axis ($\beta \neq 0$) the size of the cone of open field lines can be estimated using the following expression [4]:

$$A = \frac{9\sqrt{3} r_{LC} \sqrt{\sqrt{9-\sin^2\beta} - \cos\beta}}{\sqrt{2}\left(\cos\beta\sqrt{9-\sin^2\beta} + \sin^2\beta\right)\sqrt[4]{\left(9-\sin^2\beta\right)^3}}. \quad (8)$$

From the equation of a line of force

$$\frac{r}{\sin^2\theta} = A \quad (9)$$

we can obtain the formula for the angular width θ of the radiation cone at $r \ll r_{LC}$:

$$\theta = f(\beta)\sqrt{\frac{r}{r_{LC}}}. \quad (10)$$

The function

$$f(\beta) = \sqrt{\frac{r_{LC}}{A}} \quad (11)$$

takes values from 1.00 to 0.50 if β changes from 0° to 90°. It is assumed that the cross-section of the cone is circular, and its angular radius decreases equally in all directions with increasing β.

Factor $f(\beta)$ gives a possibility to determine the relative positions of the emitting levels at different frequencies using the expression (9). However, for estimating the absolute distance from the neutron star, this factor can be significant. For example, for angles $\beta > 60°$ at a given θ a distance r is twice more than for $\beta = 0°$.

We try to estimate the absolute values of r_{20} and r_{10}. The first way is to use statistical relationships (5)-(6). Thus, the expression

$$\theta_{10} = 6.61 P^{-0.25} \quad (12)$$

in view of (10) gives:

$$\left(\frac{r}{R_*}\right)_{10} = 63.5\frac{\sqrt{P}}{f^2(\beta)}. \quad (13)$$

Determination of the numerical coefficient in (10) is made under the assumption that $R_* = 10$ km. The emitting level at one frequency (1.5 or 3 GHz) is obtained from (10) and the level of the second frequency is given by the relation:

$$r_{20} = r_{10}\frac{n_{20}^2}{n_{10}^2}. \quad (14)$$

Another solution of the problem can be made under the assumption that the emission at this level occurs at the plasma frequency:

$$\nu = \nu_p\sqrt{\frac{2n_p e^2}{\pi m}} \quad (15)$$

as a result of two-stream instability.

In our estimation at a given frequency, we assume that the magnetic field has a dipole structure in the generation region, and much of the energy of the primary beam is transmitted to the secondary electron-positron plasma:

$$\gamma_b n_b mc^2 \approx 2\gamma_p n_p mc^2 \quad (16)$$

and the density of the primary beam density is equal to the Goldreich-Julian density:

$$n_b = \frac{B}{ceP}. \quad (17)$$

Under these assumptions, the distance to the centre of the neutron star of the corresponding level is determined by the following formula:

$$\frac{r}{R_*} = \sqrt[3]{\frac{e\gamma_b B_S}{\pi mc\gamma_p P\nu^2}}. \quad (18)$$

Here B_S is an induction of the magnetic field on the neutron star surface. Substituting the numerical values of fundamental constants, introducing the standard notation $B_{12} = B/10^{12}$, $\nu_9 = \nu/10^9$ and taking $\gamma_b = 10^6$, $\gamma_p = 10$, we obtain

$$\frac{r}{R_*} = 82.4\sqrt[3]{\frac{B_S}{P\nu_9^2}}. \quad (19)$$

Thus, $(r/R_*)_{3\,\mathrm{GHz}} = 40$, $(r/R_*)_{1.5\,\mathrm{GHz}} = 63$ for $B_{12} = 1$ and $P = 1$ sec.

The increment of the two-stream instability in the generation area at these frequencies, may can be written as [1]:

$$\Gamma = \frac{\sqrt{3n_b}\omega_p}{2\sqrt{2n_p}\gamma_b}. \quad (20)$$

The Usov's model [5] used for estimations assumes that a beam penetrating the outflowing plasma with the γ_p is the flow of secondary particles of the "tail" with $\gamma_t \approx 10^3$. The difference in rates of these two streams is approximately equal to $c/(2\gamma_p^2)$, that for values of $\gamma_p = 3 - 10$ is equal to

$c/18 - c/200$. The neutron star surface emits thin plasma layers, separated by a distance of order of the neutron star radius $R_* = 10^6$ cm. We find the distance at which the "beam" overtakes the plasma: $r = (18 - 200)R_*$, i.e, it occurs in those areas where radiation is generated at high frequencies.

To estimate the increasing of the amplitude of Langmuir waves due to the development of two-stream instability we calculate the value of

$$\tau = \int_{R_*}^{r} \Gamma \frac{\mathrm{d}r}{c} = \frac{1}{\gamma_t}\sqrt{\frac{3\pi e B_S}{mc^3 P}} \int_{R_*}^{r_2} \sqrt{\left(\frac{R_*}{r}\right)^3}\mathrm{d}r =$$

$$= 21.5\left[1 - \frac{1}{\sqrt{r/R_*}}\right]\sqrt{\frac{B_{12}}{P}}, \quad (21)$$

which describes the rate of the wave amplitude rise. The right side of (21) was obtained for $\gamma_t = 10^4$. For the estimated levels of generation $\tau_{3\,\mathrm{GHz}} = 18.1$ and $\tau_{1.5\,\mathrm{GHz}} = 18.8$, which corresponds to an increase of the wave amplitude in the tens of millions of times. These estimates give grounds to assert that the wave energy is sufficient to explain the generation of coherent radiation at frequencies considered.

RESULTS AND CONCLUSIONS

The obtained values of n are consistent with previous estimates of this parameter based on the observed pulse shape.

Statistical dependencies of $W_{10}(P)$ and the model of emission generation at the local plasma frequency are used to determine the absolute values of the distances to the areas of generation. These estimates are in a good agreement with each other and give the radii of the generation r_{10} of a few tens of the neutron star radius. At 20 cm the emission is formed at greater distances r_{20}, which are on average 1.5-2 times larger than r_{10} for the considered sample.

The effect of inclination of the radiation cone to the rotation axis of the pulsar, i. e., the distinction of the angle β from zero is taken into account when performing the calculations. It has been shown that Langmuir waves can provide energy for generation of emission at 10 and 20 cm.

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REFERENCES

[1] Bludman S. A., Watson K. M. & Rosenbluth M. N. 1960, Physics of Fluids, 3, 747
[2] Malov I. F. & Nikitina E. B. 2011, Astron. Rep., 55, 19
[3] Malov I. F. & Nikitina E. B. 2011, Astron. Rep., 55, 878
[4] Malov I. F. & Nikitina E. B. 2012, Astron. Rep., 56, 693
[5] Usov V. V. 1986, Astronomicheskii Tsirkulyar, 1431
[6] Weltevrede P. & Johnston S. 2008, MNRAS, 391, 1210