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Electronic density of states for two-dimensional system in uniform magnetic and Aharonov–Bohm fields

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We study two-dimensional electronic gas (2DEG) in the background of the Aharonov–Bohm and constant magnetic fields. The problem of ambiguity of the solutions of the Schrödinger equation is investigated by introducing a finite radius of the flux tube, which then set to zero. Wave functions and spectrum of the 2DEG Hamiltonian are used to construct an expression for the local density of states (LDOS). We obtain that LDOS has a depletion near the origin of the vortex and new peaks, which can't be explained by using Landau levels theory.

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1. Introduction

The energy dispersion $E(\mathbf{k}) = \hbar^2 k^2 / 2M$ of the quasiparticle excitations when the homogeneous magnetic field B is applied perpendicular to its two-dimensional plane transforms into the discrete Landau levels (LLs)

$$E_n = \hbar\omega_c \left(n + \frac{1}{2} \right), \quad n = 0, 1, 2 \ldots, \tag{1}$$

observed, for example, in semiconducting geterostructures. Here $\mathbf{k}$ is the momentum and ω_c is the cyclotron frequency.

In general the inhomogeneous magnetic perturbation can be presented as a sum of a constant (averaged over the system) field and field localized in some regions of the two-dimensional system. A limiting case of the perturbation can be presented by the Aharonov–Bohm field which is created by an infinitely long and infinitesimally thin solenoid.

The purpose of the present paper is to study the two-dimensional electronic excitations in the field consisting of the Aharonov–Bohm flux and a constant background magnetic field. As in the previous publication [1], where we studied the Aharonov–Bohm flux only, our main goal is the investigation of the local density of states (LDOS). We find that demonstrated in Ref. 1 rather peculiar behavior of LDOS theory with Aharonov–Bohm field persists in the presence of the constant background field. We expect that this behavior can be observed in scanning tunneling spectroscopy measurements for different metallic thin films penetrated by vortices from type-II superconductor on top of them [2].

In the previous publication we considered idealized picture when the vortex is single and there is no impact from other Abrikosov vortices. Now the constant background field is supposed to mimic the impact of the other vortices penetrating the film. It is worth to stress that devices like this with a superconducting film grown on top of a semiconducting heterojunction (such as GaAs/AlGaAs) hosting a 2DEG have in fact been fabricated twenty years ago [3,4].

The paper is organized as follows. In Sec. 2 we introduce the model Hamiltonian and discuss the configuration of the magnetic field and the regularization of the Aharonov–Bohm potential used in this work. In Sec. 3 solutions of the Schrödinger equation are obtained. In the Sec. 4 we consider the general presentation for LDOS difference. In the Sec. 5 we begin with a simpler case of the DOS and return to the LDOS in Sec. 6. In Sec. 7 our final results are summarized.

We dedicate this work to the prominent Soviet and Ukrainian theoretical physicist Emanuil Kaner, whose main research and best-known achievements are associated with the study of metals and metal systems in a magnetic field.

2. Models and main notations

The 2D nonrelativistic (Schrödinger) Hamiltonian can be expressed in the standard form

$$H_S = -\frac{\hbar^2}{2M}(D_1^2 + D_2^2), \tag{2}$$

where $D_j = \nabla_j + ie/\hbar c A_j$, $j = 1,2$ with the vector potential $\mathbf{A}$, Planck's constant $\hbar$ and the velocity of light c describes a spinless particle with a mass M and charge $-e < 0$.

As in the previous article to avoid the mathematical difficulties related to a singular nature of the Aharonov–Bohm potential at the origin, we consider a regularized potential [5,6] which depends on the dimensional parameter R:

$$\mathbf{A}(\mathbf{r}) = A_\varphi(r)\mathbf{e}_\varphi, \qquad A_\varphi(r) = \frac{Br}{2} + \frac{\Phi_0\eta}{2\pi r}\theta(r-R), \tag{3}$$

where $\mathbf{r} = (r,\varphi,z)$, $\Phi_0\eta$ is the flux of the vortex expressed via magnetic flux quantum of the electron $\Phi_0 = hc/e$ with $\eta \in [0,1)$. The value $\eta = 1/2$ corresponds to the flux of the Abrikosov's vortex. The corresponding magnetic field

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A} = \left(B + \frac{\eta\Phi_0}{2\pi R}\delta(r-R)\right)\mathbf{e}_z. \tag{4}$$

The radius R of the flux tube determines the region $r > R$ where the regularized potential coincides with the potential of the problem with Aharonov–Bohm potential, while for $r < R$ it describes a particle moving in a constant magnetic field. The solution of the problem is found by matching the solutions obtained in these regions. The limit $R \to 0$ can taken at the end and allows to avoid the formal complications. As was shown in Ref. 6, the final answer does not depend on the specific form of the regularizing potential provided that the profile of the magnetic field is nonsingular at the origin.

3. Spectrum and eigenfunctions

In this section we consider the solutions of the Schrödinger equation

$$H_S\psi(\mathbf{r}) = E\psi(\mathbf{r}) \tag{5}$$

in polar coordinates $\mathbf{r} = (r,\varphi)$ and using them to find the full and local DOS. Technically to obtain the solutions of Eq. (5) in the regularized potential (3) one should solve this equation in two regions $r < R$ and $r > R$. Since in the first domain $r < R$ the potential is nonsingular, only a regular in the limit $r \to 0$ solution of the radial differential equation is admissible. In the second domain $r > R$ the solution contains both regular and singular in the limit $r \to 0$ terms. The values of the relative weights of them can be found by matching radial components and their derivatives at $r = R$. Finally, it turns out that in the limit $R \to 0$ only the regular solution survives and the wavefunction takes the form

$$\psi_{n,m}(r,\varphi) = A_{n,m} e^{im\varphi} y^{|m+\eta|/2} e^{-y/2} L_n^{|m+\eta|}(y), \tag{6}$$

which also follows from the Schrödinger equation with a singular vortex [7]. Here the dimensionless variable $y \equiv r^2/(2l^2)$ is expressed via the magnetic length $l = (\hbar c/eB)^{1/2}$, $L_n^\alpha(y)$ is the generalized Laguerre polynomial and the normalization constant $A_{n,m}$ is given by

$$A_{n,m}^2 = \frac{n!}{2\pi l^2 \Gamma(n + |m+\eta| + 1)}. \tag{7}$$

The corresponding to the wave function (6) eigenenergy is equal to

$$E_{n,m} = \frac{\hbar\omega_c}{2}(2n + 1 + |m+\eta| + m + \eta), \tag{8}$$

where the cyclotron frequency $\omega_c = eB/Mc$, the radial quantum number $n = 0,1,\ldots$, and the azimuthal quantum number $m = -\infty,\ldots,-1,0,1,\ldots,\infty$. In what follows it is convenient to express all energies of the problem in terms of the energy $E_0 \equiv \hbar\omega_c/2$.

Having the wave function we can calculate the LDOS using the representation

$$N(\mathbf{r},E,B) = \sum_{n,m} |\psi_{n,m}(\mathbf{r})|^2 \delta(E - E_{n,m}). \tag{9}$$

In contrast to the previous article [1] the presence of a constant magnetic field makes all energy spectra discrete that demands some regularization of the δ-function in Eq. (9). For this purpose we introduce widening of the LLs to a Lorentzian shape:

$$\delta(E - E_{n,m}) \to \frac{1}{\pi}\,\mathrm{Im}\,\frac{1}{E_{n,m} - E - i\Gamma}, \tag{10}$$

where Γ is the LLs width. Such a simple broadening of LLs with a constant Γ was found to be a rather good approximation valid in not very strong magnetic fields [8].

To illustrate the method of calculation in Sec. 4 we derive the LDOS for the simplest case $(\eta = 0)$ of the constant magnetic field without vortex

$$N_0^S(E,B) = -\frac{N_0^S}{\pi}\,\mathrm{Im}\,\psi\left(\frac{1}{2} - \frac{E + i\Gamma}{\hbar\omega_c}\right). \tag{11}$$

Here $N_0^S = M/(2\pi\hbar^2)$ is a free DOS of 2DEG per spin and unit area and we omitted $\mathbf{r}$-dependence of the LDOS, because it is absent in the homogeneous field. One can readily obtain Eq. (11) in a much simplier way [9,10] starting from the usual Landau spectrum (1) which follows from the spectrum (8) for $\eta = 0$, when one relabels $n + (|m| + m)/2 \to n$. Here the relabeled n corresponds to the LL index rather than the radial quantum number. Nevertheless, in Sec. 4 we proceeded from Eq. (8) to illustrate how deal with the spectrum which is also dependent on the azimuthal quantum number m. As seen in Fig. 1,*a* on the dashed curve, Eq. (11) contains usual magnetic oscillations which result in the de Haas–van Alphen effect. One can extract them analytically using the relation

$$\psi(-z) = \psi(z) + \frac{1}{z} + \pi \cot(\pi z). \tag{12}$$

In a similar fashion we obtain in Sec. 4 the expression for the LDOS perturbation, $\Delta N_\eta^{\mathrm{S}}(\mathbf{r}, E, B) = N_\eta^{\mathrm{S}}(\mathbf{r}, E, B) - N_0^{\mathrm{S}}(\mathbf{r}, E)$ induced by the vortex

$$\Delta N_\eta^{\mathrm{S}}(\mathbf{r}, E, B) = -\frac{M}{(\pi\hbar)^2}\frac{\sin \pi\eta}{\pi}\mathrm{Im}\left[\int_0^\infty d\beta\, \mathrm{e}^{-(\delta+\beta)}\mathrm{e}^{-\beta z} \times \frac{\mathrm{e}^{-y\coth(\delta+\beta)}}{1-\mathrm{e}^{-2(\delta+\beta)}}\int_{-\infty}^{\infty} d\omega\, \mathrm{e}^{-y\cosh\omega/\sinh(\delta+\beta)}\,\frac{\mathrm{e}^{-\eta(\delta+\beta+\omega)}}{1+\mathrm{e}^{-(\delta+\beta+\omega)}}\right]. \tag{13}$$

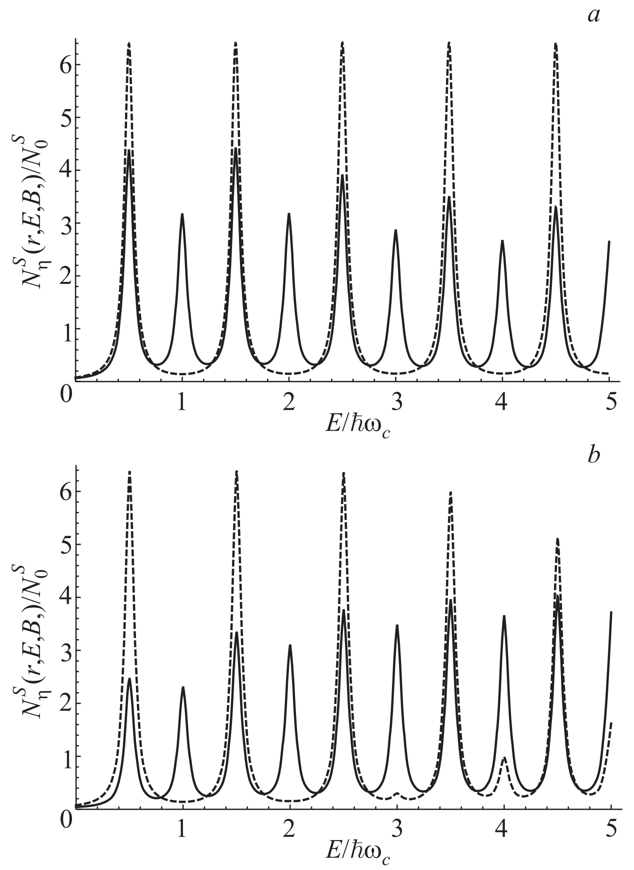

Fig. 1. The normalized full LDOS $N_\eta^{\mathrm{S}}(r, E, B)/N_0^{\mathrm{S}}$ as a function of E in the units of $\hbar\omega_c$. (*a*) For $\eta = 0$ (no vortex and LDOS is $\mathbf{r}$-independent, dashed curve) and $\eta = 1/2$ for $r = l$ (solid curve). (*b*) Both lines for $\eta = 1/2$, $r = 0.5l$ (solid curve) and $r = 5l$ (dashed curve). In all cases the width $\Gamma = 0.05\hbar\omega_c$.

Here $N_\eta^{\mathrm{S}}(\mathbf{r}, E, B)$ is the LDOS in the presence of the constant field and vortex and $N_0^{\mathrm{S}}(\mathbf{r}, E, B)$ is the LDOS in the constant magnetic field without vortex (the argument $\mathbf{r}$ is present to distinguish the LDOS from the DOS). This expression has to be calculated for $z > 0$ with the analytic continuation $z \to -(E + i\Gamma)/E_0$ dome at the end of the calculation. The representation (13) for the LDOS is our starting point for the analysis of the LDOS and DOS.

4. Preliminary calculation of the local density of states

Setting $\eta = 0$ in Eq. (6) one obtains the solution of the Schrödinger equation for $B = \mathrm{const}$ without vortex. Substituting this solution in the LDOS definition (9) and taking into account the widening of the LLs (10) we represent the LDOS as a double sum

$$N_0^{\mathrm{S}}(\mathbf{r}, E, B) = \frac{1}{\pi}\mathrm{Im}\sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty} A_{n,m}^2 y^{|m|}\frac{\mathrm{e}^{-y}[L_n^{|m|}(y)]^2}{E_{n,m} + E_0 z}, \tag{14}$$

where in the denominator we introduced the dimensionless variable $z = -(E + i\Gamma)/E_0$ with the characteristic energy E_0 defined below Eq. (8). To calculate the sum in Eq. (14) it is convenient to represent its last factor as an exponent

$$\frac{\mathrm{e}^{-\delta(2n+|m|+m+1)}}{E_{n,m} + E_0 z} = \frac{1}{E_0}\int_0^\infty d\beta\, \mathrm{e}^{-(\beta+\delta)(2n+|m|+m+1)}\mathrm{e}^{-\beta z}. \tag{15}$$

Here we also introduced the regularizing exponential factor with $\delta > 0$ which makes the sum convergent and which will be set to 0 at the end. Then the LDOS acquires the form

$$N_0^{\mathrm{S}}(\mathbf{r}, E, B) = \frac{M}{\pi^2\hbar^2}\mathrm{Im}\left[\int_0^\infty d\beta\, \mathrm{e}^{-(\delta+\beta)}\mathrm{e}^{-\beta z}\sum_{m=-\infty}^{\infty}\mathrm{e}^{-y}y^{|m|} \times \mathrm{e}^{-(\beta+\delta)(|m|+m)}\sum_{n=0}^{\infty}\frac{n!\,\mathrm{e}^{-2(\beta+\delta)n}}{\Gamma(n+|m|+1)}[L_n^{|m|}(y)]^2\right]. \tag{16}$$

We operate with the representation (16) in the following way. First we consider its analytic continuation for $z > 0$ and perform the calculation. Then to obtain the LDOS we return to the imaginary values $z \to -(E + i\Gamma)/E_0$ and evaluate the imaginary part. Using Eq. (10.12.20) from [11]

$$\sum_{n=0}^{\infty}\frac{n!}{\Gamma(n+\alpha+1)}L_n^\alpha(x)L_n^\alpha(y)z^n = (1-z)^{-1}(xyz)^{-\alpha/2} \times \exp\left(-z\frac{x+y}{1-z}\right)I_\alpha\left(2\frac{\sqrt{xyz}}{1-z}\right), \quad |z| < 1, \tag{17}$$

where I_α is modified Bessel function, we find the sum over n in Eq. (16)

$$N_0^{\mathrm{S}}(\mathbf{r},E,B)=\frac{M}{\pi^2\hbar^2}\,\mathrm{Im}\left[\int_0^\infty d\beta\,\mathrm{e}^{-(\delta+\beta)}\mathrm{e}^{-\beta z}\frac{\mathrm{e}^{-y\coth(\delta+\beta)}}{1-\mathrm{e}^{-2(\delta+\beta)}}\times\right.$$

$$\left.\times\sum_{m=-\infty}^{\infty}\mathrm{e}^{-(\delta+\beta)m}I_{|m|}\left(\frac{y}{\sinh(\delta+\beta)}\right)\right]. \qquad (18)$$

The remaining summation over m in Eq. (18) can be done using the property of the modified Bessel function $I_m(x)=I_{-m}(x)$, and that its generating function is [11]

$$\sum_{m=-\infty}^{\infty}z^m I_m(x)=\exp\left(\frac{x}{2}[z+1/z]\right). \qquad (19)$$

We obtain

$$N_0^{\mathrm{S}}(E,B)=\frac{M}{(\pi\hbar)^2}\,\mathrm{Im}\left[\int_0^\infty d\beta\,\frac{\mathrm{e}^{-(\delta+\beta)}\mathrm{e}^{-\beta z}}{1-\mathrm{e}^{-2(\delta+\beta)}}\right]. \qquad (20)$$

Notice that from the last expression one can explicitly observe that it does not depend on y, i.e., in a constant magnetic field the LDOS is position independent. Introducing a new variable $x=2(\delta+\beta)$ we can rewrite the last expression as follows

$$N_0^{\mathrm{S}}(E,B)=-\frac{M}{2(\pi\hbar)^2}\,\mathrm{Im}\left[\mathrm{e}^{\delta z}\int_{2\delta}^\infty dx\,\frac{\mathrm{e}^{-x}-\mathrm{e}^{-x(z+1)/2}}{1-\mathrm{e}^{-x}}-\right.$$

$$\left.-\mathrm{e}^{\delta z}\int_{2\delta}^\infty dx\frac{\mathrm{e}^{-x}}{1-\mathrm{e}^{-x}}\right]. \qquad (21)$$

In the limit $\delta\to 0$ the second term of Eq. (21) remains real irrespectively the value of z, while the first term gives the integral representation of the digamma function [12]

$$\psi(z)=-\gamma+\int_0^\infty dt\,\frac{\mathrm{e}^{-t}-\mathrm{e}^{-tz}}{1-\mathrm{e}^{-t}},\quad \mathrm{Re}\,z>0, \qquad (22)$$

and the final expression for the LDOS is given by Eq. (11).

Now we generalize these results for the case when the vortex is present. Repeating the steps that led us from Eq. (14) to Eq. (18) we obtain

$$N_0^{\mathrm{S}}(\mathbf{r},E,B)=\frac{M}{\pi^2\hbar^2}\,\mathrm{Im}\left[\int_0^\infty d\beta\,\mathrm{e}^{-(\delta+\beta)}\mathrm{e}^{-\beta z}\frac{\mathrm{e}^{-y\coth(\delta+\beta)}}{1-\mathrm{e}^{-2(\delta+\beta)}}\times\right.$$

$$\left.\times\sum_{m=-\infty}^{\infty}\mathrm{e}^{-(\delta+\beta)(m+\eta)}I_{|m+\eta|}\left(\frac{y}{\sinh(\delta+\beta)}\right)\right]. \qquad (23)$$

Using the method described in Ref. 13 we obtained the formula which allows to find the sum over m in Eq. (23)

$$\sum_{m=-\infty}^{\infty}\mathrm{e}^{-(\delta+\beta)(m+\eta)}I_{|m+\eta|}\left(\frac{y}{\sinh(\delta+\beta)}\right)=\mathrm{e}^{y\coth(\delta+\beta)}-\frac{\sin\pi\eta}{\pi}\times$$

$$\times\int_{-\infty}^{\infty}d\omega\,\mathrm{e}^{-y\cosh\omega/\sinh(\delta+\beta)}\frac{\mathrm{e}^{-\eta(\delta+\beta+\omega)}}{1+\mathrm{e}^{-(\delta+\beta+\omega)}}. \qquad (24)$$

The first term on the r.h.s. of the last equation corresponds to the LDOS without the vortex which was considered above, so that we can concentrate on the second term. Substituting it in Eq. (23) we arrive at Eq. (13) for $\Delta N_\eta^{\mathrm{S}}(\mathbf{r},E,B)$.

5. The density of states

While in the constant magnetic field the LDOS is position independent and is related to the full DOS by the 2D volume (area) of the system factor V_{2D}, this is not so in the presence of the vortex when the LDOS is position dependent. Then the full DOS per spin projection is obtained from the LDOS (9) by integrating over the space coordinates

$$N_\eta^{\mathrm{S}}(E,B)=\int_0^{2\pi}d\varphi\int_0^\infty rdr\,N_\eta(\mathbf{r},E,B). \qquad (25)$$

Substituting Eq. (13) in the definition (25) and integrating over the spatial coordinates we obtain

$$\Delta N_\eta^{\mathrm{S}}(E,B)=-\sin\pi\eta\frac{Ml^2}{2(\pi\hbar)^2}\mathrm{Im}\left[\int_0^\infty d\beta\int_{-\infty}^\infty d\upsilon\times\right.$$

$$\left.\times\frac{\mathrm{e}^{-\beta z}}{\cosh(\upsilon/2)\cosh(\beta+\delta-\upsilon/2)}\frac{\mathrm{e}^{-\eta\upsilon}}{1+\mathrm{e}^{-\upsilon}}\right], \qquad (26)$$

where we introduced the new variable $\upsilon=\omega+\beta+\delta$. This double integral can be rewritten using the new variables $t=\mathrm{e}^{-2\beta}$, $x=\mathrm{e}^{\upsilon}$ as follows

$$\Delta N_\eta^{\mathrm{S}}(E,B)=-\sin\pi\eta\frac{Ml^2\mathrm{e}^{-\delta}}{(\pi\hbar)^2}\times$$

$$\times\,\mathrm{Im}\left[\int_0^1 dt\,t^{(z-1)/2}\int_0^\infty\frac{dx x^{1-\eta}}{(1+x)^2(1+t\mathrm{e}^{-2\delta}x)}\right], \qquad (27)$$

where the second integral can be calculated using the residue theory

$$\int_0^\infty\frac{dx x^{1-\eta}}{(1+x)^2(1+t\mathrm{e}^{-2\delta}x)}=$$

$$=\frac{\pi}{\sin\pi\eta}\frac{1-\eta+\eta\mathrm{e}^{-2\delta}t-\mathrm{e}^{-2\eta\delta}t^\eta}{(1-\mathrm{e}^{-2\delta}t)^2}. \qquad (28)$$

Then the remaining integral is expressed via the hypergeometric function

$$\int_0^1 dt t^{(z-1)/2}\frac{1-\eta+\eta e^{-2\delta}t-e^{-2\eta\delta}t^{\eta}}{(1-e^{-2\delta}t)^2}=\frac{1-e^{-2\delta\eta}}{1-e^{-2\delta}}-$$
$$-(z+2\eta-1)\left[\frac{1}{1+z}{}_2F_1\left(1,\frac{1+z}{2};\frac{3+z}{2};e^{-2\delta}\right)-\right. \quad (29)$$
$$\left.-\frac{e^{-2\delta\eta}}{1+z+2\eta}{}_2F_1\left(1,\frac{1+z}{2}+\eta;\frac{3+z}{2}+\eta;e^{-2\delta}\right)\right].$$

Now we use the series representation of hypergeometric functions in Eq. (29)

$$\frac{e^{-2\delta\eta}}{z+2\eta+1}{}_2F_1\left(1,\frac{1+z}{2}+\eta,\frac{3+z}{2}+\eta,e^{-2\delta}\right)=$$
$$=\sum_{n=0}^{\infty}\frac{e^{-2\delta(n+\eta)}}{z+1+2\eta+2n}=e^{\delta(z+1)}\sum_{n=0}^{\infty}\int_{\delta}^{\infty}dx\, e^{-x(2n+2\eta+z+1)}=$$
$$=e^{\delta(z+1)}\int_{\delta}^{\infty}dx\,\frac{e^{-x(z+1+2\eta)}}{1-e^{-2x}}, \quad (30)$$

where the first one in Eq. (29) is recovered for $\eta=0$. We observe that the presence of finite $\delta>0$ makes the hypergeometric series well defined, but at the end of the calculation the limit $\delta\to 0$ can already be taken. Then taking into account the integral representation of the digamma function (22) [similarly to Eq. (21)] one can express the DOS (27) in the following simple form

$$\Delta N_{\eta}^{S}(E,B)=\frac{Ml^2}{2\pi\hbar^2}\mathrm{Im}\left\{(z+2\eta-1)\times\right.$$
$$\left.\times\left[\psi\left(\frac{z+1}{2}+\eta\right)-\psi\left(\frac{z+1}{2}\right)\right]\right\} \quad (31)$$

which after the analytic continuation $z\to -2(E+i\Gamma)/(\hbar\omega_c)$ takes the final form

$$\Delta N_{\eta}^{S}(E,B)=\frac{1}{\pi\hbar\omega_c}\mathrm{Im}\left\{\left(\frac{1}{2}+\frac{E+i\Gamma}{\hbar\omega_c}-\eta\right)\times\right.$$
$$\left.\times\left[\psi\left(\frac{1}{2}-\frac{E+i\Gamma}{\hbar\omega_c}\right)-\psi\left(\frac{1}{2}-\frac{E+i\Gamma}{\hbar\omega_c}+\eta\right)\right]\right\}. \quad (32)$$

Since the digamma function $\psi(z)$ has simple poles for $z=0,-1,-2,\ldots$ it is easy to see in the clean limit $\Gamma\to 0$ the DOS difference (32) reduces to a set of δ-peaks corresponding to the LLs

$$\Delta N_{\eta}^{S}(E,B)=-\sum_{n=0}^{\infty}(n+1-\eta)\delta\left(E-\hbar\omega_c\left(n+\frac{1}{2}\right)\right)+$$
$$+\sum_{n=0}^{\infty}(n+1)\delta\left(E-\hbar\omega_c\left(n+\frac{1}{2}+\eta\right)\right). \quad (33)$$

The physical meaning of (33) is that [14] on each LL $E_{n,m<0}=\hbar\omega_c(n+1/2)$, $n+1-\eta$ states disappear and $n+1$ appear at the energy $E_{n+m\to n,m\geq 0}=\hbar\omega_c(n+1/2+\eta)$.

Using the asymptotic expansion

$$\psi(z)=\ln z-\frac{1}{2z}-\frac{1}{12z^2}+O\left(\frac{1}{z^4}\right) \quad (34)$$

one can take in Eq. (32) the limit of zero field, $B\to 0$ and reproduce the Aharonov–Bohm depletion of the DOS [1,14,15] at the bottom of the spectrum

$$\Delta N_{\eta}^{S}(E,B=0)=N_{\eta}^{S}(E,B=0)-V_{2D}N_0^{S}=-\frac{1}{2}\eta(1-\eta)\delta(E) \quad (35)$$

caused by an isolated vortex. Integrating Eqs. (35) and (33) (with an appropriate regularization) one can check that the total deficit of the states induced by the vortex

$$\int_{-\infty}^{\infty}dE\,\Delta N_{\eta}^{S}(E,B)=-\frac{1}{2}\eta(1-\eta) \quad (36)$$

does not depend on the strength B of the nonsingular background field.

6. The local density of states

Although the regularization parameter δ is important for the calculation of the DOS, the integrand of Eq. (13) remains regular even in the limit $\delta\to 0$. Therefore we can take this limit and rewrite Eq. (13) as follows

$$\Delta N_{\eta}^{S}(\mathbf{r},E,B)=-\frac{M}{(\pi\hbar)^2}\frac{\sin\pi\eta}{2\pi}\times$$
$$\times\,\mathrm{Im}\left[I\left(y,z\to-\frac{E+i\Gamma}{E_0},\eta\right)\right], \quad (37)$$

where

$$I(y,z,\eta)=\int_0^{\infty}d\beta\, e^{-\beta z}\frac{e^{-y\coth\beta}}{\sinh\beta}\times$$
$$\times\int_{-\infty}^{\infty}d\omega\, e^{-y\cosh\omega/\sinh\beta}\frac{e^{-\eta(\omega+\beta)}}{1+e^{-(\omega+\beta)}}, \quad (38)$$

and the variable y describes the spatial dependence. Although the integrals in Eq. (38) can be evaluated numerically, this computation becomes troublesome when as above the analytic continuation from $z>0$ to the complex values $z\to-(E+i\Gamma)/E_0$ is done before the numerical integration.

As in Ref. 1 we observe that it is simpler to calculate integrals with the derivative $dI(y,z,\eta)/dy$ representing the function $I(y,z,\eta)$ in the form

$$I(y,z,\eta)=-\int_y^{\infty}\frac{dI(Q,z,\eta)}{dQ}, \quad (39)$$

where we used that $I(\infty, z, \eta) = 0$. The derivative $dI(Q,z,\eta)/dQ$ contains two terms

$$\frac{dI(Q,z,\eta)}{dQ} = \frac{dI_1(Q,z,\eta)}{dQ} + \frac{dI_2(Q,z,\eta)}{dQ}, \tag{40}$$

where

$$\begin{aligned}\frac{dI_1}{dQ} &= -\frac{1}{2}\int_0^\infty d\beta\, \mathrm{e}^{-\beta(z+\eta)} \frac{\mathrm{e}^{-Q\coth\beta}}{\sinh^2\beta}\times \\ &\times \int_{-\infty}^{\infty} d\omega\, \mathrm{e}^{-Q\cosh\omega/\sinh\beta}\mathrm{e}^{-(\eta-1)\omega}, \\ \frac{dI_2}{dQ} &= -\frac{1}{2}\int_0^\infty d\beta\, \mathrm{e}^{-\beta(z+\eta-1)} \frac{\mathrm{e}^{-Q\coth\beta}}{\sinh^2\beta}\times \\ &\times \int_{-\infty}^{\infty} d\omega\, \mathrm{e}^{-Q\cosh\omega/\sinh\beta}\mathrm{e}^{-\eta\omega}.\end{aligned} \tag{41}$$

Using the integral representation of the MacDonald function $K_\nu(x)$ (Ref. 11)

$$K_\nu(x) = \frac{1}{2}\int_{-\infty}^{\infty} \mathrm{e}^{-x\cosh\omega - \nu x} dx, \tag{42}$$

we obtain

$$\frac{dI_1}{dQ} = -\int_0^\infty d\beta\, \mathrm{e}^{-\beta(z+\eta)} \frac{\mathrm{e}^{-Q\coth\beta}}{\sinh^2\beta} K_{1-\eta}(Q/\sinh\beta), \tag{43}$$

and

$$\frac{dI_2}{dQ} = -\int_0^\infty d\beta\, \mathrm{e}^{-\beta(z+\eta-1)} \frac{\mathrm{e}^{-Q\coth\beta}}{\sinh^2\beta} K_{\eta}(Q/\sinh\beta). \tag{44}$$

Now introducing a new variable t via $\mathrm{e}^{-2\beta} = t/(1+t)$ we get

$$\begin{aligned}\frac{dI_1}{dQ} &= -2\mathrm{e}^{-Q}\int_0^\infty dt\, t^{(z+\eta)/2}(1+t)^{-(z+\eta)/2}\mathrm{e}^{-2Qt}\times \\ &\times K_{1-\eta}(2Q\sqrt{t(1+t)}),\end{aligned} \tag{45}$$

and

$$\begin{aligned}\frac{dI_2}{dQ} &= -2\mathrm{e}^{-Q}\int_0^\infty dt\, t^{(z+\eta-1)/2}(1+t)^{-(z+\eta-1)/2}\mathrm{e}^{-2Qt}\times \\ &\times K_{\eta}(2Q\sqrt{t(1+t)}).\end{aligned} \tag{46}$$

To integrate over t in Eqs. (45) and (46), we use the integral (2.16.10.2) from Ref. 16

$$\begin{aligned}&\int_0^\infty dx\, \frac{x^{\rho-1}}{(x+z)^\rho}\mathrm{e}^{-px}K_\nu(c\sqrt{x^2+xz}) = \\ &= \frac{1}{cz}\Gamma\left(\rho+\frac{\nu}{2}\right)\Gamma\left(\rho-\frac{\nu}{2}\right)\mathrm{e}^{pz/2}\times \\ &\times W_{1/2-\rho,\nu/2}(z_+/2)W_{1/2-\rho,\nu/2}(z_-/2), \\ &z_\pm = p \pm \sqrt{p^2-c^2}, \\ &\mathrm{Re}\,(p+c) > 0,\ \ |\arg z| < \pi,\ \ 2\,\mathrm{Re}\,\rho > |\mathrm{Re}\,\nu|,\end{aligned} \tag{47}$$

where $W_{\lambda,\mu}(z)$ is the Whittaker function. To adapt Eq. (47) to the form of Eqs. (45) and (46), we have to set $z = 1$, differentiate the result over p and then take the limit $p \to c$. This gives

$$\begin{aligned}&\int_0^\infty dx\, \frac{x^{\rho}}{(x+1)^\rho}\mathrm{e}^{-cx}K_\nu(c\sqrt{x(x+1)}) = \\ &= -\frac{1}{2}\Gamma\left(\rho+\frac{\nu}{2}\right)\Gamma\left(\rho-\frac{\nu}{2}\right)\mathrm{e}^{c/2}G_{1/2-\rho,\nu/2}\left(\frac{c}{2}\right),\end{aligned} \tag{48}$$

where the function $G_{\lambda,\mu}(Q)$ is defined as follows:

$$\begin{aligned}G_{\lambda,\mu}(Q) &= \frac{1}{2Q}W^2_{\lambda,\mu}(Q) + \frac{1}{Q}W_{\lambda,\mu}(Q)W'_{\lambda,\mu}(Q) + \\ &+ W''_{\lambda,\mu}(Q)W_{\lambda,\mu}(Q) - W'^2_{\lambda,\mu}(Q).\end{aligned} \tag{49}$$

Accordingly, we obtain that

$$\frac{dI_1}{dQ} = \Gamma\left(\frac{z+1}{2}\right)\Gamma\left(\frac{z+2\eta-1}{2}\right)G_{(1-z-\eta)/2,(1-\eta)/2}(Q) \tag{50}$$

and

$$\frac{dI_2}{dQ} = \Gamma\left(\frac{z-1}{2}\right)\Gamma\left(\frac{z+2\eta-1}{2}\right)G_{(2-z-\eta)/2,\eta/2}(Q). \tag{51}$$

Now using the differential equation

$$W''_{\lambda,\mu}(z) + \left(-\frac{1}{4}+\frac{\lambda}{z}+\frac{1/4-\mu^2}{z^2}\right)W_{\lambda,\mu}(z) = 0 \tag{52}$$

and the recursion formula

$$z\frac{d}{dz}W_{\lambda,\mu}(z) = \left(\lambda-\frac{z}{2}\right)W_{\lambda,\mu}(z) - \left[\mu^2-\left(\lambda-\frac{1}{2}\right)^2\right]W_{\lambda-1,\mu}(z) \tag{53}$$

for the Whittaker function [17] one can transform $G_{\lambda,\mu}(Q)$ to the form

$$G_{\lambda,\mu}(Q) = \frac{\mu^2 + (\lambda - 1/2)^2}{Q^2} W_{\lambda,\mu}^2(Q) - \\ - \frac{[\mu^2 - (\lambda - 1/2)^2]^2}{Q^2} W_{\lambda-1,\mu}^2(Q) - \\ - \frac{\mu^2 - (\lambda - 1/2)^2}{Q} W_{\lambda,\mu}(Q) W_{\lambda-1,\mu}(Q) - \\ - \frac{2\lambda - 1}{2Q} W_{\lambda,\mu}^2(Q) - (\lambda - 1/2)\left(\frac{W_{\lambda,\mu}^2(Q)}{Q}\right)'. \quad (54)$$

To obtain the function $F_{\lambda,\mu}(Q) = \int dQ G_{\lambda,\mu}(Q)$ we employ the relationships

$$\int \frac{dQ}{Q} W_{\lambda,\mu}(Q) W_{\rho,\mu}(Q) = \\ = \frac{1}{\rho - \lambda}[W'_{\lambda,\mu}(Q) W_{\rho,\mu}(Q) - W'_{\rho,\mu}(Q) W_{\lambda,\mu}(Q)], \\ \int \frac{dQ}{Q} W_{\lambda,\mu}(Q) W_{\lambda,\mu}(Q) = \\ = W'_{\lambda,\mu}(Q) \partial_\lambda W_{\lambda,\mu}(Q) - \partial_\lambda W'_{\lambda,\mu}(Q) W_{\lambda,\mu}(Q), \\ \int \frac{dQ}{Q^2} W_{\lambda,\nu}(Q) W_{\lambda,\nu}(Q) = \\ = \frac{1}{2\nu}(\partial_\nu W'_{\lambda,\nu}(Q) W_{\lambda,\nu}(Q) - W'_{\lambda,\nu}(Q) \partial_\nu W_{\lambda,\nu}(Q)) \quad (55)$$

which follow from the differential equation (52) for the Whittaker function. Then using the the recursion formula (53) we arrive at the following result:

$$F_{\lambda,\mu}(Q) = \frac{\mu^2 + (\lambda - 1/2)^2}{2\mu Q}[W_{\lambda+1,\mu}(Q) \partial_\mu W_{\lambda,\mu}(Q) - W_{\lambda,\mu}(Q) \partial_\mu W_{\lambda+1,\mu}(Q)] - \\ - \frac{[\mu^2 - (\lambda - 1/2)^2]^2}{2\mu Q}[W_{\lambda,\mu}(Q) \partial_\mu W_{\lambda-1,\mu}(Q) - W_{\lambda-1,\mu}(Q) \partial_\mu W_{\lambda,\mu}(Q)] + \\ + \frac{[\mu^2 - (\lambda - 1/2)^2]}{Q}[W_{\lambda,\mu}^2(Q) - W_{\lambda-1,\mu}(Q) W_{\lambda,\mu}(Q) - W_{\lambda-1,\mu}(Q) W_{\lambda+1,\mu}(Q)] - \\ - \frac{2\lambda - 1}{2Q}[2W_{\lambda,\mu}^2(Q) - W_{\lambda+1,\mu}(Q) \partial_\lambda W_{\lambda,\mu}(Q) + W_{\lambda,\mu}(Q) \partial_\lambda W_{\lambda+1,\mu}(Q)]. \quad (56)$$

The integral of each term in Eq. (40) is expressed via $F_{\lambda,\mu}(Q)$ with the prefactors given by Eqs. (50) and (51), so that we arrive at the final expression for the function $I(y, z, \eta)$ which was defined in Eq. (39).

Thus our purpose is to derive such a representation for $I(y, z, \eta)$ that it can be easily computed after the analytic continuation is done

$$I(y, z, \eta) = \Gamma\left(\frac{z+1}{2}\right)\Gamma\left(\frac{z + 2\eta - 1}{2}\right) F_{(1-z-\eta)/2,(1-\eta)/2}(y) + \\ + \Gamma\left(\frac{z-1}{2}\right)\Gamma\left(\frac{z + 2\eta - 1}{2}\right) F_{(2-z-\eta)/2,\eta/2}(y), \quad (57)$$

where the function $F_{\lambda,\mu}(y)$ is given by Eq. (56). The results of the numerical computation of the LDOS on the base of Eqs. (37) and (57) are shown in Figs. 1 and 2. We emphasize that in Fig. 1 we plot the *total* LDOS $N_\eta^S(r, E, B)$ as a function of energy E for fixed values of r and in Fig. 2 the LDOS *difference* $\Delta N_\eta^S(r, E, B)$ as a function of the distance r from the vortex center for fixed values of E is presented. Since Eq. (37) describes the perturbation of the LDOS $\Delta N_\eta^S(r, E, B)$ by the vortex, to obtain the absolute value of the LDOS $N_\eta^S(r, E, B)$ we add to ΔN_η^S its $\eta = 0$ value which is given by Eq. (11). In Fig. 1,*a* we compare already discussed after Eq. (11) case of the constant magnetic field (obviously, there is no *r*-dependence when $\eta = 0$) with the case of Abrikosov's vortex ($\eta = 1/2$) for $r = l$. Although the model we consider is suitable for all values of the distance from the center of the vortex r, there are obvious physical limitations on the possible value of r if the vortex penetrating graphene is coming from a type-II superconductor.

First of all, r cannot smaller than the vortex core which is at least on the order of magnitude larger than the lattice constant r_0. We remind that in the previous paper [1] the

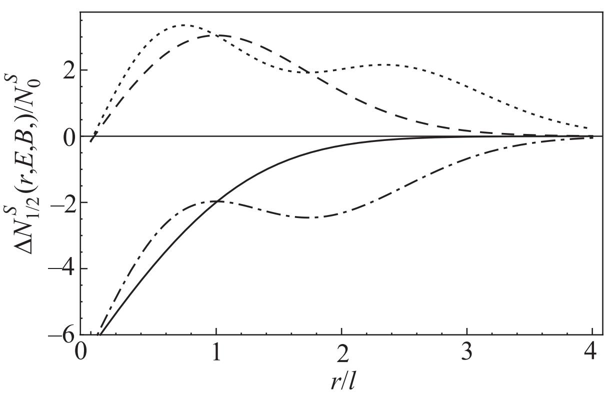

Fig. 2. The normalized LDOS difference $\Delta N_{1/2}^S(r, E, B) / N_0^S$ as a function of the distance r measured in the units of the magnetic length l for for values of $E / \hbar\omega_c =$ 0.5, 1, 1.5, 2 (solid, dashed, dot-dashed, dotted curves, correspondently). The width $\Gamma = 0.05\hbar\omega_c$.

distance r was measured in the units of r_0, because for $B = 0$ there is no such a natural scale as a magnetic length. Secondly, we replace the magnetic field created by the other vortices replacing it by a constant background magnetic field. This approximation may be appropriate if one considers a vicinity of the selected vortex which implies that r has to be less than the intervortex distance l_v. This distance is proportional to the magnetic length [18], $l_v = c\sqrt{\pi}l \approx 1.77l$, where $c \approx 1$ is the geometric factor dependent on the Abrikosov's lattice structure. Thus although one can investigate the regime $r \gg l$ theoretically, in practice it is not accessible.

In Fig. 1,*a* we compare the already discussed after Eq. (11) case of the constant magnetic field with the case when the Abrikosov vortex is also present $(\eta = 1/2)$ for $r = l$. We observe that while for $\eta = 0$ (the dashed curve is, obviously, *r*-independent) only the peaks at half-integers $E/\hbar\omega_c$ are present, for $\eta = 1/2$ the weight of these peaks is reduced and a set of the new peaks at the integers $E/\hbar\omega_c$ on the solid curve is developed. This behavior can be foreseen from the expression for the full DOS difference (33) (or Eq. (32)) discussed in the previous section. The case with the Abrikosov vortex is further explored in Fig. 1,*b*, where we plot the energy dependence of the LDOS for $r = 0.5l$ (the solid curve) and $r = 5l$ (the dashed curve). Comparing the results for $r/l = 0.5, 1, 5$ we find that as the distance r decreases, the integer $E/\hbar\omega_c$ peaks are getting stronger, while for $r = 5l$ they practically disappear. This behavior allows to attribute the corresponding energy levels to the vortex. On the other hand, the half-integer $E/\hbar\omega_c$ peaks corresponding to the usual Landau levels formed in a constant magnetic field. We stress that even for an arbitrary vortex flux η the latter levels will not change the positions, while the levels related to the vortex will shift their energies.

Analyzing Eq. (57) which was used to plot Fig. 1, we observe that the positions of all peaks are controlled by the gamma functions $\Gamma(z)$ which contain simple poles for $z = 0, -1, -2, \ldots$ However, the intensity of the peaks depends on the rather complicated modulating function $F_{\lambda,\mu}(y)$. For example, we verified that despite that the first gamma function in the second term of Eq. (57) contains the pole at the negative energy $E = -\hbar\omega_c/2$, the final LDOS does not contain this pole. To gain more insight on the behavior of the LDOS we have investigated its behavior in the limits $r \to 0$ and $r \to \infty$. Taking into account the $y \to 0$ limit of $\mathrm{Im}\, I$ given by Eq. (38), we obtain that the value $\Delta N_\eta^{\mathrm{S}}(\mathbf{r} = 0, E, B)$ is equal to the negative LDOS (11) in the constant magnetic field. This implies that the full LDOS in the center of the vortex is completely depleted,

$$N_\eta^{\mathrm{S}}(\mathbf{r} = 0, E, B) = 0. \tag{58}$$

This vortex induced depletion of the LDOS in the nonrelativistic 2DEG was already seen in Ref. 1 and now we conclude that it should also occur in the presence of the background magnetic field. This behavior we can observe from Fig. 2, where curves begin from the negative LDOS (11) in the constant magnetic field. Two of these curves, viz. the solid and the dot-dashed are for the usual Landau levels with $E/\hbar\omega_c = 0.5, 1.5,$ and the other two are for the vortex levels with $E/\hbar\omega_c = 1, 2.$ For small $r < l$ all curves increase linearly as expected from the analytic results described in Eq. (57) if we take there $\eta = 1/2$. Since for the large y the function $F_{\lambda\mu}$ decays exponentially, the LDOS difference $\Delta N_\eta^{\mathrm{S}}(\mathbf{r}, E, B) \sim \mathrm{e}^{-r^2/2l^2}$ for $r \to \infty$. We emphasize that all curves in Fig. 2 are taken by the maximum values of the LDOS in the solid curve in Fig. 1,*a*.

7. Conclusions

In this paper, we explored the electronic density of states of two-dimensional system at the presence of the Aharonov–Bohm flux tube and constant magnetic field. The expression for LDOS is found exactly as a function of energy, coordinates and a value of magnetic flux. The obtained function has a depletion near the vortex core and becomes a zero in it's center. Also the LDOS has new peaks which don't correspond to Landau levels. The appearence of these peaks can be explained in terms of the new spectrum of elementary excitations in the system. For large distance from the flux tube the LDOS difference decays exponentially. So contributions to the LDOS via Aharonov–Bohm field play a crucial role near the vortex. Since the fabrication of this system is not difficult now, one can investigate obtained results for the LDOS experimentally, using scanning tunnel microscopy.

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