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Torsional oscillation of the vortex tangle. Possible applications to oscillations of solid ^{4}He

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Torsional oscillation of the vessels with quantum fluids is one of oldest and most popular methods for the study of quantized vortices. The recent and very bright example is the discovery of the supersolidity of the solid helium. In the torsion oscillation experiments the drop in the period of oscillations with achievement of some small temperature has been observed. This effect was attributed to the appearance of the superfluid component. This phenomenon depends on many various factors and has various explanations. But, if to adopt (at least hypothetically, at this stage) that the phenomenon of “supersolidity” (dissipativeless flow) is realized, we must consider the relaxation of the vortex system (we can call it as vortex tangle, vortex fluid, chaotic set of vortices, etc.). We have to do it for the very simple reason, that the only way to involve the superfluid component into rotation is the presence of the polarized vortices (with nonzero mean polarization along the axis of rotation). In the present work we submit the approach describing the vortex tangle relaxation model for the torsional oscillation responses of quantum systems, having in mind to apply it for the study of solid ^{4}He. It is shown that the rotation of the superfluid component occurs in the relaxation-like manner with the relaxation time dependent on the amplitude of oscillation (as well as on the temperature and pressure). The study of this problem shows that there is a quasi-linear solution explaining the (amplitude dependent) shift of period. There is also an imaginary shift of the frequency (also the amplitude-dependent), which describes an additional dissipation. The results of the theory are compared with the recent measurements.

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1. Introduction and scientific background

Torsional oscillation of the vessels with quantum liquids is one of oldest and popular method to study the quantized vortices. The recent and very bright example is the discovery of the supersolidity of the solid helium [1], predicted theoretically pretty long ago [2–4]. Authors [1] observed the drop in the period of oscillations while achieved some small temperature. The measured drop of the period is expected to appear due to reduction of the momentum of inertia, which, in turn, is originated from the appearance of the superfluid component, which would not follow the rotation of the sample cell wall. Since then many various measurements had been made, where authors study the influence of various effects and factors (see, e.g., [5] and review articles [6]).

The character of the response on the torsion oscillation supposes that there are some transient, or relaxation processes inside the solid helium. The nature of this relaxation is real enigma. Unless the rigorous theory is created, any phenomenological assumption can be considered and discussed (for instance, the viscoelastic model [7], or superglass state [8]). However, if we accept (at least hypothetically, at this stage) that the phenomenon of “supersolidity” (dissipativeless flow) is indeed realized, we must consider the relaxation of the vortex system (we can call it as a vortex tangle, vortex fluid, chaotic set of vortices, etc.). We must do it for the very simple reason, since the only way to involve the superfluid component into rotation is just the existence of the polarized vortices (with nonzero mean polarization along the axis of rotation). To distinguish between the different approaches and to select out the most plausible, one has to arrange experiments with new controllable parameters. The change of the amplitude of oscillations (or the rim velocity V_{ac}) gives the unique opportunity to select out the vortex tangle model, since the latter possesses a very rich dynamics (as we will see later), resulting in many various effects. The detailed and reliable measurements for the dependence of both the drop of period and the dissipation on the amplitude of oscillations were reported in [9] (some of results are shown on Fig. 1). It can be seen from the Fig. 1 that the drop of period decreases as the applied velocity of oscillations increases. That implies that the more intensive motion is realized, the faster the superfluid part adjusts to the rotation. From the general theory of superfluidity it is known that the superfluid part can rotate only due to polarized (along the axis of rotation) vortices. That observation gave rise to hypothesis that part of the phenomena observed is due to the vortex tangle. It should be understood however that, because of the absence of the rigorous microscopic theory, there is no clear notion what quantized vortices are in solid helium and what their dynamics is. We intend to apply our knowledge on the vortex tangle in other quantum systems for this problem. We hope that this approach bears very general character, which reflects the fact that vortex tangles in different system behave in similar the manner.

In the following, we propose a phenomenological model describing the behavior of the torsional oscillations in the presence of a vortex tangle. The second section is devoted to the problem of rotation of the superfluid component due to polarization of the vortex tangle. In the third section we describe oscillation of the quantum fluids and solids in the presence of relaxating vortex fluids. In the forth section we make some comparison with experiments.

2. Unsteady rotation of superfluid component

The statement of the problem is the following. We goal to study torsion oscillations of the cylindrical vessel of the radius R filled with quantum fluid. In the fluid the vortex tangle, or chaotic set of quantized vortex lines is developed. We do not specify the origin of the tangle, it can be created by the counterflow, or flow, or it can be just a set of the thermally activated vortices. In a vortex free sample or in the case of an absolutely isotropic vortex tangle, the superfluid fraction does not participate in rotation or torsional oscillations. Therefore the momentum of inertia I_{full} acquires a deficit $I_{SF} = \rho_s V R^2/2$ where ρ_s is the superfluid density, and V is the volume of the sample. The angular momentum of the superfluid fraction appears only due to the presence of either aligned vortices (vortex array), or due to the polarized vortex tangle having nonzero total average polarization $\mathbf{P} = \mathcal{L}\langle \mathbf{s}'_z(\xi)\rangle$ along the applied external angular velocity $\mathbf{\Omega}$ (axis z, the magnitude of $\Omega_0 = V_{ac}/R$). Here $\mathcal{L}$ is the vortex line density (total length per unit volume), $\mathbf{s}(\xi)$ is the vector line position as a function of label variable ξ, $\mathbf{s}'(\xi)$ is the tangent vector. In the steady case there is a strictly fixed relation between the total polarization $\mathcal{L}\left\langle \mathbf{s}'(\xi)\right\rangle$ and applied angular velocity $\mathbf{\Omega}$,

$$\mathbf{\Omega} = \kappa \mathbf{P}/2 = \kappa \mathcal{L}\left\langle \mathbf{s}'(\xi)\right\rangle/2. \tag{1}$$

Here κ is the quantum of circulation. In the case when the vortex filaments form an array, the quantity $\mathcal{L}$ coincides with the two dimensional density n, and Eq. (1) transforms to the usual Feynman's rule. The angular momentum of the superfluid part can be written as $\mathbf{M}_{SF} = I_{SF}\mathbf{\Omega} = I_{SF}\kappa\mathbf{P}/2$. The average $\langle \mathbf{s}'(\xi)\rangle$ is defined here as $\mathcal{L}^{-1}\sum_j \int \mathbf{s}'_j(\xi_j)d\xi_j$, where index j distingushes different vortex loops.

The situation changes drastically in a nonstationary (transient or oscillating) case. The total polarization $\mathbf{P}(t)$ changes in time owing to both the vortex line density $\mathcal{L}(t)$ and the mean local polarization $\langle \mathbf{s}'(t)\rangle$ change in time according to their own, relaxation-like dynamics. Therefore, the angular momentum of the superfluid part is not $\mathbf{M}_{SF} = I_{SF}\mathbf{\Omega}$ anymore. Because of relaxation processes there is retardation between $\mathbf{\Omega}(t)$ and $\mathbf{M}_{SF}(t)$, and the connection between them is nonlocal in time. In other words the angular momentum $\mathbf{M}_{SF}(t)$ is some functional of the time dependent angular velocity $\mathbf{\Omega}(t)$. The retardation occurs due to the own dynamics of the vortex tangle. To find the connection between external applied angular velocity $\mathbf{\Omega}(t)$ and the angular momentum of the superfluid part in the unsteady case $\mathbf{M}_{SF}(t)$, we use the result from the classical theory of vorticity. The angular momentum of moving (not necessary rotating) fluid

$$\mathbf{M}_{SF} = \rho_s \int (\mathbf{r}\times\mathbf{v}_s)d^3\mathbf{r} \tag{2}$$

can be expressed via the vorticity field $\omega(\mathbf{r})$ with the use of the following formula:

$$\mathbf{M}_{SF} = \rho_s \int (\mathbf{r}\times\mathbf{v}_s)d^3\mathbf{r} =$$

$$= -\frac{1}{2}\rho_s \int r^2 \omega(\mathbf{r})d^3\mathbf{r} - \frac{1}{2}\rho_s \int r^2(\mathbf{v}_s\times\mathbf{n})d^2S. \tag{3}$$

It follows from relation (3) that the angular momentum is indeed related to the distribution $\omega(\mathbf{r})$ of vorticity inside the vessel. However this connection is ambiguously determined and depends on the surface integral. In the particular case of the uniform (solid body) rotation the connection between the angular momentum and the distribution of vortices is universal and has the following form (see, e.g., [12]):

$$\mathbf{M}_{SF} = \frac{1}{2}\rho_s \int r^2 \omega(\mathbf{r})d^3\mathbf{r}. \tag{4}$$

In the case of the discrete and quantized vortices the vorticity can be written as

$$\omega(\mathbf{r}) = \kappa \sum_j \int \delta(\mathbf{r}-\mathbf{s}_j(\xi_j))\mathbf{s}'_j(\xi_j)d\xi_j,$$

where index j is for the different vortex loops. In the what follows, we use the abbreviature

$$\sum_j \int \mathbf{s}_j(\xi_j) d\xi_j \to \int d\xi .$$

For the singular distribution of vorticity viz. for the vortex filaments, the relation for angular momentum $\mathbf{M}_{SF}$ (see (4)) can be rewritten as

$$\mathbf{M}_{SF} = \frac{1}{2}\rho_s \kappa \int (\mathbf{s}(\xi))^2 \, \mathbf{s}'(\xi) d\xi. \qquad (5)$$

Resuming, we conclude that (for uniform rotation) the angular momentum of the superfluid part $\mathbf{M}_{SF}(t)$ is firmly connected with the distribution of vortex filaments. That implies that, following the evolution of lines, we are in position to monitor the change of quantity $\mathbf{M}_{SF}(t)$. The rate of change of the angular momentum is

$$\frac{d\mathbf{M}_{SF}}{dt} = \rho_s \kappa \int (\mathbf{s}\dot{\mathbf{s}})\mathbf{s}' \, d\xi + \frac{\rho_s}{2}\kappa \int (\mathbf{s})^2 \, \dot{\mathbf{s}}' \, d\xi. \qquad (6)$$

After some long algebra Eq. (6) is transformed to relation

$$\frac{d\mathbf{M}}{dt} = \rho_s \kappa \int \mathbf{s} \times \dot{\mathbf{s}} \times \mathbf{s}' d\xi -$$

$$- \frac{\rho_s}{2} \kappa \mathbf{s}(L_j)^2 \dot{\mathbf{s}}(L_j) + \frac{\rho_s}{2} \kappa \mathbf{s}(0)^2 \dot{\mathbf{s}}(0) \, . \qquad (7)$$

It is seen from Eq. (7) that there are two mechanisms leading to the change of quantity $\mathbf{M}_{SF}(t)$. The first mechanism (expressed by the second and third terms in the rhs of (7)) arises due to reconnection of the closed vortex loops (or initially unclosed) and sliding their ends (denoted here as $\xi = 0$ and $\xi = L_j$) along the walls of the vessel in the direction of applied $\mathbf{\Omega}$. The second mechanism is more subtle and applicable only for the unsteady situation. As it is seen from the first term in the rhs of (7), it has a structure which corresponds to the area swept by vortex line element $(\dot{\mathbf{s}} \times \mathbf{s}')d\xi$. Therefore, it is just the rate of phase slippage caused by the motion of vortex lines [11]. This combination is the discrete variant of quantity $\mathbf{v}_s \times (\nabla \times \mathbf{v}_s)$, which is called a vortex force and which plays a significant role in the vortex dynamics (see, e.g., [12]). Multiplied by quantity $\mathbf{s}$, it describe the rate of change of the angular momentum due to the motion of vortex loops having the Lamb impulse in the azimuth direction.

To move further, we have to ascetain the equation of motion for elements of line $\dot{\mathbf{s}}(\xi)$ (see, e.g., [15])

$$\dot{\mathbf{s}} = \mathbf{V}_s + \beta \mathbf{s}' \times \mathbf{s}'' + \alpha \mathbf{s}' \times (\mathbf{V}_{ns} - \beta \mathbf{s}' \times \mathbf{s}'') +$$

$$+ \alpha' \mathbf{s}' \times \mathbf{s}' \times (\mathbf{V}_{ns} - \beta \mathbf{s}' \times \mathbf{s}''). \qquad (8)$$

Here $\mathbf{V}_{ns}(\mathbf{r}) = (\mathbf{\Omega} - \mathbf{\Omega}_s) \times \mathbf{r}$ is the local relative velocity. Applying (8) to (7), we can rewrite the latter in the form

$$\frac{d\mathbf{M}_s}{dt} = \int (\mathbf{s} \times \mathbf{F}_{sn}) d\xi - \frac{\rho_s}{2} \mathbf{s}(L_j)^2 \dot{\mathbf{s}}(L_j) + \frac{\rho_s}{2} \mathbf{s}(0)^2 \dot{\mathbf{s}}(0). \qquad (9)$$

Here $\mathbf{F}_{sn}$ is the mutual friction. The integral term in (9) expressed the obvious fact that the rate of change of the angular momentum is just the full momentum of force. Supposing further the uniform distribution of lines and performing the averaging over various vortex loops configuration, we arrive at the result

$$I_s \frac{d\mathbf{\Omega}_s}{dt} = G\mathcal{L}(\mathbf{\Omega}(t) - \mathbf{\Omega}_s). \qquad (10)$$

In Eq. (10), $G = g\kappa\rho_s \alpha V$, where α is the mutual friction coefficient, g is the geometric factor of the order of unity (multiplied by the squared radius R^2 of the vessel) arising from the spacial averaging. Equation (10) shows that the angular velocity of the superfluid part rate adjusts to the changes of the external velocity $\mathbf{\Omega}(t)$ in the relaxation manner. However, the things are not too simple. The point is that the vortex line density $\mathcal{L}$ is not a fixed quantity. On the contrary, it is the time-dependent function, which is determined by the whole process. This circumstance makes the problem highly nonlinear and involved. The situation is complicated by the fact that there is no reliable theory for the behavior of the $\dot{\mathcal{L}(t)}$ in the vortex under unsteady rotation. There is a number of various approaches and views but, in general, the situation is extremely vague. For instance, when the vortex tangle is initially created by the counterflow, there is a generalization of the Vinen equation, which takes into account the angular velocity (see, e.g., [13,14])

$$\frac{d\mathcal{L}}{dt} = -\beta\kappa\mathcal{L}^2 + \left[\alpha_1 V_{ns} + \beta_2 \sqrt{\kappa\Omega}\right]\mathcal{L}^{3/2} -$$

$$- \left[\beta_1 \Omega + \beta_4 V_{ns} \sqrt{\frac{\Omega}{\kappa}}\right]\mathcal{L}, \qquad (11)$$

where α and β are some phenomenologically introduced coefficients. The set of Eq. (10), (11) with respect to two variables $\mathbf{\Omega}_s$ (superfluid angular velocity) and $\mathcal{L}$ (vortex line density) describes the evolution of system under transient rotation $\mathbf{\Omega}(t)$. In the next Section we will use this result to describe the torsion oscillations of the vortex tangle.

3. Torsion oscillations

In this Section we study torsion oscillations of the cylindrical vessel with radius R filled with quantum fluid with the developed vortex tangle. At this stage we choose a simplified version. We accept that the instantaneous value of the vortex line density relaxes faster then the superfluid angular velocity, so the total $\mathcal{L} = \mathcal{L}(t) + \mathcal{L}_{\text{pre-existing}}$, where $\mathcal{L}_{\text{pre-existing}}$ is the initial value of the vortex line density, and $\mathcal{L}(t)$ is the variable part. We also take that the variable part is proportional to the amplitude of oscillation

$\mathcal{L}(t) \propto \Omega_0 = V_{ac}/R$. In view of the said above, Eq. (10) can be rewritten as

$$\frac{d\mathbf{\Omega}_s}{dt} = \tau^{-1}(\mathbf{\Omega}(t) - \mathbf{\Omega}_s). \tag{12}$$

Equation (12) has the pure relaxation form with the characteristic time satisfying

$$\tau^{-1} = \alpha(T)\Omega_0 + \beta(T).$$

Thus we have typical relaxation process. The intensity of relaxation (the inverse characteristic time) consists of two parts. One of them is related to the initial level of the total vorticity, and the second one is due to variation of the vortex line density under transient rotation. Equation (12) is the linear equation. However, the relaxation time depends on the magnitude of oscillation (not on the instantaneous value). Therefore this problem takes into account a nonlinear effect in this shortened form. We will call this statement as a seminonlinear relaxation. Let us note that, under stepwise behavior $\mathbf{\Omega}(t)$ (so-called spin up, or spin down) the function $\mathbf{\Omega}_s$ is approaching to its equilibrium value in the relaxation manner with the pure exponential behavior $\varphi(t'/\tau) \sim \exp(t'/\tau)$.

In the presence of relaxation, the angular momentum $\mathbf{M}(t)$ of the superfluid part is related to the applied angular velocity $\mathbf{\Omega}(t)$ by the nonlocal relation,

$$\mathbf{M} = a\mathbf{\Omega}(t) + b\int_0^\infty \mathbf{\Omega}(t-t')\varphi\left(\frac{t'}{\tau}\right)\frac{dt'}{\tau}. \tag{13}$$

Relation (13) implies that the angular momentum $\mathbf{M}(t)$ depends on the applied angular velocity $\mathbf{\Omega}(t)$ taken in the all previous moments of time with the weight $\exp(-t/\tau)$. To clarify the physical meaning of constants a and b, we consider the limiting cases of very small and very large frequencies. In the case $\omega \to 0$ the slowly changing function $\mathbf{\Omega}(t-t')$ can be considered as a constant and be taken out of the integral, whereupon the rest of integral becomes unity and we have $\mathbf{M}_{\omega\to 0} = (a+b)\mathbf{\Omega}$. But at the same time, both components participate in the solid body rotation, thus $(a+b) = I_{\text{tot}}$. In the opposite case of very large frequencies, $\omega \to \infty$, the integral from rapidly oscillating functions $\mathbf{\Omega}(t-t')$ vanishes, so $\mathbf{M}_{\omega\to\infty} = a\mathbf{\Omega}$. Since under these conditions the superfluid component does not participate in motion at all, we conclude that the constant a is nothing but the full moment of inertia I_N of the sample without the superfluid part (which includes the moment of inertia of the empty cell I_{empty}). Thus, the quantity b is the moment of inertia I_{SF} of the superfluid part. Substituting (13) with $a = I_N$ and $b = I_{SF}$ into the equation of motion of the TO, we get

$$\frac{d}{dt}\left[I_N\Omega(t) + I_{SF}\int_0^\infty \Omega(t-t')\varphi\left(\frac{t'}{\tau}\right)\frac{dt'}{\tau}\right] + k\theta = 0. \tag{14}$$

Here $\theta(t)$ is the angle of rotation of the oscillator, k is the spring constant. Equation (14) says the elementary thing that the rate of the change of the angular momentum is equal to the applied torque (from the spring). Relation (14) is an integro-differential equation and, in general, requires a special treatment. However, by the use of the circumstance that $\varphi(t'/\tau)$ is an exponential function, we can eliminate the integral term. Omitting details, we arrive at the case where Eq. (14) is reduced to an ordinary differential equation of third order

$$\tau\frac{d}{dt}\left(I_N\frac{d^2\theta(t)}{dt^2} + k\theta(t)\right) + \left(I_N\frac{d^2\theta(t)}{dt^2} + k\theta(t)\right) + I_{SF}\frac{d^2\theta(t)}{dt^2} = 0. \tag{15}$$

We are looking for solution in the form $\theta(t) = \theta_0\exp(i\omega t)$. The frequency ω satisfies the algebraic cubic equation, which can written in the following form:

$$\omega = \sqrt{\frac{k}{I_{\text{full}}}}\left[1 + \frac{I_{SF}}{2I_{\text{full}}}\frac{(\omega\tau)^2}{(\omega\tau)^2+1} + \frac{I_{SF}}{2I_{\text{full}}}\frac{i\omega\tau}{(\omega\tau)^2+1}\right].$$

Using the smallness of the $I_{SF} << I_{\text{full}}$ we can develope the perturbation techniques. Practically it is reduced to that we put $\omega = \omega_{\text{full}} = \sqrt{k/I_{\text{full}}}$ in the right hand side (further the index in ω_{full} is omitted). Thus, the frequency of the oscillation consists of three parts. The first one $\omega_0 = \sqrt{k/I_{\text{full}}}$ describes the oscillation with the full moment of the inertia I_{full} as if all constituents (empty cell, normal part, superfluid component) fully participate in motion. The second term is responsible for increase of the frequency because the superfluid component only partly participates in the torsional oscillation. The third term is the imaginary one. It describes the attenuation of the oscillation amplitude, i.e., it describes the energy dissipation. The amplitude decreases (in time) as $\exp[-\Im(\omega)t]$, and the inverse quality factor is $Q^{-1} = 2\Im(\omega)/\omega$. Let us rewrite the described result in the following form:

$$\frac{\Delta P}{P} = -\frac{1}{2}\frac{I_{SF}}{I_{\text{full}}}\frac{(\omega\tau)^2}{(\omega\tau)^2+1}, \tag{16}$$

$$\Delta Q^{-1} = \frac{2\Im(\omega)}{\omega} = \frac{I_{SF}}{I_{\text{full}}}\frac{\omega\tau}{\omega^2\tau^2+1}. \tag{17}$$

Relations (16) and (17) are the final solution to the problem of the torsional oscillation when the superfluid component is involved in rotation via polarized vortex fluids, and polarization occurs in the relaxation-like manner. In the next section we apply the theory developed to oscillations of solid helium.

4. Torsion oscillations of solid helium

As is discussed in the Introduction we apply the developed theory to oscillations of solid helium, concenrating on the amplitude dependence. We choose the detailed study on the amplitude dependence submitted in Ref. 5. In Fig. 1 there are data on the relative shift of period (authors call it as nonlinear rotational susceptibility (NLRS)) and relative dissipation (inverse quality factor).

Being phenomenological, the approach developed does not allow determining some quantities entering the formalism. Thus the parameters $\alpha(T, p)$ and $\beta(T, p)$ responsible for the relaxation of the vortex tangle should be also obtained on the basis more rigorous microscopic theory, which is absent so far. Nevertheless, comparison of our results with the experimental data allows us to explain a series of experimental results and to get some quantitative information and insights. Let us analyze relations (16) and (17). From relations (16), (17) of our paper it follows that $(\Delta P/P)/\Delta Q^{-1}$ is equal to $(1/2)(\omega\tau)$. It can take any value depending on the arrangement of the experiment. But usually observations were fulfilled under conditions with $\omega\tau$ of the order of unity. Therefore in many experiments the $\Delta P/P$ and ΔQ^{-1} are of the same order of the magnitude, although sometimes they can be significantly different (see [5]). Dividing the first relation by the second one and taking the zero V_{ac} limit in the relation $(\Delta P/P)/\Delta Q^{-1} = (1/2)(\omega\tau)$, we get an expression for relaxation time $\beta(T)$ due to diffusion of vortices. Taking further the zero V_{ac} limit for the period drop, and assuming that $\beta(T)$ abruptly vanishes below the "critical velocity" (which is equivalent to the absence of the pre-existing vortices), we find the superfluid momentum of inertia I_{SF} and, consequently, superfluid density ρ_s can be extracted from the graphs for $\Delta P/P$. Knowing $I_{SF}(\rho_s(T))$, $\beta(T)$ and fitting the curves $\Delta P/P$ as functions of V_{ac}, it is possible to determine the inverse relaxation time due to aligning $\tau_1^{-1}(V_{ac}) \sim \alpha(T)V_{ac}/R$ and quantity $\alpha(T)$. In Fig. 2 we show ΔQ^{-1} and $\Delta P/P$ = NLRS as functions of V_{ac}, plotted with the use of relations (16) and (17) and extracted experimental data. It can be seen that the shapes of curves and their response to the change of T correspond to the curves shown in Fig. 1. It is seen that in the limit $V_{ac} \to 0$, or $\omega \to \infty$, or $\alpha(T) \to 0$, NLRS reaches the maximum value. Physically it is clear, since under these conditions the superfluid part cannot participate in

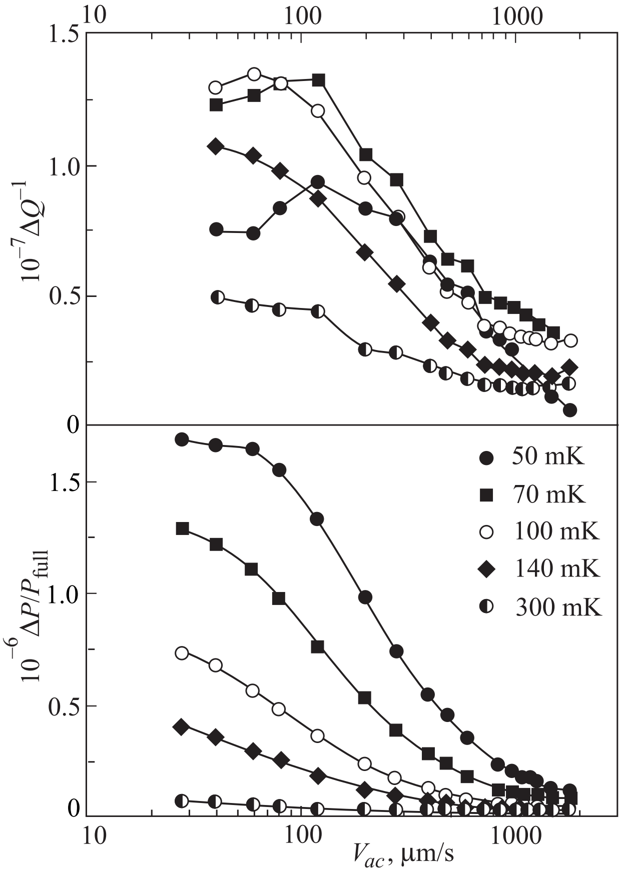

Fig. 1. The data on the torsion oscillation responses as the functions of the rim velocity V_{ac}. The upper column indicates energy dissipation ΔQ^{-1} and the lower column shows the nonlinear rotational susceptibility, NLRS = $\Delta P/P$ for the 49 bar hcp ^{4}He at different T's as functions of V_{ac}.

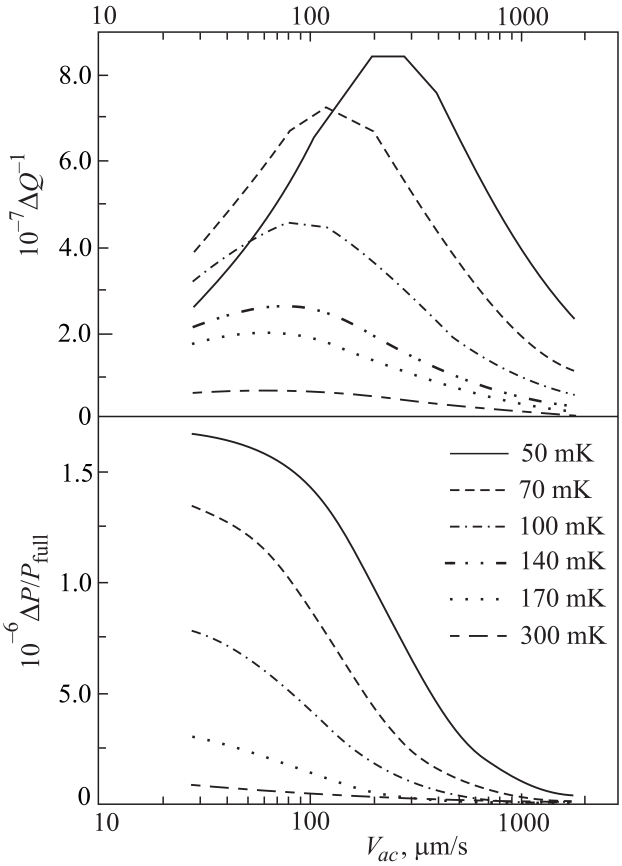

Fig. 2. Energy dissipation ΔQ^{-1} and nonlinear rotational susceptibility at different temperatures as a function of V_{ac}, obtained using relations (16), (17) with the fitting parameters taken from the procedure described in the text.

rotation at all. The other limits $V_{ac} \to \infty$, or $\omega \to 0$, or $\alpha(T) \to \infty$ correspond to vanishing the effect. This is also reasonable since under these conditions the superfluid part participates in the solid body rotation, and no effect appears.

If relaxation due to the pre-existing vortices is weak (this should occur at small temperature), the dependence of dissipation becomes non-monotonic. The analysis shows that the critical value of $\tau_2^{-1} = \beta(T)$ is equal to the frequency ω. In fact, this can differ by some factor of the order of unity. One of the possible reasons for this difference is that we calculate using purely exponential relaxation process, whereas in reality it can be described by a more complicated dependence. The maximum value of dissipation $\Delta Q_{\rm peak}^{-1}$ should be at $(1/2)(\Delta P/P)$ and it should be reached at values of the rim velocity $V_{ac} = R[\omega - \beta(T)]/\alpha(T)$. This tendency is easily seen in Fig. 1, $\Delta Q_{\rm peak}^{-1}$ decreases with T and shifts in the direction of small V_{ac}. For some "critical temperature" when $\tau_2^{-1} = \beta(T)$, the peak disappears completely. It happens at T about 120 mK.

Comparing with the experimental data, one can conclude that the behavior described above takes indeed place for T above about 75 mK, but the agreement fails for lower T. This can be explained by the fact that the real nonlinear relaxation is omitted in our semi-nonlinear problem. Relations (16) and (17) can also explain the $f = \omega/2\pi$ dependence of NLTS and ΔQ^{-1} observed in [16].

In summary, the phenomenological model of relaxation processes of the vortex tangle has been introduced. Unsteady rotation and torsional oscillation have been studied. Dependence of both the NLRS and the ΔQ^{-1} on V_{ac} and f has been studied. The results obtained may serve as a good qualitative description of the corresponding measurements in the solid ^{4}He. Combining theoretical predictions with experimental data, it became possible to obtain some quantitative results.

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