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Charge and spin currents in the ballistic SNS Josephson junction between *p*-wave superconductors

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Charge and spin transport properties of a clean TS–N–TS Josephson junction (triplet superconductor-normal metal-triplet superconductor) are studied using the quasiclassical Eilenberger equation for Green's function. Effects of thickness of normal layer between superconductors on the spin and charge currents are investigated. The effect of a misorientation between triplet superconductors which creates the spin current is the main subject of this paper. It is shown that for some values of phase difference between superconductors the spin current exists in the absence of charge current and vice versa.

PACS: **74.50.+r** Tunneling phenomena; Josephson effects;
74.70.Pq Ruthenates;
74.70.Tx Heavy-fermion superconductors;
72.25.–b Spin polarized transport.

Keywords: triplet superconductor, Josephson junction, spin current.

Nowadays, the spin-triplet superconductivity is an interesting and important subject in condensed matter physics [1,2]. It has been observed in Sr_2RuO_4 Refs. 3, 4, UPt_3 and some other heavy fermion complexes Ref. 5–9. The superconductivity of ferromagnetics (UGe_2 , URhGe and ZrZn) is a new result of discovery of triplet pairing symmetry [10]. Possessing physical properties, which are essentially different from singlet superconductors, the triplet superconductors (TS's) attract a large attention of both experementalists and theorists. Specifically, much attention is paid to investigations of Josephson junctions inclusive triplet superconductors in consequence of their practical applications.

Josephson effect in the junction between singlet and triplet superconductors [11–13], misorientated TS's [14–19], TS's separated by diffusive normal layer [20], point-contact [21] had been investigated. In Ref. 22 the general formula for the dc Josepson current between superconductors with arbitrary (singlet or triplet) symmetry had been derived. In Refs. 16, 19, 23 a polarized dissipation-less supercurrent of spins had been predicted. It was shown that the current-phase dependencies are totally different from the current-phase dependencies of the junction between conventional *s*-wave superconductors [24] and high temperature *d*-wave superconductors [25].

In the present paper we investigate a Josephson junction between misorientated crystals of p-wave TS which sandwich the mesoscopic normal metal layer. The model of the *p*-wave triplet pairing symmetry, which possibly describes the superconductivity of Sr_2RuO_4 crystals [26,27], is used to illustrate general results.

Let us consider a normal metal of the thickness l between two misoriented *p*-wave TS's (see Fig. 1). For the cases $l \gg \lambda_F$ and l much smaller than electron mean free path, we can use the quasiclassical Eilenberger equation for Green function in the ballistic regime [28].

$$\mathbf{v}_F \nabla \breve{g} + \left[\varepsilon_m \breve{\sigma}_3 + i\breve{\Delta}, \breve{g} \right] = 0 \qquad (1)$$

where $\varepsilon_m = \pi T(2m+1)$ are discrete Matsubara energies $m = 0,1,2...$, T is the temperature, $\mathbf{v}_F$ is the Fermi velocity and $\breve{\sigma}_3 = \hat{\sigma}_3 \otimes \hat{I}$ in which $\hat{\sigma}_j \left(j = 1,2,3 \right)$ are Pauli matrices, $\hat{I}$ is unit matrix. Also, Green's function has to satisfy the normalization condition $\breve{g}\breve{g} = \breve{I}$.

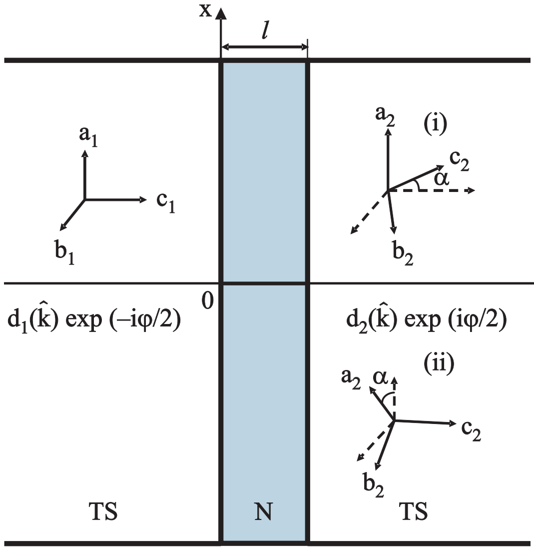

Fig. 1. Scheme of a flat interface between two superconducting bulks. Here, n = 1, 2 label the left and right half-spaces. Two superconducting bulks which are separated by a layer of normal metal have a misorientation α. Two different geometries are corresponding to the different orientations of the crystals in the right and left sides of the interface. In geometry (i), the *bc*-plane in the right side has been rotated around the *a*-axis by α. In geometry (ii) the *ab*-plane in the right side has been rotated around the *c*-axis normal to the interface (*z*-axis) by α. For both of geometries (i) and (ii), we consider a rotation only in the right superconductor and the crystallographic *a*-axis in the left half-space is selected parallel to the partition between the superconductors (*x*-axis).

The Matsubara propagator $\breve{g}$ can be written in the standard form:

$$\breve{g} = \begin{pmatrix} g_1 + \mathbf{g}_1\hat{\boldsymbol{\sigma}} & (g_2 + \mathbf{g}_2\hat{\boldsymbol{\sigma}})i\hat{\sigma}_2 \\ i\hat{\sigma}_2(g_3 + \mathbf{g}_3\hat{\boldsymbol{\sigma}}) & g_4 - \hat{\sigma}_2\mathbf{g}_4\hat{\boldsymbol{\sigma}}\hat{\sigma}_2 \end{pmatrix}, \quad (2)$$

where $\hat{\boldsymbol{\sigma}}$ is a vector with component as the Pauli matrices. The matrix structure of the off-diagonal self energy $\breve{\Delta}$ in the Nambu space is

$$\breve{\Delta} = \begin{pmatrix} 0 & \mathbf{d}\hat{\boldsymbol{\sigma}}i\hat{\sigma}_2 \\ i\hat{\sigma}_2\mathbf{d}^*\hat{\boldsymbol{\sigma}} & 0 \end{pmatrix}. \quad (3)$$

$\mathbf{d}$ is a vector order parameter of TS. In this paper, the unitary state, for which $\mathbf{d}\times\mathbf{d}^* = 0$, is investigated. Also, the unitary states vectors $\mathbf{d}_{1,2}$ can be written as $\mathbf{d}_{1,2} = \boldsymbol{\Delta}_{1,2}\exp i\psi_n$, where $\boldsymbol{\Delta}_{1,2}$ are the real vectors in the left (1) and right (2) superconductors. The Eq. (1) should be supplemented by the self-consistency equation for the vector $\mathbf{d}$

$$\mathbf{d}(\hat{\mathbf{v}}_F, \mathbf{r}) = \pi T N(0)\sum_m \left\langle V(\hat{\mathbf{v}}_F, \hat{\mathbf{v}}_F')\mathbf{g}_2(\hat{\mathbf{v}}_F', \mathbf{r}, \varepsilon_m)\right\rangle \quad (4)$$

where $V(\hat{\mathbf{v}}_F, \hat{\mathbf{v}}_F')$ is a potential of pairing interaction, $\langle ... \rangle$ stands for averaging over the directions of an electron momentum on the Fermi surface, and $N(0)$ is the electron density of states at the Fermi level of energy for one spin direction. Also we can use above equations for normal metal with considering $\mathbf{d} = 0$ at $0 < z < l$. Solutions of Eqs. (1) and (4) must satisfy the boundary conditions for Green's functions and for vector $\mathbf{d}$ in the bulks of the superconductors as follow:

$$\breve{g}(\pm\infty) = \frac{\varepsilon_m\breve{\sigma}_3 + i\breve{\Delta}_{2,1}}{\sqrt{\varepsilon_m^2 + |\mathbf{d}_{2,1}|^2}}; \quad (5)$$

$$\mathbf{d}(0 \le z \le l) = 0, \quad (6)$$

$$\mathbf{d}(\pm\infty) = \mathbf{d}_{2,1}\exp\left(\pm\frac{i\phi}{2}\right), \quad (7)$$

where ϕ is the external phase difference between the order parameters of the bulks of superconductors. Equations (1) and (4) have to be supplemented by the continuity conditions at the interface between metal and superconductors. For all quasiparticle trajectories, the Green's functions satisfy the boundary conditions in both right and left bulks as well as at the interfaces $z = 0$, $z = l$.

The system of equations (1) and (4) can be solved only numerically. Using self-consistency equation, Eq. (4), we can calculate spatial variation of order parameter. It has been shown that the absolute value of order parameter (gap vector) near the interface is suppressed, while its dependence on the direction in the momentum space remains unaltered [29]. Consequently, this suppression does not influence the Josephson effect drastically, keeping the current-phase dependence qualitatively unchanged but, it changes the amplitude value of the current.

For example, a good agreement between self-consistent and non-self-consistent results has been obtained in Ref. 25 and Refs. 29, 30. Also, it has been observed that the results of the non-self-consistent investigation of *d*-wave superconductor-ferromagnet-superconductor proximity structure in the paper [31] agree with the experimental results of the paper [32]. Consequently, the non-self-consistent formalism can be used for the junction between unconventional superconducting bulks.

In further evaluations we use the model of the constant order parameter up to the interfaces, which equals to its value (7) far from the interface in the left or right bulks. We believe that under this assumptions our results describe the real situation qualitatively. In the framework of such model, the analytical expressions for the charge and spin current can be obtained for an arbitrary symmetry of the triplet order parameter.

The solution of Eqs. (1) and (4) allows us to calculate the charge and spin current densities in normal metal. Following the Ref. 33, the expression for the charge current is:

$$\mathbf{j}_e(\mathbf{r}) = 2i\pi eTN(0)\sum_m \left\langle \mathbf{v}_F g_1(\hat{\mathbf{v}}_F, \mathbf{r}, \varepsilon_m)\right\rangle \quad (8)$$

and also for the spin current we use:

$$\mathbf{j}_{s_i}(\mathbf{r}) = i\pi\hbar TN(0)\sum_m \left\langle \mathbf{v}_F \left(\hat{\mathbf{e}}_i \mathbf{g}_1 \left(\hat{\mathbf{v}}_F, \mathbf{r}, \varepsilon_m\right)\right)\right\rangle \quad (9)$$

where $\hat{\mathbf{e}}_i = \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$.

The Eilenberger equations can be decomposed on the blocks of equations as following:

$$v_z \frac{\partial g_1}{\partial z} = i\left(\mathbf{d}\cdot\mathbf{g}_3 - \mathbf{d}^*\cdot\mathbf{g}_2\right), \quad (10)$$

$$v_z \frac{\partial \mathbf{g}_1}{\partial z} = \mathbf{g}_3 \times \mathbf{d} + \mathbf{g}_2 \times \mathbf{d}^*, \quad (11)$$

$$v_z \frac{\partial \mathbf{g}_2}{\partial z} = 2\mathbf{d}\times\mathbf{g}_1 - 2\varepsilon_m \mathbf{g}_2 + 2ig_1\mathbf{d}, \quad (12)$$

$$v_z \frac{\partial \mathbf{g}_3}{\partial z} = 2\mathbf{d}^*\times\mathbf{g}_1 + 2\varepsilon_m \mathbf{g}_3 - 2ig_1\mathbf{d}^*. \quad (13)$$

The general solution satisfying the boundary conditions (5) and Eqs. (10)–(13) in the superconducting regions are as follow:

$$g_1^{(1,2)} = \frac{\varepsilon_m}{\Omega_{1,2}} + C_{1,2} f_{1,2}, \quad (14)$$

$$\mathbf{g}_1^{(1,2)} = \mathbf{C}_{1,2} f_{1,2}, \quad (15)$$

$$\mathbf{g}_2^{(1,2)} = \frac{i\mathbf{d}_{1,2}}{\Omega_{1,2}} + \frac{2iC_{1,2}\mathbf{d}_{1,2} + \mathbf{d}_{1,2}\times\mathbf{C}_{1,2}}{\pm 2\eta\Omega_{1,2} + 2\varepsilon_m} f_{1,2}, \quad (16)$$

$$\mathbf{g}_3^{(1,2)} = \frac{i\mathbf{d}^*_{1,2}}{\Omega_{1,2}} + \frac{2iC_{1,2}\mathbf{d}^*_{1,2} - \mathbf{d}^*_{1,2}\times\mathbf{C}_{1,2}}{\mp 2\eta\Omega_{1,2} + 2\varepsilon_m} f_{1,2}, \quad (17)$$

where, $f_{1,2}(z, \varepsilon_m, \mathbf{v}_F) = \exp(-2\Omega_{1,2}\,|z/v_z|)$. Also, for the normal region:

$$g_{1N} = \tilde{g}, \quad (18)$$

$$\mathbf{g}_{1N} = \tilde{\mathbf{g}}, \quad (19)$$

$$\mathbf{g}_{2N} = \tilde{\mathbf{g}}_2 \exp\left(-2\varepsilon_m z / v_z\right), \quad (20)$$

$$\mathbf{g}_{3N} = \tilde{\mathbf{g}}_3 \exp\left(2\varepsilon_m z / v_z\right), \quad (21)$$

where $\eta = \mathrm{sgn}(v_z)$ and $\Omega_{1,2} = \sqrt{\varepsilon_m^2 + \left|\mathbf{d}_{1,2}\right|^2}$. By matching the solutions (14)–(17) at the interfaces ($z = 0$ and $z = l$), we obtain constants $C_{1,2}$, $\mathbf{C}_{1,2}$, $\tilde{g}$, $\tilde{\mathbf{g}}$, $\tilde{\mathbf{g}}_2$ and $\tilde{\mathbf{g}}_3$. Then we obtain the components of Green's functions g_1 and $\mathbf{g}_1$.

In the normal region, $\mathbf{d} = 0$, the terms g_{1N} and $\mathbf{g}_{1N}$ are constant. Using the continuity conditions, bulk solutions and Eilenberger equation we obtain new terms of Green's functions for normal metal and also at two interfaces as follow:

$$g_{1N} = \frac{i\varepsilon_m(\Omega_1+\Omega_2)\cos\beta - \eta\sin\beta(\Omega_1\Omega_2+\varepsilon_m^2)}{i\eta\sin\beta\,\varepsilon_m(\Omega_1+\Omega_2) + \cos\beta(\Omega_1\Omega_2+\varepsilon_m^2) + \mathbf{\Delta}_1\mathbf{\Delta}_2}, \quad (22)$$

$$\mathbf{g}_{1N} = \frac{\eta\mathbf{\Delta}_1\times\mathbf{\Delta}_2}{i\eta\sin\beta\,\varepsilon_m(\Omega_1+\Omega_2) + \cos\beta(\Omega_1\Omega_2+\varepsilon_m^2) + \mathbf{\Delta}_1\mathbf{\Delta}_2}, \quad (23)$$

where

$$\beta = \psi_2 - \psi_1 + \phi - \frac{2i\varepsilon_m l}{|v_z|}.$$

At the high temperatures, $T \to T_C$, simplified Green's function charge and spin terms are

$$g_{1N} = i\left(1 - \frac{\mathbf{\Delta}_1\mathbf{\Delta}_2 \mathrm{e}^{-i\eta\beta}}{2\varepsilon_m^2}\right) \text{ and } \mathbf{g}_{1N} = \frac{\eta\mathbf{\Delta}_1\times\mathbf{\Delta}_2 \mathrm{e}^{-i\eta\beta}}{2\varepsilon_m^2}. \quad (24)$$

Using the above terms of Green's function, charge and spin currents can be calculated at $T \to T_c$ analytically.

In this paper, a *p*-wave model of order parameter is considered as:

$$\mathbf{d}(T, \mathbf{v}_F) = \Delta_0(T)\hat{\mathbf{z}}\left(k_x + ik_y\right). \quad (25)$$

Here, coordinate axes $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are chosen along the crystallographic axes $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ in the left side of Fig. 1. The function $\Delta_0 = \Delta_0(T)$ in Eq. (25) describes the dependence of the order parameter $\mathbf{d}$ on the temperature T. The numerical solution of Eq. (4) in the bulk of TS gives the temperature dependence of $\Delta_0(T)$. At $T = 0$, $\Delta_0(0) = 2.05 T_c$ and at the temperatures close to $T = T_c$

$$\Delta_0(T \to T_c) = \sqrt{\frac{10\pi^2}{7\zeta(3)} T_c (T_c - T)}. \quad (26)$$

Using the suitable terms of Green's function (22), (23) we plot the charge and spin current for Josephson junction between *p*-wave superconducting crystals with pairing symmetry defined by Eq. (25). The currents are periodic functions of phase difference between superconductors. Two superconducting bulks may have misorientation which we consider in geometries (i) and (ii). For two geometries and specific model of *p*-wave pairing symmetry we have plotted the currents as the function of the phase difference in Figs. 3 for $\alpha = 0, \pi/6, \pi/4$.

Calculated current of the present triplet-normal-triplet junction is totally different from the current of singlet-triplet junction in [13], triplet-ferromagnet-triplet junction in [23] and triplet-normal-triplet junction in [20]. Figures 2, 4 illustrate the decreasing of Josephson current with the increasing the thickness of normal layer. The most important case is illustrated in Fig. 4, where the spin current is plotted as the function of the phase difference. It shows that the spin current in the Josephson junction be-

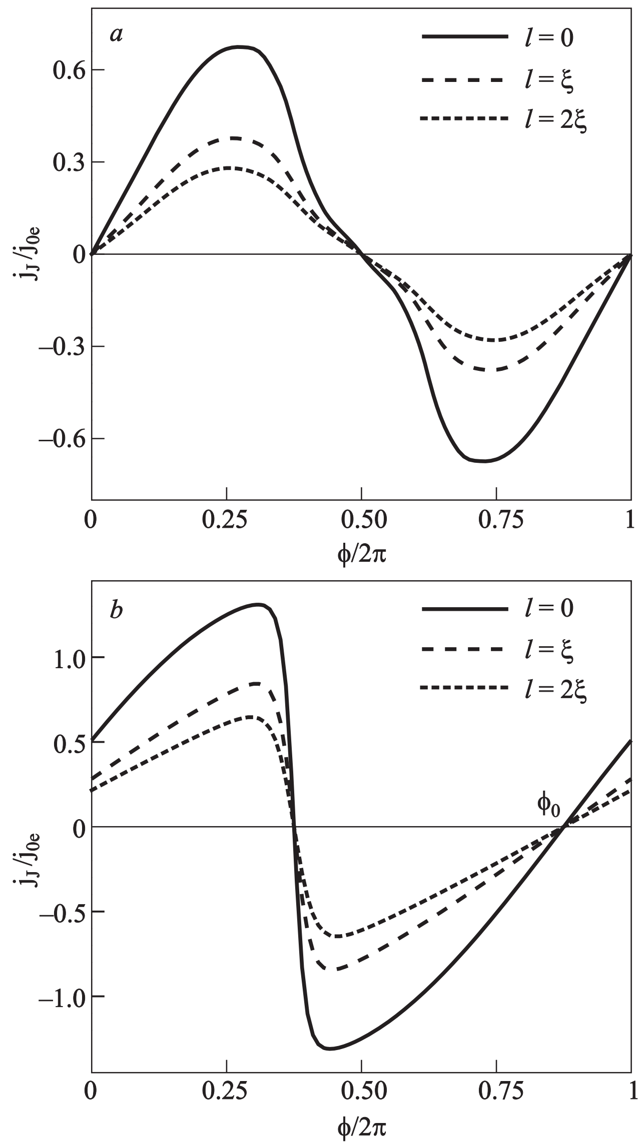

Fig. 2. The z component of the charge current (Josephson current) versus the phase difference ϕ for the order parameter (25), $\alpha = \pi/4$, $T = 0.05T_c$ and the different thicknesses of normal metal between superconductors. Here (*a*) part of figure is for geometry (i) and (*b*) for geometry (ii) and currents are given in units of $j_{0,e} = \frac{\pi}{8} eN(0)v_F\Delta_0(0)$.

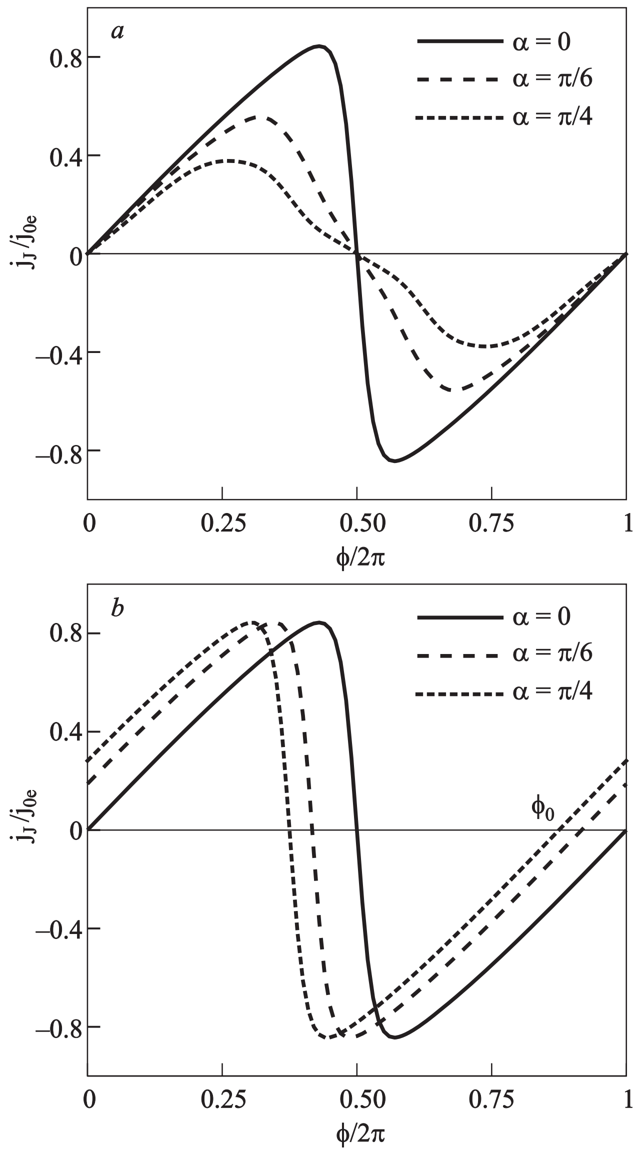

Fig. 3. The normal component (z component) of the charge current versus the phase difference ϕ, thickness of normal layer $\ell = \xi$, $T = 0.05T_c$ and the different misorientations between superconductors α. Here (*a*) part of figure is for geometry (i) and (*b*) for geometry (ii).

tween two TS's is due to the misorientation between gap vectors of superconductors (Fig. 4 and Eq. (23)). For the case of geometry (ii) we do not have spin current because both gap vectors are at the same direction but they have different values,

$$\mathbf{d}_1(\theta,\varphi) = \Delta_0(T)\hat{\mathbf{z}}\sin\theta\exp i\varphi$$

and

$$\mathbf{d}_2 = \Delta_0(T)\hat{\mathbf{z}}\sin\theta\exp i(\varphi-\alpha) .$$

Here θ and φ are polar and azimuthal angles of quasiparticle velocity (trajectory) at the Fermi energy and z axis has been specified in Fig. 1. Also, for geometry (i) we have left and right gap vectors as follow: $\mathbf{d}_1 = \Delta_0(T)(k_x + ik_y)\hat{\mathbf{z}}$ and $\mathbf{d}_2 = \Delta_0(T)(k'_x + ik'_y)\hat{\mathbf{z}}'$, in which primed notation has been used for rotated variables around the *x*-axis. We obtain that for TS–N–TS junction the polarized spin current s_x exists in the z direction normal to the interface. The question is how misorientation and normal layer do effect on spin and charge currents. Our calculations show that, misorientation creates the spin current (only for geometry (i)) and for zero misorientation, $\alpha = 0$, situation is the same as for *s*-wave junction and spin current is absent. From Eq. (23) it is clear that in the absence of misorientation (cross product of gap vectors being zero) the spin current is absent. Increasing the normal layer decreases the currents as we expect. A very interesting result is for the phase $\phi = \pi$: while the spin current exists, the charge current disappears although carriers of both of charge and spin

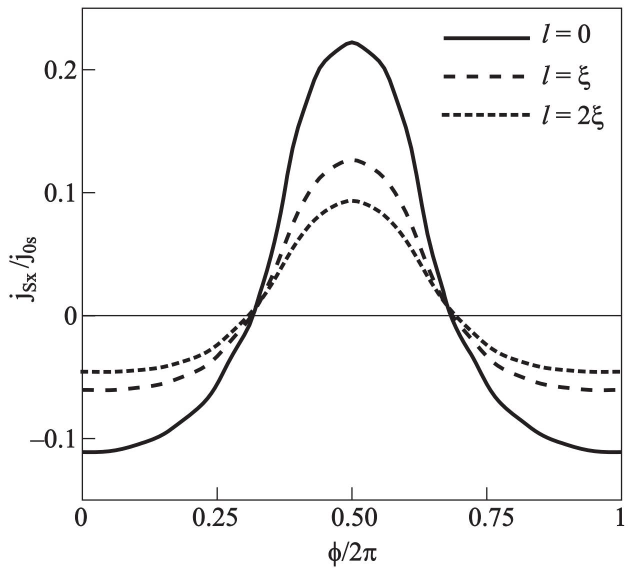

Fig. 4. The normal, z, component of the spin current (s_x) versus the phase difference ϕ for geometry (i), $\alpha = \pi/4$, $T = 0.05T_c$ and the different thicknesses l of normal metal layer. Currents are calculated in units of $j_{0,s} = \pi/8\, N(0) v_F \Delta(0) \hbar$.

are the same (paired electrons). Important to note that the spin current changes the direction in a vicinity of phase difference close to π (see Fig. 4). The similar current — phase dependence was obtained for the tangential component of the current in Josephson junction between current — carrying singlet superconductors [34,35]. In Figs. 2, 3, the current-phase dependence is plotted and it is shown that the current-phase diagrams are totally different from the case of *s*-wave Josephson junction. While the spin current (Fig. 4) is an even function of phase difference ϕ the charge current is odd one. At some value of phases ϕ the charge current is zero but spin current exists and vice versa. In Fig. 2,*b* and Fig. 3,*b*, the current will disappear at $\phi = \phi_0$ and this is a spontaneous phase difference in equilibrium state of the Josephson contact. The value of the phase ϕ_0 is determined by minimization of the total energy of the system. Also, the state with a zero Josephson current and minimum of the free energy corresponds to a finite phase difference and is not $\phi = 0$ and $\phi = \pi$. In Fig. 3,*b* the spontaneous phase difference depends on the misorientation angle and does not depend on the thickness of normal layer (Fig. 2,*b*).

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