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Levitation of delocalized states at weak magnetic field: critical exponents and phase diagram

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We study numerically the form of the critical line in the disorder–magnetic field phase diagram of the *p–q* network model, constructed to study the levitation of extended states at weak magnetic fields. We use one-parameter scaling, keeping either *q* (related to magnetic field) or *p* (related to energy) constant, to calculate two critical exponents, describing the divergence of the localization length in each case. The ratio of those two exponents defines the form of the critical line close to zero magnetic field.

PACS: 72.15.Rn Localization effects (Anderson or weak localization);
73.20.Fz Weak or Anderson localization;
73.43.–f Quantum Hall effects.

Keywords: magnetic field, phase diagram, delocalized states.

The levitation scenario, describing the divergence of extended state energy at zero magnetic field, was proposed by Khmelnitskii in 1984 [1]. It was introduced to reconcile the result of the scaling theory for 2d systems [2], that there are no extended states, and the necessity of a delocalized state for a quantum Hall (QH) transition [3]. Several approaches to prove that conjecture were performed during last 25 years experimentally, numerically and analytically (see [4] and references therein for more details), resulting only in establishing the tendency of the extended states energy to increase with the decrease of magnetic field. In order to describe the motion of electron at really low magnetic field one has to allow backscattering which immediately breaks the chirality of the Chalker–Coddington (CC) network model [5], constructed to study inter-plateaux QH transitions in strong magnetic field. It was achieved in the *p–q* network model [6] with point contacts on the links describing the backscattering by disorder and bend-junctions at the nodes describing the orbital action of magnetic field. It was demonstrated that, in restricted geometry, electron motion reduces to two CC networks, with opposite directions of propagation along the links, which are weakly coupled by disorder. Interplay of backscattering and bending results in the quantum Hall transition in a non-quantizing magnetic field, which decreases with increasing mobility. This is in accord with scenario of floating up delocalized states.

The main goal of that model was to separate in space the regions with *phase* action of magnetic field, where it affects interference in course of multiple disorder scattering, and the regions with *orbital* action of magnetic field, where it bends electron trajectories. In *p–q* model, the disorder mixes counter-propagating channels on the links (the probability of backscattering is p), while scattering matrices at the nodes describe exclusively the bending of electron trajectories (magnetic field is proportional to $(1/2-q)$). The form of the *disorder–magnetic field* phase diagram was predicted (see Fig. 1) and checked numerically. This diagram contains the regions with and without edge states, i.e., the regions with zero and quantized Hall conductivities. Taking into account that, for a given disor-

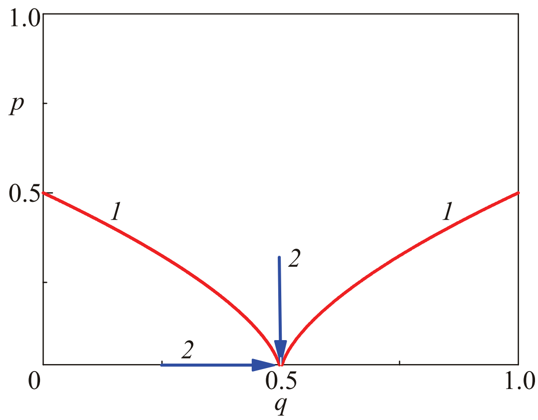

Fig. 1. (Color online) Critical red lines (*1*) on the phase diagram of the *p–q* model. Blue arrows (*2*) show two lines to approach a critical point of infinite energy at zero magnetic field, studied in this paper.

der, the scattering strength scales as inverse electron energy, the agreement of this phase diagram with levitation scenario was found: energy separating the Anderson and quantum Hall insulating phases floats up to infinity upon decreasing magnetic field. From numerical study, based on the analysis of quantum transmission of the network with random phases on the links, it was concluded that the positions of the weak-field quantum Hall transitions on the phase diagram are very close to the classical-percolation results. It was checked that, in accord with the Pruisken theory [7], presence or absence of time reversal symmetry *on the links* has no effect on the line of delocalization transitions. It was also found that floating up of delocalized states in energy is accompanied by *doubling* of the critical exponent of the localization radius.

In this brief report we study numerically the divergence of the localization length when the parameters approach a tricritical point $p = 0$, $q = 1/2$ corresponding to an infinite critical energy at zero magnetic field. We compute the normalized localization length ξ_M / M in the same manner as in original CC model [5] for strips of width $M = 16, 32, 64, 128$ with periodic boundary conditions. We propose that near this critical point the normalized localization length ξ_M / M is described by a two-parameter scaling function

$$\xi_M / M = f(|1/2 - q| M^{1/\nu}, pM^{1/\mu}) . \qquad (1)$$

When there is no backscattering, $p = 0$ (horizontal blue arrow in Fig. 1) the network splits into two independent CC networks with electron propagating in the opposite directions, producing the standard QH critical exponent $\nu \approx 2.6$ (see [8]). Numerical results of the renormalized localization lengths ξ_M / M as function of parameter p at zero magnetic field, $q = 1/2$ (vertical blue arrow (*2*) in Fig. 1) for different system widths M are presented in Fig. 2. Note, that in a limiting case, $p \to 0$ the data strongly fluctuate. For very small values of p the off-diagonal terms in the transfer matrix are close to 0 (they are $\sim\sqrt{p}$), leading to strong fluctuations in numerical results. Physically, enhancement of fluctuations near $p = 0$ is a result of proximity to two critical points, q_c and $1 - q_c$ (see Fig. 1), where the doubling of critical exponent takes place [6]. Nevertheless, the data satisfies rather convincing one-parameter scaling, presented in Fig. 3. Numerical analysis shows that the critical exponent along the line $q = 1/2$ is $\mu \approx 4$. This value is in a qualitative agreement with arguments on doubling of the critical exponent presented in [6].

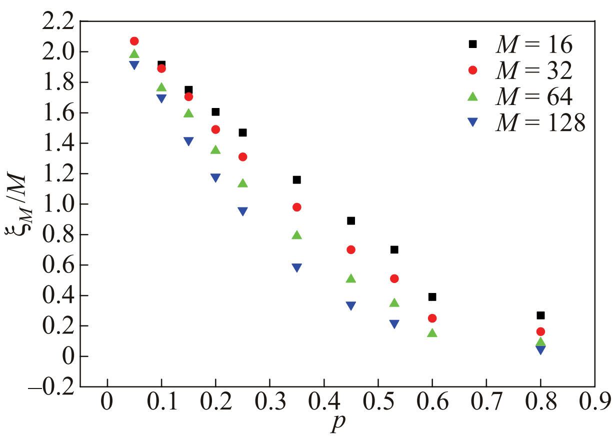

Fig. 2. Renormalized localization length ξ_M / M as function of parameter p at fixed $q = 1/2$ (zero magnetic field).

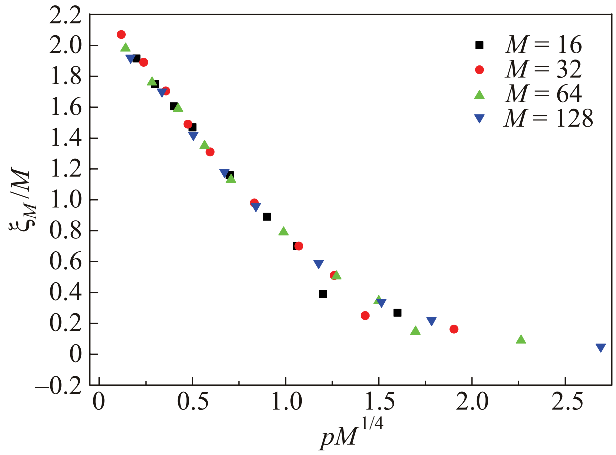

Fig. 3. A one-parameter scaling $\xi_M / M = f(pM^{1/\mu})$ of data presented in Fig. 2. Critical exponent $\mu = 4$.

On a critical line the values of renormalized localization lengths ξ_M / M are expected to be independent on width M, and therefore the parameters $|1/2 - q| M^{1/\nu}$ and $pM^{1/\mu}$ serve to define a one-parameter curve

$$p \sim |1/2 - q| M^{\nu/\mu} . \qquad (2)$$

The form of the curve presented in Fig. 1 is indeed described by Eq. (2). To summarize, using one-parameter scaling, we have studied numerically the critical exponents, describing divergence of localization length along $p = 0$ and $q = 1/2$ lines, and have found that the critical line in *p*–*q* phase space obtained from these values, is in agreement with analytical predictions and direct numerical calculations [6].

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