Nonlinear anticyclone structures in the Earth's atmosphere

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We present some results of the study of nonlinear vortex structures in the Earth's atmosphere known as blocking-anticyclones. It is shown that synoptic-scale structure can be considered in the geostrophic approximation in terms of the Charney-Obukhov equation. The numerical scheme based on the full reduction method is proposed to study the dynamics of the initial perturbations. The dominant role of the vector nonlinearity for the vortex structures stability is shown. In the absence of the nonlinearity the vortex structures are unstable and quickly decompose into the linear Rossby waves. The linear and nonlinear anticyclone structures dynamics in the Earth's atmosphere were studied using the data from NCEP(National Centers for Environmental Prediction) / NCAR (National Center for Atmospheric Research) data base — pressure, geopotential, temperature, wind velocity. Blocking anticyclones exist during a long time (typical lifetime is from 5 days to a month), while linear anticyclone exist few days (up to 5 days). Conventional synoptic anticyclone usually moves eastward but blocking anticyclones usually move westward (or stay on the same place in the zonal flow). It was also found that blocking anticyclones have larger amplitude of temperature and pressure perturbations, so the impact of the blocking anticyclone on the formation of weather is larger.

Introduction

Dynamics of the waves in the atmosphere of a rotating planet can be described with the following system of equations

$$\left\{ \begin{array}{lcl} \frac{d\mathbf{V}}{dt} & = & -\nabla (gH) + \Omega [\mathbf{V}, \zeta], \\ \frac{\partial H}{\partial t} & = & \operatorname{div}(\mathbf{V}H), \end{array} \right. \tag{1}$$

where $\mathbf{V}$ is the horizontal component of the flow velocity in the atmosphere, ζ is the vector normal to the planet's surface, Ω is the planet's rotation frequency, g is the acceleration of gravity near the planet surface, H is the pressure scale height, which depends on the sea level pressure (for the atmosphere with molecules of effective masses M and temperatures T, undisturbed pressure scale height is $H_0 = \frac{k_B T}{Mg}$, where k_B is the Boltzmann constant; for Earth H_0 is about 8 km, for Jupiter H_0 is about 25 km). Taking into account the approximation for the atmosphere as an incompressible layer of liquid with depth equal to H one can obtain dispersion equation for oscillations with small amplitudes

$$\omega \left(1 + k^2 r_R^2 - \frac{\omega^2}{\Omega^2} \right) = -k_\varphi V_*, \tag{2}$$

where $r_R = \frac{(gH_0)^{\frac{1}{2}}}{\Omega}$ is the barotropic Rossby radius (it is about 2000 km for the Earth and 6000 km for Jupiter), where $\mathbf{k}$ is the wave vector, V_* is the Rossby velocity, k_φ is the longitudinal projection of the wave vector. For frequencies much less then Ω the dispersion equation for Rossby waves can be found as:

$$\omega = -\frac{k_\varphi V_*}{1 + k^2 r_R^2}.$$

Thus linear barotropic Rossby waves have the westward phase velocity of [3, 5].

$$V_{ph} = -\frac{V_*}{1 + k^2 r_R^2}.$$

When the wavelength tends to infinity, the phase velocity tends to the Rossby velocity. Taking into account Ertel theorem one can decompose Eq. (1) in the small parameter and obtain [4]:

$$\frac{\partial}{\partial t}\left(h - r_R^2 \Delta h\right) - \frac{V_*}{R}\frac{\partial h}{\partial y}\left(h + \frac{h^2}{2}\right) = \Omega r_R^4 \left[\nabla h, \nabla \Delta h\right], \tag{3}$$

where $h = \frac{H-H_0}{H}$. Then, using well known β-plane approximation, we introduce the two-dimensional local Cartesian system of coordinates (x, y) with longitude and latitude being x and y correspondingly. x_0 and y_0 are longitude and latitude of the plane contact point. In this coordinate system the x-axis is directed from the west to the east and the y-axis points to the north. Then with the β-plane approximation we get non-dimensional Charney-Obukhov equation

$$\frac{\partial(\Delta h - h)}{\partial t} + \beta\frac{\partial h}{\partial y} + h\frac{\partial h}{\partial y} + \{h, \Delta_\perp h\} = 0, \tag{4}$$

where $V_* = \frac{gH_0}{2\omega_0 R \sin^2 a}$ is the non-dimensional Rossby velocity and $\{h, \Delta_\perp h\} = \partial_x h \partial_y \Delta h - \partial_y h \partial_x \Delta h$ denotes the Poisson bracket. The second and third parts of Eq. (4) include two types of nonlinearities: the scalar and the vector one. The scalar nonlinearity is directly related to the changes of the thickness of H layer. It is usually included in the equation for nonlinear waves, such as the Korteweg–de Vries equation obtained from finite amplitude waves on the shallow water — the first soliton in the history of science, observed by Scott Russel about 170 years ago. Vector nonlinearity may not be related to changes of H. Two kinds of nonlinearities may be subdivided only asymptotically. Scalar nonlinearity vanishes in the absence of liquid free surface, while the vector nonlinearity disappears under two conditions: axial symmetry and the Rossby velocity independence on latitude. The equation (4) has a form similar to the Hasegawa-Mina equation for an inhomogeneous plasma [2]. Taking the ratio of the third term to the fourth one one can obtain the following relation [4, 5]:

$$\frac{scalar}{vector} \approx \frac{a^2}{r_R^2}. \tag{5}$$

In the case of $a > r_R$ the scalar nonlinearity dominates, while for $a < r_R$ the vector nonlinearity dominates. The ratio (5) are estimative and useful for experimental search of the conditions under which we can expect the prevalence of the first or the second kind of nonlinearity.

From equation (4) the asymmetry of cyclones and anticyclones properties follows. The scalar nonlinearity can be in a balance with the anticyclone dispersion according to equation (4). Cyclones have the same signs in the dispersion and scalar nonlinearity and therefore can not be mutually compensated. This cyclone-antycyclone asymmetry mainly determines the possibility (or impossibility) of formation of single vortices of different polarity.

Algorithm of the numerical simulations

We consider the numerical scheme for study the dynamics of the time-dependent perturbations of a system described by equation Charney-Obukhov (or its generalization taking into account the scalar type of linearity in a form of the Korteveg-de Vries equation):

$$\frac{\partial\left(h - \Delta_\perp h\right)}{\partial t} - \beta\frac{\partial h}{\partial y} = \left\{h, \Delta_\perp h\right\}. \tag{6}$$

We used the Arakawa numerical scheme for the spatial derivatives [1]. The `IDL` procedure of numerical integration algorithm on a grid size of 50×50 and 100×100 with increments of 0.1 in time and space was used for simulations. The initial conditions are defined by a two dimensional Gaussian with characteristic scale about Rossby radius. The nature of the solution depends on the ratio between linear and nonlinear component in the equation. In a case of scalar component predominance the initial disturbance in a form of the monopole vortex is unstable. It decomposes to the linear Rossby waves due to dispersion of the drift velocity. In a case of the significant nonlinear component occurrence the nonlinear rotational solution is stable. It is also somewhat dispersed during the drift, but remains localized in space and retains the characteristic features.

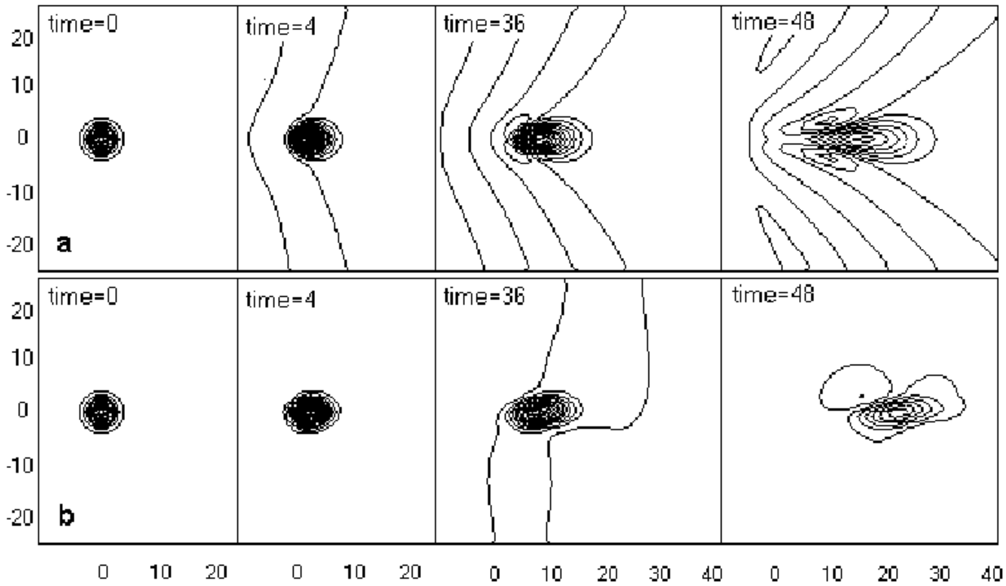

Figure 1: Dynamics of the initial perturbation in the form of the monopoly vortex with the Gaussian potential: dynamics of the linear system (*top panel:* the initial structure decomposes to the linear waves) and dynamics of the nonlinear system (*bottom panel:* it retains the basic The spatial scale is in units of Rossby radius r_R. Time is in r_R/v^*

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Evolution in time of the initial conditions on the grid is presented in Fig. 1, where the atmosphere disturbance h is shown at times moments 0, 4, 36 and 48 of r_R/v^* units. The linear case can also be used as a test case to verify the algorithm. The results are in a good agreement with the properties of small scale anticyclones in the Earth atmosphere described in [4, 5]. For experimental verification we used the data from National Center for Environmental Prediction (NCEP) and National Center for Atmospheric Research (NCAR) data bases. These data are public available[1] in NetCDF format. The files contain the data with step 2.5° in latitude and longitude, divided into 17 levels in height from 1000 mB to 10 mB, where each level contains six variables that represent geopotential altitude, temperature, v-and u-wind components, and other variables with a time step of 1 day. In general, the data has dimension [144,73,17,365] for 144 longitudes values, 73 latitude values and 365 days.The small scale vortex (spatial scale is about r_R) structure observed on September 17, 2001 is shown in Fig. 2 in the geopotential values. The structure was observed during more then 10 days on the same place. The background disturbances were much more dynamic with characteristic time scale about 3–4 days.

Results and conclusions

The study of the nonlinear vortex formation in the Earth's atmosphere known as a blocking anticyclone is proposed. It is shown that this synoptic-scale structure can be considered in the geostrophic approximation in terms of the Charney-Obukhov equation. The numerical scheme based on the full reduction method is proposed to study the dynamics of the initial perturbations. The dominant role of the vector nonlinearity for the vortex structures stability is shown. In the absence of the nonlinearity the vortex structures are unstable and quickly decompose into the linear Rossby waves. The differences between linear and nonlinear anticyclone structures were studied using the data from Atmospheric Research NCEP/NCAR data base. Blocking anticyclones exist during a long time (typical lifetime is from 5 days to a month), while linear anticyclones exist during few days (up to 5 days). Blocking anticyclones can stay for a long time on the same place (Fig. 2). Conventional synoptic anticyclones usually move eastward, while blocking anticyclones

[1] http://www.cdc.noaa.gov/cdc/data.ncep.reanalysis.pressure.html

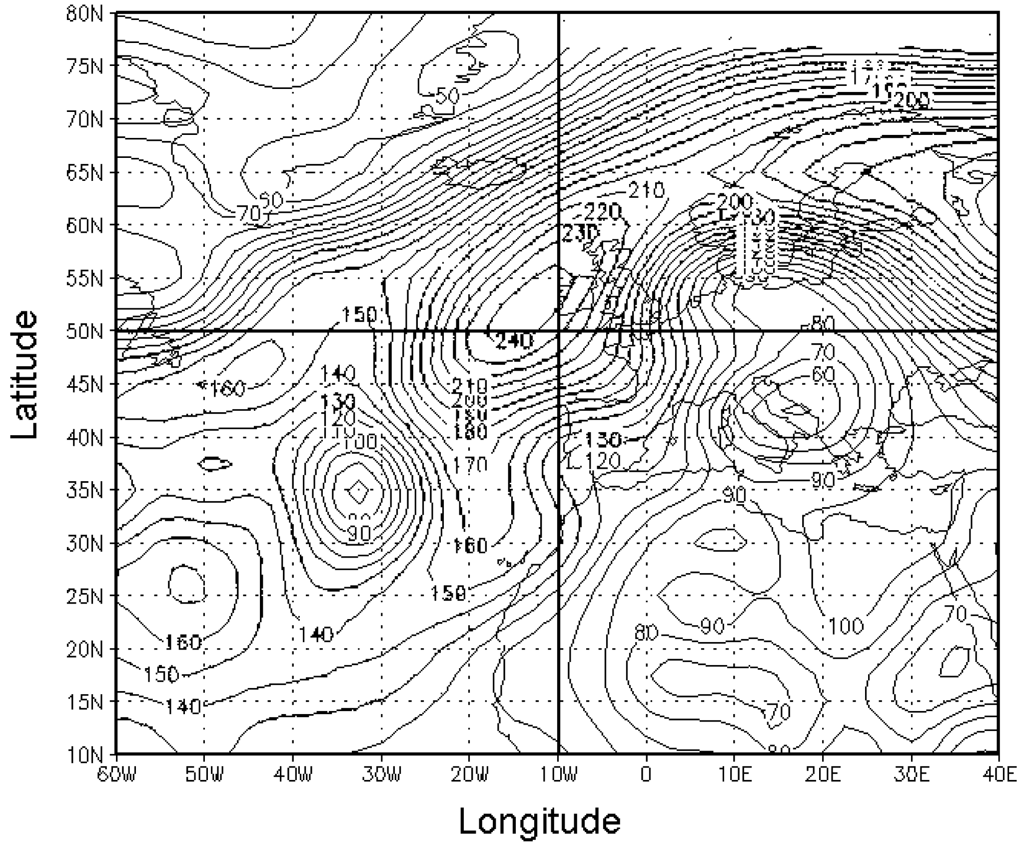

Figure 2: The geopotetial map that shows location of blocking anticyclone on 17.09.2001.

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Acknowledgement

We would like to thank Atmospheric Research NCEP / NCAR data service for atmospheric data.

References

[1] Arakawa A. J. Comput. Phys., V. 135, pp. 103-114 (1997)

[2] Horton W., Hasegawa A. Chaos: An Interdisciplinary J. of Nonlinear Science, V. 4, No. 2, pp. 227-251 (1994)

[3] Monin A. S., Zhikharev G. M. Advances in Physical Sciences, V. 160, no. 5, pp. 1-48 (1990)

[4] Nezlin M.V. Advances in Physical Sciences, V. 150, no. 1, pp. 3-60 (1986)

[5] Petviashvili V. I., Pokhotelov V. A. Solitary waves in planetary atmospheres and oceans. Energoatomizdat, Moscow (1989)