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Superconducting properties of a two-dimensional doped semiconductor

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In this paper, we study the superconducting properties of a two-dimensional model with an additional (insulating) gap Δ_{ins}, which depends on temperature and doping. In particular, we study the doping evolution of the Berezinskii–Kosterlitz–Thouless critical temperature T_c and the superconducting pseudogap temperature T_c^{MF} at different values of Δ_{ins} by taking into account the hydrodynamic fluctuations of the superconducting order parameter. We demonstrate that the presence of the gap Δ_{ins} affect the values of the superconducting gap and temperatures T_c and T_c^{MF} in the region of the carrier densities where the gap Δ_{ins} approaches to zero. In particular, the derivatives of these quantities have a jump in this region. We discuss possible relevance of the results to high-temperature superconductors.

PACS: **74.20.–z** Theories and models of superconducting state;
74.20.Fg BCS theory and its development;
74.20.Rp Pairing symmetries (other than *s*-wave);
74.72.–h Cuprate superconductors.

Keywords: pseudogap, cuprate superconductors, spin-wave pairing.

1. Introduction

Despite more than 20 years of efforts, theoretical understanding of high-temperature superconductors (HTSCs) remains an unsolved problem. One of the crucial tasks is to understand the so-called pseudogap phase, observed in hole-doped cuprates at low carrier densities. In this phase, the quasiparticle density of states demonstrates a pseudogap near the Fermi level even in the case when there is no superconductivity at high temperatures and/or very low doping (see, for example, Ref. 1). In this phase, the materials demonstrate many properties different from the Fermi-liquid behavior. Usually, two types of scenario to explain this phenomenon are used: the superconducting, or Emery–Kivelson [2], scenario and the scenario when an additional nonsuperconducting gap is present. In the first case, the pseudogap is a result of the fluctuation of the superconducting order parameter at high temperatures. Namely, it is assumed that the noncorrelated superconducting pairs are formed below some temperature, which we call the mean-field critical temperature T_c^{MF}. The order parameters (their phases) become correlated (algebraically ordered) and the system undergoes the superconducting Berezinskii–Kosterlitz–Thouless transition at critical temperature $T_c < T_{PG}$ (for over-review, see, e.g., [3]). The existence of such a pseudogap phase at $T_c < T$ has been confirmed experimentally in several cuprates, where the Nernst effect was observed (see, for example, Refs. 4, 5). However, the anomalous properties (the pseudogap phase) in the cuprates are observed even at much higher temperatures than «the Nernst» temperature $T_0 = T_c^{MF}$, which is of order T_c.

According to the second scenario, there exists an additional gap, which is nonzero even in the nonsuperconducting state (see, for example, Ref. 4). The origin of this gap is a topic of hot debates, though many researchers tend to believe that it is defined by properties of the magnetic subsystem. In this paper, we consider the superconducting properties of a model with a second gap, which we call an insulating gap, Δ_{ins}. In particular, we study how the value of this gap affects the superconducting properties of the system, like the doping dependence of the zero-temperature superconducting gap and the critical temperatures T_c and T_c^{MF}. We assume that $\Delta_{\rm ins}$ depends on the values of temperature and doping δ, and it is zero above the pseudogap temperature curve $T^*(\delta)$. The model we use is a more realistic generalization of the model proposed by Nozieres and Pistolesi [6], who considered the case of constant $\Delta_{\rm ins}$. We show that the temperature and doping dependence of $\Delta_{\rm ins}$ affects the superconducting properties of the system, in particular it can leads to a jump of the gap and temperature derivatives with respect to the doping. The experimental observance of such jump may indicate presence of the second gap. The value of the gap $\Delta_{\rm ins}$ can be estimated by using such a phenomenological model.

We are happy to contribute with this paper to a special issue of Low Temperarture Physics devoted to the 75th anniversary of Professor I.O. Kulik, a world-renowned expert in the fields of weak superconductivity and HTSC.

The paper is organized as follows. The model and the main equations are presented in Sec. 2. In Sec. 3, we solve the model at different values of the insulating gap and show the temperature diagram of the system. A summary and conclusions are given in Sec. 4.

2. The model and main equations

Here we consider a phenomenological model of Nozieres and Pistolesi for a doped semiconductor with the free-electron dispersion $\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \mu$, which begins at $\pm\Delta_{\rm ins}$ ($-W/2-\Delta_{\rm ins} < \varepsilon_{\mathbf{k}} < -\Delta_{\rm ins}$ and $\Delta_{\rm ins} < \varepsilon_{\mathbf{k}} < W/2+\Delta_{\rm ins}$, where W is the free quasiparticle bandwidth, μ is the chemical potential). We consider a generalized case when $\Delta_{\rm ins}$ is temperature- and doping-dependent. Namely, we put

$$\Delta_{\rm ins}(T,\delta) = \Delta_{\rm ins}\left[1-\left(\frac{T}{T_{PG}}\right)^2-\left(\frac{\delta}{\delta_{\rm cr}}\right)^2\right]\times \theta\left[1-\left(\frac{T}{T_{PG}}\right)^2-\left(\frac{\delta}{\delta_{\rm opt}}\right)^2\right]. \quad (1)$$

This phenomenological dependence is chosen to model semi-quantitatively the doping dependence of the pseudogap temperature which begins at some magnetic temperature T_{PG} at zero doping and goes to zero at some value of the doping $\delta_{\rm opt}$, which is close to the optimal doping and corresponds to the maximum of T_c. In HTSCs, the pseudogap temperature T_{PG} is usually defined by the magnetic subsystem. Below, we put $T_{PG} = 0.03$ eV, $\delta_{\rm opt} = 0.165$ in most cases. For simplicity, we also assume that the electron attraction is local with energy V. In the case one takes into account the hydrodynamic (second order) fluctuations of the phase of the superconducting order parameter, it is possible to get the following system of equations for the superconducting gap $\Delta_{\mathbf{k}}$, chemical potential and the critical temperature T_c (see, e.g., Refs. 7–9):

$$1 = V\int \frac{d^2k}{(2\pi)^2}\gamma_k^2 \frac{\tanh\frac{E_{\mathbf{k}}}{2T}}{2E_{\mathbf{k}}}, \quad (2)$$

$$\delta = \int \frac{d^2k}{(2\pi)^2}\left[1-\tanh\left(\frac{E_{\mathbf{k}}}{2T}\right)\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right], \quad (3)$$

$$T = \frac{\pi\delta}{8m} - \frac{\pi}{32m^2}\frac{1}{T}\int \frac{d^2k}{(2\pi)^2}\frac{k^2}{\cosh^2(E_{\mathbf{k}}/2T)}, \quad (4)$$

where $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$ is the quasiparticle spectrum in the superconducting state, $\Delta_{\mathbf{k}} = \Delta\gamma_{\mathbf{k}}$ is the Fourier transform of the amplitude of the order parameter, $\gamma_{\mathbf{k}}$ is the order parameter symmetry factor (equals 1 in the *s*- and $\cos kx - \cos ky$ in the *d*-wave case, for example). δ is the doping (carrier density in the system) and m is the quasiparticle mass. In the two dimensional case, the doping is linearly proportional the Fermi energy $\epsilon_F = \pi\delta/m$ (for simplicity, below we put $\pi/m = 1$).

3. Superconducting properties of the model

3.1. Zero-temperature limit

For simplicity we consider the *s*-pairing case. In the zero-temperature limit, the system of equations for the gap and the chemical potential

$$1 = V\int \frac{d^2k}{(2\pi)^2}\gamma_{\mathbf{k}}^2 \frac{1}{2E_{\mathbf{k}}}, \quad (5)$$

$$\delta = \int \frac{d^2k}{(2\pi)^2}\left[1-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right], \quad (6)$$

which follows from Eqs. (2) and (3) can be easily solved (for square dispersion). In particular, in the case with no insulating gap ($\Delta_{\rm ins} = 0$) and a wide bandwidth W, the gap and the chemical potential monotonously increase with doping:

$$\Delta = \sqrt{2|\varepsilon_b|\epsilon_F}, \quad \mu = -\frac{|\varepsilon_b|}{2} + \epsilon_F, \quad (7)$$

where $\varepsilon_b = -2W\exp(-4\pi/mV)$ is the two-particle bound state energy. The chemical potential changes sign at $\epsilon_F = |\varepsilon_b|/2$, thus a crossover from superfluidity of local boson pairs to superconductivity of Cooper pairs takes place (see, e.g, Ref. 10). Similar situation takes place when the gap $\Delta_{\rm ins}(T=0)$ is included. Moreover, as it was shown in Ref. 6, an insulator–superconductor crossover takes place when $\Delta_{\rm ins}(T=0, \delta=0)$ is equal one half of the superconducting (BCS) gap in the highly-doped (metallic) case.

In general, when the gap $\Delta_{\rm ins}$ is doping-independent, it leads to a decrease of the superconducting gap, but the values of the gap are not decreasing dramatically. In our case, when

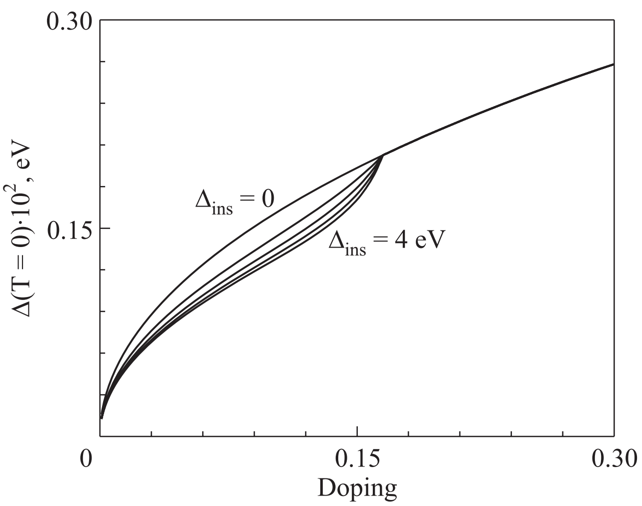

Fig. 1. The doping-dependence of the superconducting gap in the case of $V = 0.15$ eV, $W = 4$ eV and different values of the gap $\Delta_{ins} = 0$; 1; 2; 3 and 4 eV (from left to right). The discontinuity in the derivative $\partial\Delta / \partial\delta$ at $\delta = \delta_{opt} = 0.165$ correspond to the critical density when the gap Δ_{ins} disappears. Here and in other figures by doping we mean the Fermi energy.

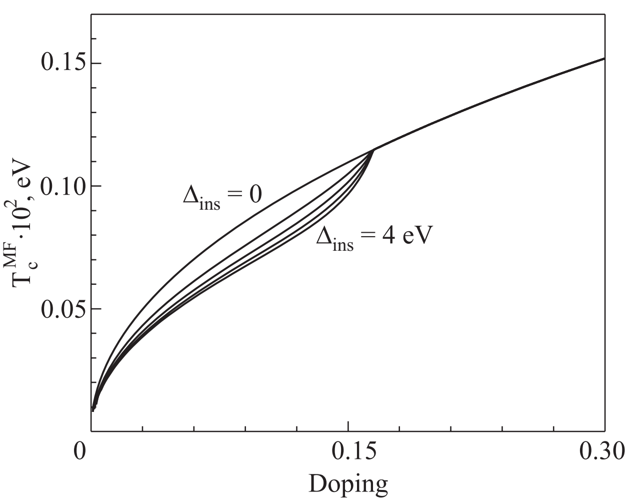

Fig. 2. The same as in Fig. 1 for the mean-field critical temperature.

$$\Delta_{ins}(0,\delta) = \Delta_{ins}\left[1-\left(\frac{\delta}{\delta_{opt}}\right)^2\right]\theta\left[1-\left(\frac{\delta}{\delta_{opt}}\right)^2\right],$$

one can find a jump of the gap derivative at critical density δ_{opt} (Fig. 1). The effect increases with Δ_{ins} growth. This effect is more important, and it can be checked experimentally. The discontinuity of the gap derivative is a consequence of the discontinuity of the derivative of $\Delta_{ins}(T = 0,\delta)$ at $\delta = \delta_{opt}$. Therefore, in the case of presence of the second gap Δ_{ins}, the doping-dependence of superconducting gap must show a feature near the critical doping.

3.2. The case of finite temperatures

The doping-dependence of the critical temperatures at different values of the gap Δ_{ins} is given in Figs. 2, 3 and 5.

As it follows from these figures, this gap affects the values of the temperatures, especially T_c^{MF}, similar to the gap. In the region of nonzero insulating gap ($\delta < \delta_{opt}$), the mean-field critical temperature is reduced comparing to the $\Delta_{ins} = 0$ case (Fig. 2).

However, the decrease of the temperature with the gap increasing is not very strong for a reasonable values of the insulating gap (of order or less than 0.01 eV). What is more remarkable, the critical temperature experiences a jump of the derivative near the doping where Δ_{ins} goes to zero. This is a consequence of such a jump of the derivative for Δ_{ins} near the optimal doping. We have chosen the doping dependence of the insulating gap in form (1) in such a way that its derivative is a discontinuous function of doping at δ_{opt}. The question whether such a doping dependence of the second gap is real is an open one, due to experimental difficulties of measuring it in the region near the optimal doping. However, as it follows from the experimental extrapolation of the doping-dependence of the PG temperature curve (see Fig. 5), this dependence is quite possible. Calculations of the Berezinskii–Kosterlitz–Thouless critical temperature show that this temperature also weakly depends on Δ_{ins} (Fig. 3). It is approximately equal to $\epsilon_F / 8$ for almost all values of doping. Contrary to the case of T_c^{MF}, we have found that T_c slightly increases with growth of Δ_{ins}. However, the derivative of this temperature with respect to the doping has a large jump near δ_{opt} (Fig. 4), which could be measured experimentally. Typical phase diagram of the model is presented in Fig. 5. As it follows from this figure, the superconducting pseudogap region ($T_c < T < T_c^{MF}$) is pretty wide and it increases

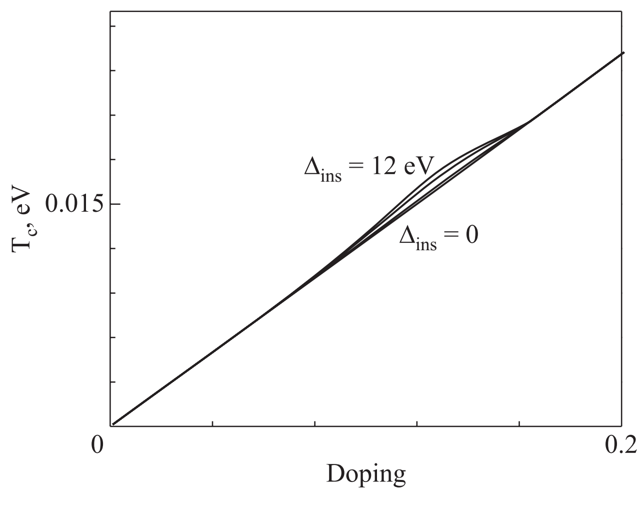

Fig. 3. The doping-dependence of the superconducting critical temperature in the case of $V = 0.5$ eV, $W = 4$ eV and different values of the gap $\Delta_{ins} = 0$; 5; 10 and 12 eV (from right to left).

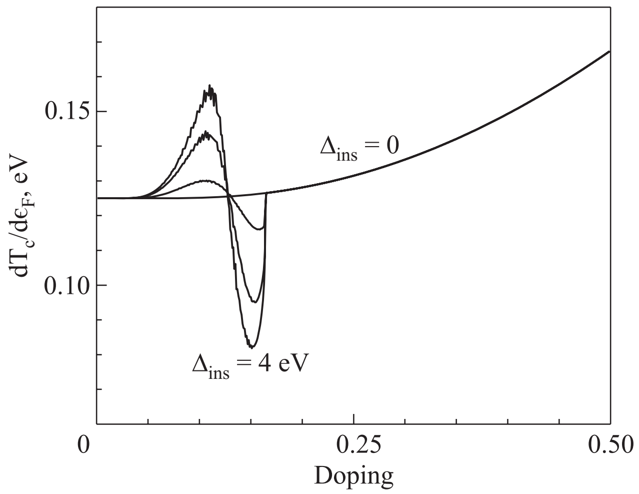

Fig. 4. The derivative of the superconducting critical temperature with respect to the doping as function of doping. The values of the parameters are given in caption to Fig. 3.

with doping increasing, what is in a qualitative agreement with the experiments. We did not obtain a suppression of superconductivity in the over-doped regime in this simple model. It can be obtained by assuming a decrease of the pairing interaction V with doping increasing. Such a situation corresponds to a decrease of the spin ordering and spin-wave pairing with the doping growth, for example.

4. Conclusions

In this paper, we have considered the superconducting properties of a model of doped semiconductor with the gap which depends on temperature and doping. This dependence is chosen in such a way that it can reproduce semi-quantitatively the pseudogap phase of cuprate superconductors. Namely, this gap disappears when temperature is higher than the doping dependent PG temperature T_{PG}. We did not find a dramatic change of the superconducting gap and critical temperatures with Δ_{ins} increasing. However, it is found that in the temperature-carrier-density region where Δ_{ins} disappears all superconducting properties demonstrate a change in the dependence on doping. In particular, the mean-field critical temperature and zero-temperature superconducting gap decrease with Δ_{ins} increasing in this region. The critical temperature slightly increases at values of the doping close to the critical one. However, its derivative with respect to doping, similar to the derivatives of the zero-temperature gap and the mean-field critical temperature, demonstrate a large jump in this region. This is the main result of the paper. Experimental observation of such nonmonotonic dependencies of the superconducting quantities on the doping can indirectly show the existence of such a second gap in the spectrum, which is often related to the pseudogap in underdoped cuprates. Moreover, by using such a simple phenomenological model one can estimate possible values of this gap.

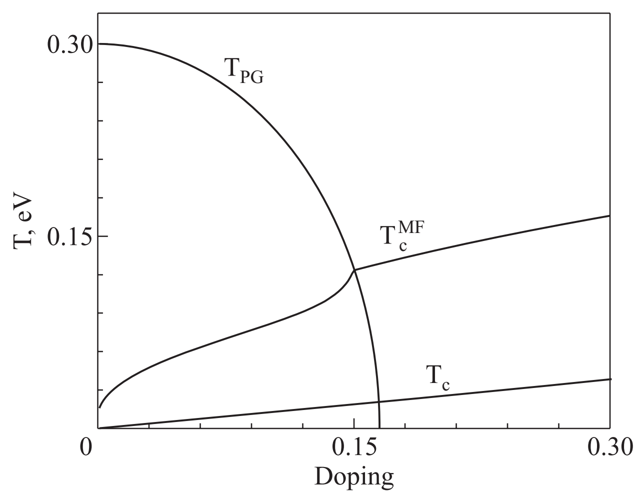

Fig. 5. The temperature-doping phase diagram of the model with two gaps at $V = 0.5$ eV, $W = 4$ eV, $T_{PG} = 0.3$ eV and $\Delta_{ins} =$ $= 4$ eV.

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