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The scattering of surface plasmon polaritons by nanoscale surface defects

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A rigorous computational approach based on Green's second integral identity in the plane is used to calculate the transmission, reflection, and conversion into volume electromagnetic waves of a surface plasmon polariton incident on a nanoscale one-dimensional surface defect on an otherwise planar interface between vacuum and a lossy metal.

PACS: 73.20.Mf Collective excitations (including excitons, polarons, plasmons and other charge-density excitations);

78.66.–w Optical properties of specific thin films.

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The ability to control the propagation of surface plasmon polaritons is important for their use in nanoscale devices [1–3]. A way in which the propagation of these surface electromagnetic waves can be controlled is to scatter them from one- and two-dimensional nanoscale surface defects. In the case of one-dimensional defects, the surface plasmon polariton transmission and reflection coefficients, and the strength and angular dependence of the volume electromagnetic waves radiated into the vacuum above the surface, can be controlled by varying the size and shape of the defect [2]. The scattering of surface plasmon polaritons from isolated one-dimensional surface defects [4–9], and from arrays of a finite number of defects [10], has been studied in several recent theoretical investigations.

All of these calculations have two features in common. They are based on the use of an impedance boundary condition of one form or another, so that only the electromagnetic field in the vacuum above the metal surface needs to be considered, and they all assume that the dielectric function of the metal is real.

In this paper we study the scattering of surface plasmon polaritons incident normally on nanoscale one-dimensional surface defects on an otherwise planar lossy metal surface without invoking either of these approximations.

The physical system we consider consists of vacuum in the region $x_3 > \zeta(x_1)$, and a metal, characterized by an isotropic, frequency-dependent, complex dielectric function $\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega)$ in the region $x_3 < \zeta(x_1)$. We are interested in the frequency range within which the real part of $\varepsilon(\omega)$, $\varepsilon_1(\omega)$, satisfies $\varepsilon_1(\omega) < -1$, which is the range in which surface plasmon polaritons exist. The imaginary part of $\varepsilon(\omega)$, $\varepsilon_2(\omega)$, is non-negative for all frequencies. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1, that is twice differentiable, and is nonzero only in the interval $-L/2 < x_1 < L/2$.

We assume that a p-polarized surface plasmon polariton of frequency ω, whose sagittal plane is the x_1x_3 plane, is incident on the surface defect from the planar region of the surface where $x_1 < -L/2$. When the metal surface supporting it is lossy, the surface plasmon polariton is attenuated as it propagates in the $+x_1$ direction. But this result also means that the amplitude of this wave grows exponentially as $x_1 \to -\infty$. This causes some of the integrals that arise in the scattering theory to diverge. We can avoid these unphysical divergences by creating a source in the region $x_1 < -L/2$ that launches surface plasmon polaritons propagating in both the $+x_1$ direction and the $-x_1$ direction, and therefore decaying in both directions.

To do so we begin by writing the single nonzero component of the magnetic field in the vacuum region $x_3 > \zeta(x_1)$ as the sum of an incident and a scattered field

$$H_2^{>}(x_1,x_3\,|\,\omega) = H_2^{>}(x_1,x_3\,|\,\omega)_{\rm inc} + H_2^{>}(x_1,x_3\,|\,\omega)_{\rm sc}, \quad (1)$$

and in the region of the metal $x_3 < \zeta(x_1)$ as

$$H_2^{<}(x_1,x_3\,|\,\omega) = H_2^{<}(x_1,x_3\,|\,\omega)_{\rm inc} + H_2^{<}(x_1,x_3\,|\,\omega)_{\rm sc}, \quad (2)$$

where

$$H_2^{>}(x_1,x_3\,|\,\omega)_{\rm inc} = \{\theta(x_1+L_0)\exp[ik(\omega)(x_1+L_0)-\beta_0(\omega)x_3]+ \\ +\theta(-x_1-L_0)\exp[-ik(\omega)(x_1+L_0)-\beta_0(\omega)x_3]\}, \quad (3)$$

$$H_2^{<}(x_1,x_3\,|\,\omega)_{\rm inc} = \{\theta(x_1+L_0)\exp[ik(\omega)(x_1+L_0)+\beta(\omega)x_3]+ \\ +\theta(-x_1-L_0)\exp[-ik(\omega)(x_1+L_0)+\beta(\omega)x_3]\}, \quad (4)$$

with $L_0 > L/2$. In these expressions $\theta(x)$ is the Heaviside unit step function,

$$k(\omega) = (\omega/c)[\varepsilon(\omega)/(\varepsilon(\omega)+1)]^{1/2} = k_1(\omega) + ik_2(\omega),$$

with $k_1(\omega) > 0$, $k_2(\omega) > 0$, is the wavenumber of the surface plasmon polariton of frequency ω at the planar interface between vacuum and a metal whose dielectric function is $\varepsilon(\omega)$, while $\beta_0(\omega) = (\omega/c)[-1/(\varepsilon(\omega)+1)]^{1/2}$ and $\beta(\omega) = -\varepsilon(\omega)(\omega/c)[-1/(\varepsilon(\omega)+1)]^{1/2}$ are the inverse decay lengths of the electromagnetic field of the surface wave into the vacuum and the metal, respectively.

The incident fields $H_2^{>,<}(x_1,x_3\,|\,\omega)_{\rm inc}$ given by Eqs. (3) and (4) satisfy the equations

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\omega^2}{c^2}\right)H_2^{>}(x_1,x_3\,|\,\omega)_{\rm inc} = \\ = 2ik(\omega)\delta(x_1+L_0)\exp[-\beta_0(\omega)x_3], \quad x_3 > \zeta(x_1), \quad (5)$$

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}+\varepsilon(\omega)\frac{\omega^2}{c^2}\right)H_2^{<}(x_1,x_3\,|\,\omega)_{\rm inc} = \\ = 2ik(\omega)\delta(x_1+L_0)\exp[\beta(\omega)x_3], \quad x_3 < \zeta(x_1). \quad (6)$$

The scattered fields $H_2^{>,<}(x_1,x_3\,|\,\omega)_{sc}$ satisfy the homogeneous forms of Eqs. (5) and (6), respectively.

Thus, we have introduced a planar source perpendicular to the x_1 axis at $x_1 = -L_0$, whose strength decreases exponentially with increasing distance from the interface into the vacuum and the metal.

We now define two Green's functions $G_0(x_1,x_3\,|\,x_1',x_3')$ and $G_\varepsilon(x_1,x_3\,|\,x_1',x_3')$ as the solutions of the equations

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\omega^2}{c^2}\right)G_0(x_1,x_3\,|\,x_1',x_3') = \\ = -4\pi\delta(x_1-x_1')\delta(x_3-x_3'), \quad (7)$$

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}+\varepsilon(\omega)\frac{\omega^2}{c^2}\right)G_\varepsilon(x_1,x_3\,|\,x_1',x_3') = \\ = -4\pi\delta(x_1-x_1')\delta(x_3-x_3'), \quad (8)$$

subject to outgoing wave boundary conditions at infinity. These functions can be represented by

$$G_0(x_1,x_3|x_1',x_3') = i\pi H_0^{(1)}\{(\omega/c)[(x_1-x_1')^2+(x_3-x_3')^2]^{1/2}\} = \\ = \int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{2\pi i}{\alpha_0(q)}\exp[iq(x_1-x_1')+i\alpha_0(q)|x_3-x_3'|] \quad (9)$$

and

$$G_\varepsilon(x_1,x_3\,|\,x_1',x_3') = \\ = i\pi H_0^{(1)}(\sqrt{\varepsilon(\omega)}(\omega/c)[(x_1-x_1')^2+(x_3-x_3')^2]^{1/2}) = \\ = \int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{2\pi i}{\alpha(q)}\exp[iq(x_1-x_1')+i\alpha(q)\,|\,x_3-x_3'\,|], \quad (10)$$

where $H_0^{(1)}(x)$ is the Hankel function of the first kind and zero order, $\alpha_0(q) = [(\omega/c)^2-q^2]^{1/2}$, with $\mathrm{Re}\,\alpha_0(q) > 0$, $\mathrm{Im}\,\alpha_0(q) > 0$, and $\alpha(q) = [\varepsilon(\omega)(\omega/c)^2-q^2]^{1/2}$, with $\mathrm{Re}\,\alpha(q) > 0$, $\mathrm{Im}\,\alpha(q) > 0$.

When we apply Green's second integral identity in the plane [11] to the regions $x_3 > \zeta(x_1)$ and $x_3 < \zeta(x_1)$ in turn, the preceding results enable us to write the equations satisfied by $H_2^{>,<}(x_1,x_3\,|\,\omega)$, respectively, as

$$\theta(x_3-\zeta(x_1))H_2^{>}(x_1,x_3\,|\,\omega) = H_2^{>}(x_1,x_3\,|\,\omega)_{\rm inc} + \\ + k(\omega)\int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{\exp[iq(x_1+L_0)+i\alpha_0(q)x_3]}{\alpha_0(q)[\beta_0(\omega)+i\alpha_0(q)]} + \\ +\frac{1}{4\pi}\int_{-\infty}^{\infty}dx_1'\left\{\left[\frac{\partial}{\partial N'}G_0(x_1,x_3\,|\,x_1',x_3')\right]_{x_3'=\zeta(x_1')}H(x_1'\,|\,\omega) - \\ - [G_0(x_1,x_3\,|\,x_1',x_3')]_{x_3'=\zeta(x_1')}L(x_1'\,|\,\omega)\right\}, \quad (11)$$

$$\theta(\zeta(x_1)-x_3)H_2^{<}(x_1,x_3\,|\,\omega) = H_2^{<}(x_1,x_3\,|\,\omega)_{\rm inc} + \\ + k(\omega)\int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{\exp[iq(x_1+L_0)-i\alpha(q)x_3]}{\alpha(q)[\beta(\omega)+i\alpha(q))]} - \\ -\frac{1}{4\pi}\int_{-\infty}^{\infty}dx_1'\left\{\left[\frac{\partial}{\partial N'}G_\varepsilon(x_1,x_3\,|\,x_1',x_3')\right]_{x_3'=\zeta(x_1')}H(x_1'\,|\,\omega) - \\ - \varepsilon(\omega)\left[G_\varepsilon(x_1,x_3\,|\,x_1',x_3')\right]_{x_3'=\zeta x_1')}L(x_1'\,|\,\omega)\right\}. \quad (12)$$

In writing these equations we have introduced the source functions $H(x_1 \mid \omega)$ and $L(x_1 \mid \omega)$ that are defined by

$$H(x_1 \mid \omega) = H_2^{>}(x_1, x_3 \mid \omega)\,|_{x_3=\zeta(x_1)}\,, \tag{13}$$

$$L(x_1 \mid \omega) = \frac{\partial}{\partial N} H_2^{>}(x_1, x_3 \mid \omega)\,|_{x_3=\zeta(x_1)}, \tag{14}$$

where $\partial/\partial N = -\zeta'(x_1)(\partial/\partial x_1) + (\partial/\partial x_3)$. We have also used the boundary conditions at the interface $x_3 = \zeta(x_1)$ namely $H_2^{>}(x_1, x_3 \mid \omega) = H_2^{<}(x_1, x_3 \mid \omega)$, and $\partial H_2^{>}(x_1, x_3 \mid \omega)/\partial N = \varepsilon^{-1}(\omega)(\partial H_2^{<}(x_1, x_3 \mid \omega)/\partial N)$.

The equations satisfied by the source functions $H(x_1 \mid \omega)$ and $L(x_1 \mid \omega)$ are obtained by setting $x_3 = \zeta(x_1) + \eta$, where η is a positive infinitesimal, in Eqs. (11) and (12). The resulting equations are

$$\begin{aligned} H(x_1 \mid \omega) &= H(x_1 \mid \omega)_{\text{inc}} + \\ &+ k(\omega)\int_{-\infty}^{\infty} \frac{dq}{2\pi} \frac{\exp[iq(x_1 + L_0) + i\alpha_0(q)\zeta(x_1)]}{\alpha_0(q)[\beta_0(\omega) + i\alpha_0(q)]} + \\ &+ \frac{1}{4\pi}\int_{-\infty}^{\infty} dx_1' \left\{ \left[\frac{\partial}{\partial N'} G_0(x_1, x_3 \mid x_1', x_3') \right]_{\substack{x_3' = \zeta(x_1') \\ x_3 = \zeta(x_1) + \eta}} H(x_1' \mid \omega) - \right. \\ &\left. - [G_0(x_1, x_3 \mid x_1', x_3')]_{\substack{x_3' = \zeta(x_1') \\ x_3 = \zeta(x_1) + \eta}} L(x_1' \mid \omega) \right\} \end{aligned} \tag{15}$$

and

$$\begin{aligned} 0 &= H(x_1 \mid \omega)_{\text{inc}} + k(\omega)\int_{-\infty}^{\infty} \frac{dq}{2\pi} \frac{\exp[iq(x_1 + L_0) - i\alpha(q)\zeta(x_1)]}{\alpha(q)[\beta(\omega) + i\alpha(q)]} - \\ &- \frac{1}{4\pi}\int_{-\infty}^{\infty} dx_1' \left\{ \left[\frac{\partial}{\partial N'} G_\varepsilon(x_1, x_3 \mid x_1', x_3') \right]_{\substack{x_3' = \zeta(x_1') \\ x_3 = \zeta(x_1) + \eta}} H(x_1' \mid \omega) - \right. \\ &\left. - \varepsilon(\omega)[G_\varepsilon(x_1, x_3 \mid x_1', x_3')]_{\substack{x_3' = \zeta(x_1') \\ x_3 = \zeta(x_1) + \eta}} L(x_1' \mid \omega) \right\}, \end{aligned} \tag{16}$$

where $H(x_1 \mid \omega)_{\text{inc}} = H_2^{>}(x_1, x_3 \mid \omega)_{\text{inc}}\,|_{x_3=\zeta(x_1)}$.

Equations (15) and (16) are solved numerically for $H(x_1 \mid \omega)$ and $L(x_1 \mid \omega)$ in exactly the same way as this was done in [12].

We wish to obtain the fraction $R(\omega)$ of the power in the incident surface plasmon polariton that is converted into the reflected surface plasmon polariton, the fraction $T(\omega)$ transmitted beyond the defect in the form of a surface plasmon polariton, and the fraction $S(\omega)$ converted into volume electromagnetic waves in the vacuum.

We begin with a calculation of the total time-averaged flux incident on the defect. This is given by

$$P_{\text{inc}}(x_1) = \int_{-L_2/2}^{L_2/2} dx_2 \left[\int_0^{\infty} dx_3 \,\text{Re}\, S_1^c(x_1, x_3 \mid \omega)_{\text{inc}}^{>} + \int_{-\infty}^{0} dx_3 \,\text{Re}\, S_1^c(x_1, x_3 \mid \omega)_{\text{inc}}^{<} \right], \tag{17}$$

where $L_i (i = 1, 2)$ is the length of the surface along the x_i axis, and where $S_1^c(x_1, x_3 \mid \omega)_{\text{inc}}^{>,<}$ is the 1-component of the complex Poynting vector,

$$S_1^c(x_1, x_3 \mid \omega))_{\text{inc}}^{>,<} = -i\frac{c^2}{8\pi\omega\varepsilon} \frac{\partial H_2^{>,<}(x_1, x_3 \mid \omega)_{\text{inc}}}{\partial x_1} H_2^{>,<}(x_1, x_3 \mid \omega)_{\text{inc}}^{*}\,, \tag{18}$$

where ε the dielectric constant of the medium in which the field is calculated. Thus, the incident flux in the region $-L_0 < x_1 \le -L/2$, obtained from the first term on the right-hand sides of Eqs. (3) and (4), together with Eqs. (17) and (18), is

$$\begin{aligned} P_{\text{inc}}(x_1) &= L_2 \frac{c^2}{8\pi\omega} \left[\frac{k_1(\omega)}{\beta_0(\omega) + \beta_0^*(\omega)} + \right. \\ &\left. + \frac{k_1(\omega)\varepsilon_1(\omega) + k_2(\omega)\varepsilon_2(\omega)}{\varepsilon_1^2(\omega) + \varepsilon_2^2(\omega)} \frac{1}{\beta(\omega) + \beta^*(\omega)} \right] \times \\ &\times \exp[-2k_2(\omega)(x_1 + L_0)], \quad -L_0 < x_1 \le -L/2. \end{aligned} \tag{19}$$

In the region $x_1 \le -L/2$, $x_3 > 0$ in front of the defect the magnetic field of the reflected surface plasmon polariton has the form

$$H_2^{>}(x_1, x_3 \mid \omega)_{\text{ref}} = r(\omega) \exp[-ik(\omega)x_1 - \beta_0(\omega)x_3], \tag{20}$$

while in the region $x_1 \le -L/2,\ x_3 < 0$ it is given by

$$H_2^{<}(x_1, x_3 \mid \omega)_{\text{ref}} = r(\omega) \exp[-ik(\omega)x_1 + \beta(\omega)x_3]. \tag{21}$$

The total time-averaged reflected flux is then obtained from Eqs. (17) and (18) with «inc» replaced by «ref», with the result that

$$\begin{aligned} P_{\text{ref}}(x_1) &= -L_2 |r(\omega)|^2 \frac{c^2}{8\pi\omega} \left[\frac{k_1(\omega)}{\beta_0(\omega) + \beta_0^*(\omega)} + \right. \\ &\left. + \frac{k_1(\omega)\varepsilon_1(\omega) + k_2(\omega)\varepsilon_2(\omega)}{\varepsilon_1^2(\omega) + \varepsilon_2^2(\omega)} \frac{1}{\beta(\omega) + \beta^*(\omega)} \right] \times \\ &\times \exp[2k_2(\omega)x_1], \qquad x_1 < -L/2. \end{aligned} \tag{22}$$

In the region $x_1 \ge L/2, x_3 \ge 0$ beyond the defect the total magnetic field of the surface plasmon polariton has the form

$$H_2^{>}(x_1, x_3 \mid \omega)_{\text{tr}} = t(\omega) \exp[ik(\omega)x_1 - \beta_0(\omega)x_3], \tag{23}$$

while in the region $x_1 \ge L/2, x_3 \le 0$ it has the form

$$H_2^{<}(x_1, x_3 \mid \omega)_{\text{tr}} = t(\omega) \exp[ik(\omega)x_1 + \beta(\omega)x_3]. \tag{24}$$

The total time-averaged power carried by the surface plasmon polariton in the region $x_1 \ge L/2$ is then given by

$$P_{\rm tr}(x_1) = L_2\, |t(\omega)|^2 \frac{c^2}{8\pi\omega}\left\{\frac{k_1(\omega)}{\beta_0(\omega)+\beta_0^*(\omega)} + \right.$$
$$\left. + \frac{k_1(\omega)\varepsilon_1(\omega)+k_2(\omega)\varepsilon_2(\omega)}{\varepsilon_1^2(\omega)+\varepsilon_2^2(\omega)}\frac{1}{\beta(\omega)+\beta^*(\omega)}\right\}\times$$
$$\times\exp[-2k_2(\omega)x_1], \qquad x_1 \ge L/2. \qquad (25)$$

The magnetic component of the field scattered into the vacuum region can be written in the far zone as

$$H_2^{>}(x_1,x_3\,|\,\omega)_{\rm rad} = \int_{-\infty}^{\infty}\frac{dq}{2\pi}R(q,\omega)\exp[iqx_1 + i\alpha_0(q)x_3], \quad (26)$$

where from Eq. (11) we find that

$$R(q,\omega) = k(\omega)\frac{\exp(iqL_0)}{\alpha_0(q)[\beta_0(\omega)+i\alpha_0(q)]} +$$
$$+\frac{i}{2\alpha_0(q)}\int_{-\infty}^{\infty}dx_1\{i[q\zeta'(x_1)-\alpha_0(q)]H(x_1\,|\,\omega)-L(x_1\,|\,\omega)\}\times$$
$$\times\exp[-iqx_1 - i\alpha_0(q)\zeta(x_1)]. \qquad (27)$$

The 3-component of the time-averaged Poynting vector of the field scattered into the vacuum is

$$P_{\rm rad} = {\rm Re}\int_{-L_1/2}^{L_1/2}dx_1\times$$
$$\times\int_{-L_2/2}^{L_2/2}dx_2\left[-\frac{ic^2}{8\pi\omega}\frac{\partial H_2^{>}(x_1,x_3\,|\,\omega)_{\rm rad}}{\partial x_3}H_2^{>}(x_1,x_3\,|\,\omega)^*_{\rm rad}\right] =$$
$$= L_2\frac{c^2}{8\pi\omega}\int_{-\omega/c}^{\omega/c}\frac{dq}{2\pi}\alpha_0(q)\,|R(q,\omega)|^2 \qquad (28)$$

in the limit as $L_1 \to \infty$. With the change of variable $q = (\omega/c)\sin\theta_s$ Eq. (28) becomes

$$P_{\rm rad} = \int_{-\pi/2}^{\pi/2}d\theta_s\,P_{\rm rad}(\theta_s), \qquad (29)$$

where

$$P_{\rm rad}(\theta_s) = L_2\frac{\omega}{16\pi^2}\cos^2\theta_s\,|R((\omega/c)\sin\theta_s,\omega)|^2. \quad (30)$$

We can now obtain expressions for the surface plasmon polariton reflection, transmission, and radiation coefficients, $R(\omega)$, $T(\omega)$, and $S(\omega)$, respectively. These are given by the total time-averaged powers in the reflected and transmitted surface plasmon polaritons, and in the volume waves in the vacuum, normalized by the incident power. However, due to the presence of damping in the metal, each of these fluxes depends on the coordinate x_1. Therefore we have to indicate at which value of x_1 each flux is calculated. For the incident flux it seems natural to evaluate it at $x_1 = -L/2$, i.e., at the point where the incident surface plasmon polariton meets the defect. We will evaluate the reflected flux at the same value of x_1, namely at the value $x_1 = -L/2$ at which the reflected surface plasmon polariton leaves the region of the defect. We will evaluate the transmitted flux at $x_1 = L/2$, namely at the value of x_1 at which the transmitted surface wave leaves the region of the defect.

With these choices the surface plasmon polariton reflection coefficient $R(\omega)$ is given by

$$R(\omega) = \frac{|P_{\rm ref}(-L/2)|}{P_{\rm inc}(L/2)} =$$
$$= |r(\omega)|^2\exp[2k_2(\omega)(L_0 - L)]. \qquad (31)$$

The surface plasmon polariton transmission coefficient becomes

$$T(\omega) = \frac{P_{\rm tr}(L/2)}{P_{\rm inc}(-L/2)} =$$
$$= |t(\omega)|^2\exp[2k_2(\omega)(L_0 - L)]. \qquad (32)$$

The fraction of the total time-averaged incident flux that is converted into bulk electromagnetic waves in the vacuum is then given by

$$S = \frac{P_{\rm rad}}{P_{\rm inc}(-L/2)}, \qquad (33)$$

while the fraction of the total time-averaged incident flux that is converted into bulk electromagnetic waves propagating in the angular interval $(\theta_s, \theta_s + d\theta_s)$ is

$$S(\theta_s) = \frac{P_{\rm rad}(\theta_s)}{P_{\rm inc}(-L/2)}. \qquad (34)$$

It remains only to determine the reflection and transmission amplitudes $r(\omega)$ and $t(\omega)$, respectively. Far from the surface defect, where $x_1 = -L_1$, with $|x_1| \gg L_0$, the total field evaluated on the surface $x_3 = 0$, $H_2^{>}(-L_1, 0\,|\,\omega) \equiv H(-L_1\,|\,\omega)$ is the sum of the fields of the incident and reflected surface plasmon polaritons, $H(-L_1|\omega) = \exp[ik(\omega)(L_1 - L_0)] + r(\omega)\exp[ik(\omega)L_1]$. From this result we obtain

$$r(\omega) = H(-L_1\,|\,\omega)\exp[-ik(\omega)L_1] - \exp[-ik(\omega)L_0]. \qquad (35)$$

In a similar fashion we argue that far from the defect, where $x_1 = L_1 \gg L/2$, the total field evaluated on the surface $x_3 = 0$, $H_2^{>}(L_1, 0\,|\,\omega) \equiv H(L_1\,|\,\omega)$ is given by $H(L_1\,|\,\omega) = t(\omega)\exp[ik(\omega)L_1]$. It follows, therefore, that

$$t(\omega) = H(L_1\,|\,\omega)\exp[-ik(\omega)L_1]. \qquad (36)$$

We now illustrate the preceding results by applying them to several examples.

We consider first the scattering of a surface plasmon polariton from a defect defined by a Gaussian surface profile function

$$\zeta(x_1) = \delta \exp(-x_1^2 / a^2) \quad (37)$$

on a silver surface. This defect is a ridge if δ is positive, and a groove if δ is negative.

In the numerical calculations the frequency of the incident surface plasmon polariton was assumed to be given by $\hbar\omega = 1.96$ eV, which corresponds to a vacuum wavelength $\lambda = 632.8$ nm. This is the wavelength of the light used in photon scanning tunneling microscope (PSTM) experiments [13–16] to excite surface plasmon polaritons on silver surfaces. The dielectric constant of silver at this wavelength is $\varepsilon(\omega) = -17.2 + i0.479$ [17]. The values of L, L_0, and L_1 assumed in carrying out these calculations were $L = 240\ \mu\text{m}$, $L_0 = 30\ \mu\text{m}$, and $L_1 = 8\ \mu\text{m}$.

In Fig. 1 we plot the dependence on a/λ of the coefficients $R(\omega)$, $T(\omega)$, and $S(\omega)$ for Gaussian ridges and grooves of amplitudes $\delta/\lambda = 0.05$ and 0.2. From the plot it seen that for wide defects all of these coefficients practically coincide for the ridges and grooves of the same δ; the difference between the results for the ridges and grooves displays itself only for narrow defects $a < \lambda/2$.

The reflection coefficient $R(\omega)$ of surface plasmon polaritons from ridges or grooves is significant only for very narrow surface defects. In Fig. 1 it is seen to attain its maximum value for a value of a of the order of $a_{\text{opt}} \approx 0.1\lambda$, irrespective of the value of δ, but shifted slightly to smaller values of a for ridges. The maximum reflectivity, however, increases with increasing δ, and is slightly larger for ridges than for grooves. Such defects have been called plasmon mirrors [4,15,16].

The transmissivity $T(\omega)$ of a surface plasmon polariton propagating through a ridge or a groove, after a drop for very narrow defects, increases monotonically with increasing defect width. This is in contrast to the results of Ref. 5, where the transmissivity of a surface plasmon polariton propagating through a groove increased in an oscillatory fashion with increasing defect width.

The total normalized power $S(\omega)$ scattered from a surface defect, also in contrast to Ref. 5, has a maximum for a very narrow defect, and decreases monotonically with increasing defect width. It is larger for larger δ, but the overall behavior of $S(\omega)$ is the same for grooves and ridges. Thus both narrow ridges and narrow grooves can act as light emitters, i.e., as surface defects that convert a large fraction of the power in the incident surface plasmon polariton into volume electromagnetic waves in the vacuum region.

The absence of the oscillations observed in Ref. 5 in our results for $T(\omega)$ and $S(\omega)$ for the deep groove could be due to the presence of losses that lead to the overdamping of local shape resonances

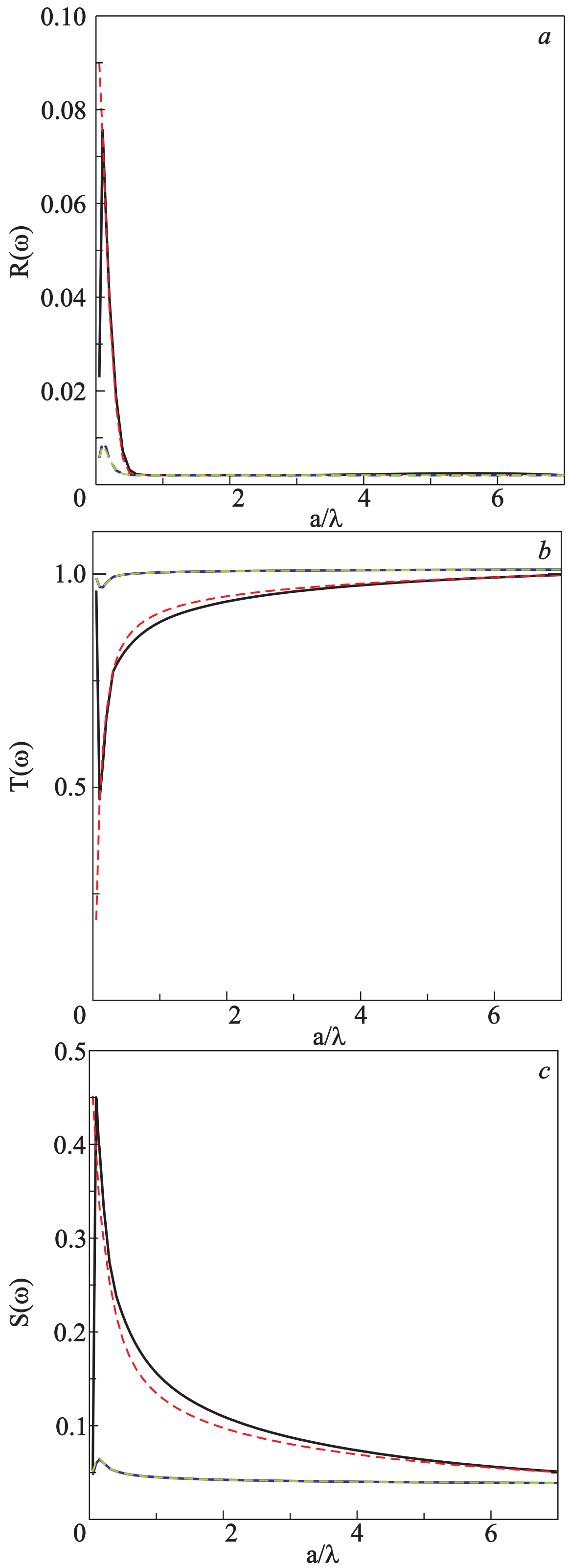

Fig. 1. Surface plasmon polariton reflection (*a*) and transmission (*b*) coefficients $R(\omega)$ and $T(\omega)$, respectively, and the total normalized scattered power $S(\omega)$ (*c*), as functions of the $1/e$ half width of a Gaussian surface defect: $\lambda = 632.8$ nm and $\varepsilon(\omega) = -17.2 + i0.479$. Long dashed curve: $\delta = 0.2\lambda$; solid curve $\delta = -0.2\lambda$; dot-dashed curve: $\delta = -0.05\lambda$; dotted curve: $\delta = 0.05\lambda$.

The frequency dependencies of $R(\omega)$, $T(\omega)$, and $S(\omega)$ were studied in Ref. 6 and display interesting features. In Fig. 2 we have plotted the analogous dependencies for a Gaussian ridge and groove, defined by Eq. (37), with $\delta = 785$ nm and $a = 157$ nm. In obtaining these results the frequency dependence of $\varepsilon(\omega)$ was assumed to be given by $\varepsilon(\omega) = 1 - \omega_p^2 / (\omega(\omega + i\gamma))$, with $\lambda_p = 157$ nm and $\gamma = 0.009681\omega_p$. In the case of a Gaussian ridge all of these coefficients display a weak dependence on the frequency: the transmissivity $T(\omega)$ decreases monotonically, with increasing frequency, while $R(\omega)$ and $S(\omega)$ increase with increasing frequency, displaying weak structure. We note that the in this case most of the energy of the incident surface plasmon polariton is radiated into the vacuum. In contrast, in the case of a Gaussian groove all of these coefficients are strongly frequency dependent, with large oscillations (resonances) corresponding to strong transmission or radiation within some frequency ranges. The reflectivity $R(\omega)$ oscillates weakly but increases with increasing frequency.

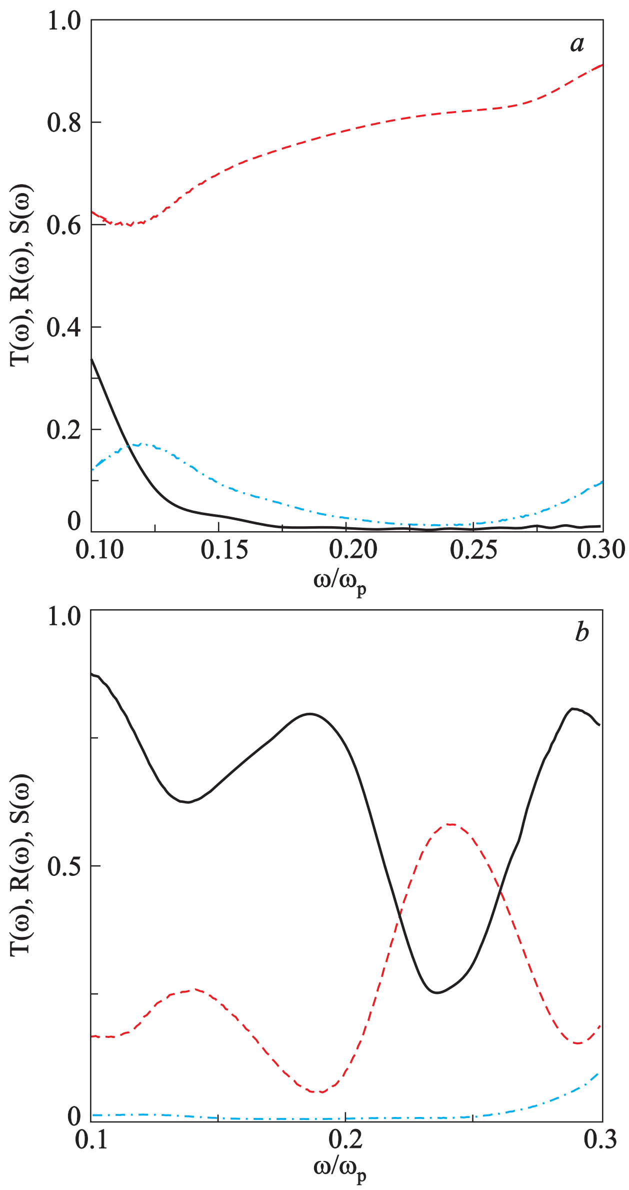

Fig. 2. The frequency dependence of the surface plasmon polariton reflection (dash-dotted curve) and transmission (solid curve) coefficients $R(\omega)$ and $T(\omega)$, respectively, and the total normalized scattered power $S(\omega)$ (dashed curve), of a Gaussian surface defect of $1/e$ half width $a = 157$ nm: ridge, $\delta = 785$ nm (*a*); groove, $\delta = -785$ nm (*b*). $\omega_p = 2\pi c / \lambda_p$, where $\lambda_p = 157$ nm (Ag).

In this paper we have presented a rigorous approach to the scattering of a surface plasmon polariton incident normally on a one-dimensional defect on the otherwise planar surface of a metal that also takes into account ohmic losses in the metal. Expressions for the surface plasmon polariton reflection ($R(\omega)$) and transmission ($T(\omega)$) coefficients, and for the total normalized power of the volume electromagnetic waves radiated into the vacuum above the metal surface ($S(\omega)$), have been obtained in terms of the solutions of a pair of coupled inhomogeneous integral equations. These equations have to be solved numerically, but this can be done by standard methods.

This approach has been illustrated by applying it to the calculation of $R(\omega)$, $T(\omega)$, and $S(\omega)$ for the scattering of a monochromatic surface plasmon polariton from a nanoscale Gaussian ridge or groove on a silver surface. These results demonstrate that our approach is computationally tractable. When they are compared with the results of earlier calculations of these coefficients, in which an impedance boundary condition was used to simplify the calculations, and the metal supporting the surface plasmon polariton was assumed to be lossless, qualitative agreement is found. However, the two sets of results also reveal some qualitative discrepancies.

The theoretical/computational study of properties of surface plasmon polaritons have now advanced to such a level that simplifications of the kind used in earlier scattering calculations are no longer needed. The success of the approach to such calculations presented here would seem to validate this point of view. It is expected that it will be useful in studies of other scattering problems, such as the scattering of surface plasmon polariton pulses from one-dimensional surface defects [6,7], or scattering from an array of such defects [10], which until now have been investigated only by approximate methods.

Acknowledgments

It is a pleasure to dedicate this paper to Viktor Baryakhtar, a distinguished scholar and old friend, on the occasion of his 80th birthday, as a token of our esteem and affection. We wish him many more years of health, happiness, and success in all his endeavors.

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