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Soliton transmission through disordered system

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An exact formula for the transmission time in the disordered nonlinear soliton-bearing classical one-dimensional system is obtained.

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1. Introduction

Transmission experiments are very powerful tools for probing various excitations of solid state systems. In linear ordered systems they enable one to restore the band structure of elementary excitations (electrons, phonons, magnons ets.). In the disordered systems they allow to study Anderson localization of these excitations. One of the simplest ways to incorporate the disorder is to introduce into the system some kind of short-range (point-like) defects. If the wavelength of the bare excitation is mach larger than the average distance between defects, the disorder can be treated in the continuous limit. Here the dynamics of the system can be described by means of some kind of macroscopic approach. In the opposite limiting case the excitations behave themselves mostly as the bare ones between collisions with the defects. The defects manifest themselves as the point scattering centers only.

The situation with the nonlinear system is rather similar. The crucial difference is that here the bare elementary excitations of its linear prototype may form bound states (envelope solitons). Now in the disordered case we deal with the soliton transmission through a disordered segment or a piece of a layer. In practical applications one mostly deals with the case, where an excited soliton pulse is transmitting through a medium with random point defects [1–6]. However, here the characteristic length of the bare excitation is not the wavelength but the soliton spatial size. Therefore two solvable limits mentioned in the previous paragraph, take plase (i) when the spatial size of the soliton is much larger than average distance between defects [3] (large density of the defects), or (ii) when the size of soliton is much smaller then this distance.

This paper is devoted to the second case (small density of the defects). Here the soliton scattering on a single defect leads to the modification of soliton parameters (this problem was partly solved in [2]). But the shifts of the soliton parameters cannot be observed experimentally. The quantities which can be observed are the total change of soliton energy and the shift of its position (or the shift

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of the corresponding propagation time) after transmission of the soliton through the macroscopic region of the disordered medium. The study of these quantities is very topical for information transmission through the optical fibers [7]. Because of the random distribution of defects all these quantities are random. Therefore the subjects of interest are their probabilistic characteristics (mean values, various moments, probability densities and so on).

In this paper we study the shift of the soliton transmission time in the framework of the one-dimensional nonlinear Schrodinger equation (NLSE). In homogeneous (ordered) systems, this equation is completely integrable and possesses the stable robust soliton solutions [8,9]. The NLSE describes many problems of solid state physics: nonlinear magnetization dynamics in ferro-magnets with the easy-axis type anisotropy [10], the soliton motion along the quasi-one dimensional molecular chain, the dynamics of the envelope of phonon excitations produced by an intensive pulse pumping of a crystal [11] etc. Here we obtain, in the second order of weak scattering intensity, the mean value and the variance of the transmission time shift through a disordered segment.

The structure of the paper is following. In the next Sec. 2 we describe the model discussed and introduce all needed notations. Sec. 3 is devoted to the soliton scattering on a single defect. The two first moments of the transmission time shift are calculated in Sec. 4. In Conclusion we summarize the results obtained.

2. The model

The model under consideration is described by the nonuniform NLSE

$$i u_t + u_{xx} + 2|u|^2 u = u\,\varepsilon \sum_{k=1}^{n} \delta(x - x_k)\,, (0 \le x \le L)\,. \quad (1)$$

Here $u(x,t)$ is the field variable which may have a number of physical meanings (e.g. the density of the spin waves) and the subscripts denote the partial derivatives with respect to the corresponding variables (time t and coordinate x). The right hand side describes the influence of the point defects with the intensity ε, placed at the points $\{x_k\}$, $1 \le k \le n$, $0 \le x_n \le x_{n-1} \le \ldots \le x_2 \le x_1 \le L$. We are interested in the case of weak scatterers, so ε is thought to be small (this condition will be clarified later).

In what follows, we assume that the defects are independently and uniformly distributed within the segment $[0, L]$ with mean distance l between the adjacent defects. This means that the number n of defects inside the segment $[0, L]$ is random, inside and the probability p_n to find exactly n defects within the segment, is

$$p_n = \frac{\Lambda^n}{n!} e^{-\Lambda}, \qquad \sum_{n=0}^{\infty} p_n = 1\,, \quad (2)$$

where $\Lambda = L / l$ is the average number of defects on the segment. The conditional probability density to find these n defects at the points $\{x_k\}$ is

$$\rho_n(\{z\}) = n!\,\theta(1 - \sum_{k=1}^{n} z_k) \prod_{k=1}^{n} \theta(z_k) =$$

$$= n!\,\theta(x_n) \prod_{k=2}^{n} \theta(x_{k-1} - x_k)\,\theta(L - x_1),$$

where (3)

$$z_k = L^{-1}(x_{k-1} - x_k)\,, \quad 2 \le k \le n\,,$$

are the scaled distances between adjacent defects and $z_1 = 1 - L^{-1} x_1$. The probability density (3) corresponds to canonical ensemble (a fixed number of the scatterers on the segment). The probability density

$$\rho(n, \{z\}) = p_n\, \rho_n(\{z\}) \quad (4)$$

with p_n from (2) and $\rho_n(\{z\})$ from (3) corresponds to a grand canonical ensemble. In this case we have an infinite line with the scatterers distributed independently along the line with density l and then we cut off a segment with length L from this line. The number of scatterers on the segment fluctuates and its mean number is Λ.

In what follows the symbols $\langle f_n \rangle$ and $\langle f \rangle$ will be used for canonical (with probability density (3)) and grand canonical (with probability density (4)) averaging of the function f correspondently:

$$\langle f_n(\{z\}) \rangle = \int \rho(\{z\})\, f_n(\{z\})\, dz_1 \ldots dz_n,$$

$$\langle f(\{z\}) \rangle = \sum_{n=0}^{\infty} p_n \langle f_n(\{z\}) \rangle\,.$$

The variances of the averaging functions are introduced in a usual way:

$$\delta f_n = \sqrt{\langle f_n^2(\{z\}) \rangle / \langle f_n(\{z\}) \rangle^2 - 1}$$

for canonical ensemble and

$$\delta f = \sqrt{\langle f^2(\{z\}) \rangle / \langle f(\{z\}) \rangle^2 - 1}$$

for grand canonical ensemble.

In infinite system $(-\infty < x < \infty)$ without any perturbations $(\varepsilon = 0)$, the NLSE possesses the well known fundamental four-parametric soliton solution (see e.g. [9])

$$u_s(x,t) = 2i\eta \frac{\exp[-2i\xi x - i\Delta(t) - i\varphi_0]}{\cosh[2\eta\{x - x_0(t) - i\varphi_1\}]}\,,$$

$$\Delta(t) = 4(\xi^2 - \eta^2)t, \quad x_0(t) = -4\xi t\,. \quad (5)$$

The parameter η determines the soliton amplitude $A_s = 2\eta$ and its width $\sim \eta^{-1}$, while ξ determines the velocity of its envelope $V_s = -4\xi$. The choice of η and ξ as the main soliton parameters is related to the fact that the complex value $\lambda_s = \xi + i\eta$ for soliton is the eigen-value of the linear problem associated with NLSE [9]. The parameters η and ξ are related to two integrals of motion, the number of quasi-particles bound in the soliton $N_s = 4\eta = \int_{-\infty}^{\infty} |u|^2 dx$, and the soliton momentum

$$P_s = 8\xi\eta = i\int_{-\infty}^{\infty} u\partial_x u^* dx = V_s N_s / 2.$$

We suppose that parameter ξ is positive and soliton transmits from the right to the left with the velocity $V_s < 0$.

The soliton energy can be expressed via parameters ξ and η or via the integrals of motion

$$E_s = 8\xi^2\eta\left[1 - \frac{2}{3}\left(\frac{\eta}{\xi}\right)^2\right] = \frac{N_s V_s^2}{4} - \frac{N_s^3}{12} = \frac{P_s^2}{N_s} - \frac{N_s^3}{12}. \quad (6)$$

The first term in each of these expressions is the kinetic energy while the second one represents the potential energy of attraction of quasi-particles bound in soliton. The number of these quasi-particles N_s plays a role of the soliton mass. With the help of new parameter $\alpha = -N_s / V_s = \eta / \xi > 0$ we distinguish two limiting cases. The first one, $\alpha << 1$, is the case of «light» soliton, which kinetic energy essentially exceeds the binding energy of quasi-particles forming the soliton. In the opposite case, $\alpha >> 1$, potential energy is much larger then kinetic one. This is the case of «heavy» soliton.

Two other parameters — the phases φ_0, φ_1 — are not so important in homogeneous (ordered) system but in the inhomogeneous case they must be taken into account.

Scenario (which will be justified in the next Section) of soliton transmission through the disordered segment is the following. Consider an unperturbed at $t \to -\infty$ soliton (5) characterized by its energy E_s, number of quasi-particles N_s and its velocity $V_s = -\sqrt{4E_s / V_s + N_s^2 / 3}$, and incident from the right of the segment $[0, L]$. Passing through the first (from the right) scatterer placed at the point x_1, the soliton changes its parameters and reaches the segment $[0, x_2]$ with another energy E_s', number of quasi-particles N_s' and velocity $V_s' = -\sqrt{4E_s' / N_s' + N_s'^2 / 3}$. Passing through the second scatterer, the soliton changes its parameters once again and so on. Because of change of the soliton velocity on each step, its total transmitting time through the entire segment differs from that in the ideal system without scatterers. The value of this shift depends on a realization of our random system, i.e. on the number of scatterers n, their positions $\{x_i\}$ and their intensity ε. So, our first step is to study the soliton transmission through a single defect and to find the corresponding transformation $(N_s, E_s, V_s) \to (N_s', E_s', V_s')$ of solitonic parameters.

Here we must pay attention to the fact that in homogeneous system the NLSE has infinite number of integrals of motion, the first three are N, P and E. In the inhomogeneous system with defects the values N and E remain to be the integrals of motion, but P does not. That is why we choose the integrals N and E as the parameters of soliton. We do not take into account another integrals of motion and that means that we suppose that under the process of soliton scattering another moving solitons and solitons bounded with the defects do not appear.

3. Soliton scattering on a single defect

3.1. Emission of quasi-particles

The scattering of a soliton on a single point-like defect with small intensity ε, placed at origin of infinite system, can be considered within the framework of perturbation theory with respect to defect intensity. It was shown [2] that soliton passes through the defect only when the condition $4\xi^2 > \varepsilon\eta$ is fulfilled. But in linear approximation (in the first order in ε) soliton demonstrates only small $\sim \varepsilon$ and physically inessential phase shifts of φ_0 and φ_1 while the soliton velocity V_s and its amplitude A_s do not change at all. Their changes (or changes of N_s and E_s) appear only in the second order of perturbation theory $\sim \varepsilon^2$ where the emission of elementary (linear) excitations is taken into account. In the case of «rather fast» soliton $\varepsilon\eta << \xi^2$ or $\alpha << \xi / \varepsilon$ the velocity change due to interaction with a defect is small, and the problem can be solved analytically.

NLSE is exactly integrable by the inverse scattering technique [9]. It is naturally to study the problem in terms of perturbation theory based on this technique [12]. Within this approach, one deals with the linear problem associated with NLSE. However considering the soliton scattering on the defect, we should take into account the continuous spectrum as well. It consists of real values $\lambda = -k / 2$ simply related to the wave number k of linear waves with dispersion relation $\omega = k^2$.

In the presence of linear quasi-particles and solitons the integrals of motion are modified

$$N = N_s + \int_{-\infty}^{\infty} n(k)\, dk,$$
$$E = E_s + \int_{-\infty}^{\infty} k^2 n(k)\, dk. \quad (7)$$

In our case when at $t = -\infty$ we have the pure soliton (5), $n(k)$ corresponds to the density of radiated quasi-particles at $t=\infty$. For the fast solitons, this value was calculated in [2]

$$n(k)=\frac{\pi}{2^7}\frac{\varepsilon^2}{\xi^6}\frac{\left[\left(\frac{k}{2}+\xi\right)^2+\eta^2\right]^2}{\cosh^2\left[\frac{\pi}{4\eta\xi}\left(\frac{k}{2}\right)^2-\xi^2+\eta^2\right]}\,. \tag{8}$$

During the interaction the soliton emits $N_f=\int_{-\infty}^{0}n(k)$ quasi-particles in the forward (left) direction and $N_b=\int_{0}^{\infty}n(k)$ quasi-particles are reflected from the defect and go backward (right) — we remind that our soliton moves to the left. The total number N_e of quasi-particles emitted by the soliton passing through the defect from right ($x>0$) to left ($x<0$) is

$$N_e=N_f+N_b=\int_{-\infty}^{\infty}n(k)\,dk\,. \tag{9}$$

Correspondingly, the total energy lost by the soliton is

$$E_e=E_f+E_b=\int_{-\infty}^{\infty}k^2n(k)\,dk\,. \tag{10}$$

3.2. Change of the soliton velocity and amplitude

Substituting expressions (9), (10) into conservation laws $N_s=N_s'+N_e$ and $E_s=E_s'+E_e$ we obtain

$$N_s'=N_s-\int_{-\infty}^{\infty}n(k)\,dk\,,$$
$$E_s'=E_s-\int_{-\infty}^{\infty}k^2n(k)\,dk\,. \tag{11}$$

From these equations, with the help of relation $V_s=-\sqrt{4E_s/N_s+N_s^2/3}$ valid for the fundamental soliton (5), we find the change of the soliton velocity

$$V_s'^2=V_s^2-\frac{4E_e/N_s+(2N_s/3-4E_s/N_s^2)N_e-N_e^2+N_e^3/N_s}{1-N_e/N_s}\,. \tag{12}$$

The number N_s and energy E_e of emitted quasi-particles have the order of magnitude ε^2 and are much smaller then N_s and E_s. Therefore the latter equation with ε^2 accuracy reads

$$V_s'-V_s\approx-F(N_e,E_e,N_s,V_s)\,, \tag{13}$$

where

$$F(N_e,E_e,N_s,V_s)=-\frac{N_eV_s}{2N_s}+\frac{4E_e+N_eN_s^2}{2N_sV_s}\,. \tag{14}$$

Equations (11),(13),(14) generally solve the problem posed. More detailed and explicit results can be obtained in the limiting cases of light and heavy solitons. In both these cases we suppose that the condition of the «rather fast soliton» $\alpha<<\xi/\varepsilon$ is valid. For the light solitons it means that $\alpha<<1$ and $\alpha<<|V|/\varepsilon$. *In this limit the soliton is similar to wave packet with $V\sim\varepsilon$ and we in some cases may use the analogy with the interaction of a single quasi-particle with the δ-function defect.* For the heavy soliton the double inequality must be fulfilled $1<<\alpha<<|V|/\varepsilon$.

We start from the case of light soliton. Here the density $n(k)$ of emitted quasi-particles (8) is strong enough and has two well pronounced peaks with widths $\Delta k\sim\eta$ centered near the points $k\approx\pm2\xi$. The amplitude of the right peak at $k\approx2\xi$ is of order ε^2/ξ^2 and essentially exceeds that of the left peak at $k\approx-2\xi$ which is α^4 times smaller. As a result, the main part of the particles is reflected from the defect $N_b^l>>N_f^l$ and number of emitting quasi-particles is of order $N_e^l\sim\varepsilon^2\alpha/\xi$. More detailed calculations lead to the following results for the number of emitted quasi-particles and the total emitting energy

$$N_e^l\approx\frac{\varepsilon^2\alpha}{4\xi}=\frac{\varepsilon^2N_s^l}{(V_s^l)^2}\,, \tag{15}$$

$$E_s^l\approx4\varepsilon^2\alpha\xi=\varepsilon^2N_s^l\,. \tag{16}$$

Corresponding results for soliton amplitude $A_s^l=N_s^l/2$ and velocity $V_s^l=-\sqrt{4E_s^l/N_s^l+(N_s^l)^2/3}$ with the same accuracy read

$$A_s^{l'}\approx A_s^l(1-\varepsilon^2/(V_s^l)^2)\,, \tag{17}$$

$$V_s^{l'}\approx V_s^l(1-3\varepsilon^2/2(V_s^l)^2)\,. \tag{18}$$

The limits of applicability of these results are $|V_s|>>\max\{\varepsilon,|A_s|\}$. We emphasize (i) that velocity transformation law for fast light soliton does not include soliton amplitude and (ii) that soliton delays, passing (from the right to the left) through the defect $|V_s^{l'}|<|V_s^l|$.

In the opposite limiting case of heavy soliton with $\alpha>>1$ the density of emitted quasi-particles is exponentially small $\sim\exp(-\pi\alpha/2)$. It is almost symmetric with the width of order $\xi\sqrt{\alpha}\sim\sqrt{A_s^h|V_s^h|}$. The numbers of quasi-particles emitted backward exceeds only slightly

$$0<N_b^h-N_f^h\approx\frac{\varepsilon^2\alpha^3}{8\xi}\exp(-\pi\alpha/2)<<N_e^h\,, \tag{19}$$

where the total number of emitted particles is equal to

$$N_{\rm e}^{h} \approx \frac{\varepsilon^2\pi\sqrt{2}\alpha^{9/2}}{2^6\xi}\exp(-\pi\varepsilon/2)\,. \qquad (20)$$

In the same approximation the total energy of the emitted quasi-particles is positive and equal to

$$E_{\rm e}^{h} \approx \frac{\varepsilon^2\pi\xi\alpha^{11/2}}{4}\exp(-\pi\varepsilon/2)\,. \qquad (21)$$

For heavy soliton, transformation laws of its amplitude and velocity have more complicated form:

$$A_s^{h'} \approx A_s^h(1-\varepsilon^2\lambda^h/(V_s^h)^2)\,, \qquad (22)$$

$$V_s^{h'} \approx V_s^h - 2\varepsilon^2\lambda^h(A_s^h/V_s^h)^2/V_s^h\,, \qquad (23)$$

where $\lambda^h(A_s^h,V_s^h) \approx \pi(A_s^h/|V_s^h|)^{7/2}\exp(\pi A_s^h/V_s^h) << 1$. Analogously to the light soliton, the heavy soliton also slows when passing the defect. However its velocity transformation strongly depends on soliton amplitude.

4. Soliton passing over random medium

4.1. Amplitude and velocity

Passing through the disordered segment, soliton changes its amplitude and velocity. Under some assumptions, these changes can be easily calculated. Suppose that the density of the defects is small so that the average distance between the defects is much larger then the soliton size. This enables us to use the results (17)–(23) obtained in the previous Sec. 3 for infinite system. Then considering each act of scattering we neglect the quasi-particles emitted during the previous acts, and exclude the possibility of the additional solitons excitation. Finally, assuming that the total number of scatterers is not enormously large $n << \varepsilon^{-2}$, we can perform all calculations with the ε^2 accuracy. In this case the total changes of soliton amplitude $\Delta A_s = A_s|_{x=0} - A_s|_{x=L}$ and velocity $\Delta V_s = V_s|_{x=0} - V_s|_{x=L}$ are additive and do not depend on the spatial realization of the defects. For example, after passing k-th scatterer the light soliton loses the velocity (18) $\delta V_s^k = V_s^{k+1} - V_s^k \approx \approx -3\varepsilon^2/2V_s^k$. In the right-hand side $V_s^k \approx V_s^{k-1} - -3\varepsilon^2/2V_s^{k-1}$, but we must not take into account the second term as it gives the correction to δV_s^k of order $\sim\varepsilon^4$. After k steps of this procedure we obtain $\delta V_s^k \approx -3\varepsilon^2/2V_s^1 = = -3\varepsilon^2/2V_s^{\rm input}$.

For the light soliton, the total shifts of the soliton amplitude and velocity are

$$\Delta A_s^l \approx -\varepsilon^2 nA_s^l/(V_s^l)^2, \quad \Delta V_s^l \approx -3\varepsilon^2 n/2V_s^l\,. \qquad (24)$$

Corresponding shifts for the heavy soliton have the form:

$$\Delta V_s^h \approx -\varepsilon^2 nA_s^h\lambda^h/(V_s^h)^2, \qquad \Delta V_s^h \approx -2\varepsilon^2 n(A_s^h)^2\lambda^h/(V_s^h)^3. \qquad (25)$$

All quantities entering the right hand sides of (24,25) are taken at $x = L$ and correspond to their input values.

4.2. Transmission time

From the physical point of view, much more important quantity is the shift of the soliton transmission time. Let $v_1 > 0$ be the absolute value of the input velocity of the soliton incident from the right on the segment $[0,L]$. Then, let $v_k > 0$, $1 \le k \le n-1$ be the absolute value of the soliton velocity between $(k-1)$ 1-th and k-th scatterers, and v_{n+1} — the output velocity of the soliton which passed through the last n-th scatterer. The total transmission time in homogeneous case equals $T^0 = L/v_1$ while the transmission time through disordered segment containing n scatterers

$$T_n = \frac{L-x_1}{v_1} + \frac{x_1-x_2}{v_2} + \ldots + \frac{x_{n-1}-x_n}{v_n} + \frac{x_n}{v_{n+1}} \qquad (26)$$

is bigger then T^0 because of soliton delay after each act of scattering $v_1 > v_2 > \ldots > v_n > v_{n+1}$. The transmission time shift $\Delta T = T_n - T^0 > 0$ is very important characteristic of the various delay lines. But this shift is not additive one and strongly depends on the realization of the defects. The calculation of its statistical characteristics is more complicated problem. Let start with the light soliton. In this case according to (18) we have a recurrency relation for the soliton velocities within two neighboring intervals between adjacent defects

$$v_{k+1} = v_k - 3\varepsilon^2/2v_k, \quad 1 \le k \le n\,. \qquad (27)$$

The solution of the latter equation in the main approximation (taking into account only terms of order of ε^2) has the form

$$v_k = v_1 - \zeta^*(k-1), \qquad (28)$$

where

$$\zeta = \zeta^*/v_1 = 3\varepsilon^2/2v_1^2 << 1 \qquad (29)$$

is the basic small parameter of the theory. In terms of dimensionless distances z_k between adjacent scatterers (see (4)), the total transmission time ΔT_n reads

$$\Delta T_n = \zeta T^0[n - \sum_{k=1}^{n}(n-k+1)z_k]\,. \qquad (30)$$

Averaging the shift ΔT_n and its square with the probability density (3) (for canonical ensemble with fixed number n) we obtain the mean transmission time shift $\langle\Delta T_n\rangle$ and its variance δT_n^l for light soliton

$$\langle \Delta T_n^l \rangle = \frac{n}{2}\zeta T^0 = \frac{3nL\varepsilon^2}{4v_1^3}, \quad \delta T_n^l = \frac{\langle \Delta T_n^l \rangle_n}{\sqrt{3n}}. \quad (31)$$

The variance smallness is provided by large number of scatterers $n >> 1$, while transmission time shift is small if $n\zeta << 1$. After the next averaging over the number of scatterers we obtain for the grand canonical ensemble

$$\langle \Delta T^l \rangle = \frac{\Lambda}{2}\zeta T^0 = \frac{3L^2\varepsilon^2}{4lv_1^3}, \quad \delta T^l = \langle \Delta T^l \rangle \sqrt{\frac{4l}{3L}}. \quad (32)$$

These results qualitatively are the same as those for a fixed number of defects. The difference is that here an average number of scatterers $\Lambda = L/l$ stands instead of n and variance numerical coefficient is changed.

For the first sight, it seems that the same problem for heavy soliton is much more complicated because in this case we should deal with double recurrency for both soliton velocity and amplitude. However in the main approximation $\propto \varepsilon^2$ the only change is appearance in recurrence (23) for velocity additional multiplier μ

$$v_k = v_1 - \mu\zeta(k-1), \quad \mu = \frac{4}{3}\pi\left(\frac{A_1}{v_1}\right)^{11/2} \exp\left(-\frac{\pi A_1}{v_1}\right). \quad (33)$$

Corresponding changes should be introduced into the final results for both fixed number of scatterers

$$\langle \Delta T_n^h \rangle = \frac{n}{2}\zeta\mu T^0 = 2\pi\frac{nL\varepsilon^2}{v_1^3}\left(\frac{A_1}{v_1}\right)^{11/2} \exp(-\pi A_1/v_1),$$
$$\delta T_n^h = \frac{\langle \Delta T_n^h \rangle_n}{\sqrt{3n}}, \quad (34)$$

and for those corresponding to the grand canonical ensemble

$$\langle \Delta T^h \rangle = \frac{\Lambda}{2}\zeta\mu T^0 = 2\pi\frac{L^2\varepsilon^2}{lv_1^3}\left(\frac{A_1}{v_1}\right)^{11/2} \exp(-\pi A_1/v_1),$$
$$\delta T^h = \langle \Delta T_n^h \rangle \sqrt{\frac{4l}{3L}}. \quad (35)$$

In this paper, we neglected the phase shifts $\varphi_{0,1}$ (5). However they also lead to transmission time shift which is of order

$$\Delta T_n^\varphi \sim \frac{n\varepsilon}{v_1^3}. \quad (36)$$

Therefore results (31) for the fixed number of scatterers are valid for sufficiently long segment $L >> 1/\varepsilon$, the validity of the results (32) for grand canonical ensemble needs the following inequality to hold: $L >> (nl/\varepsilon)^{1/2}$. Corresponding conditions for heavy soliton due to the presence of additional small parameter μ in (34), (35) look much stronger: $L >> (\varepsilon\mu)^{-1}$ (fixed number of scatterers) and $L >> (nl/\varepsilon\mu)^{1/2}$ (grand canonical ensemble). If inequalities mentioned above are not fulfilled, transmission time shift is described by (36) and is related to the main contribution to the shifts of phases.

5. Conclusion

In the framework of one-dimensional NLSE the propagation of envelope soliton through disordered system with δ-function defects is investigated in the case when the spatial size of soliton is much smaller then the average distance between the defects. In limiting cases of light and heavy solitons the shifts of soliton amplitude, velocity and transmission time after propagation through the finite segment of disordered media with fixed number of disordered impurities (canonical ensemble) and fixed average distance between disordered defects (grand canonical ensemble) are calculated.

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