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# Equations of motions and velocities of longitudinal waves for superfluid $^3$He–$A$ filled aerogel in the presence of finite magnetic field

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We have determined the dynamic equations for superfluid $^3$He–$A$ filled aerogel in finite magnetic fields. The speeds of propagating longitudinal modes for this system have been found for $A_1$ and $A$ phases. We have shown that sound phenomena in case $^3$He–$A_1$ superfluid are modified from those for He II superfluid.

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## 1. Introduction

Porous media filled with fluid have been intensively studied experimentally and theoretically due to their application importance in various fields. An important theme that threads through many areas of current interest in condensed matter physics is the effect of broken relative symmetry, randomness and disorder. Especially, the investigation of porous media filled with superfluid helium is a rapidly developing research [1–9]. Especially much attention has been devoted to studying the behavior of a superfluid liquid under conditions of a restricted geometry, for example, in porous nanostructures of the aerogel type.

The element helium comes in two stable forms, $^4$He and $^3$He. At low temperatures and pressures both form liquids rather than solids. The atoms of $^3$He have spin 1/2 and obey Fermi–Dirac statistics. By contrast, the atoms of $^4$He have spin zero and should therefore obey Bose–Einstein statistics. That means, that liquid $^4$He (bosons) and $^3$He (fermions) exhibit distinct mechanisms for condensation into there superfluid states. The liquid phase of the isotope $^4$He at temperatures below about 2 K shows the property of superfluidity — the ability to flow through the narrowest capillaries without friction. The mechanism of superfluidity is condensation of bosons in basic state that is forbidden by Pauli's principle for fermions.

At temperatures of the order of millikelvins, quasiparticles in liquid $^3$He are paired in the triplet state with the relative orbital angular momentum $L = 1$ and with spin $S = 1$ experience Cooper pairing leading to a transition to the superfluid state. In zero magnetic field, liquid $^3$He exists in two superfluid phases known as the $A$ and $B$ phases. $B$ phase is coherent mixture of $\uparrow\uparrow (S_z = 1)$, $\downarrow\downarrow (S_z = -1)$ and $\uparrow\downarrow (S_z = 0)$ states, with equal amplitudes of pairing $\Delta_{\uparrow\uparrow} = \Delta_{\uparrow\downarrow} = \Delta_{\downarrow\downarrow}$. In $A$ phase we have pairing in two states: $\uparrow\uparrow$ and $\downarrow\downarrow$, besides $\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow}$. In a magnetic field, $A$ phase modifies in $A_2$ phase with $\Delta_{\uparrow\uparrow} > \Delta_{\downarrow\downarrow} \neq 0$ and near normal-superfluid transition curve exists superfluid phase $^3$He$-A_1$ with $\Delta_{\uparrow\uparrow} = 1$ and $\Delta_{\downarrow\downarrow} = 0$, which has particular properties [10].

In superfluid phases of $^3$He can propagate various types of waves [11]. Normal acoustic modes in unbounded $^3$He are the first and second sounds as longitudinal spin waves, while in $^3$He$-A_1$ such modes are first sound and a spin-temperature wave. In the case of complete stagnation of the normal component in capillaries, normal modes are the fourth sound and the longitudinal spin wave in $^3$He$-A$ and a magnetoacoustic wave in $^3$He$-A_1$. In addition, transverse spin waves, orbital diffusive wave and viscous diffuse mode, in which only the normal component oscillates (if we neglect the tensor nature of kinetic coefficients appearing in the hydrody-

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namic equations), also exist in $^3$He$-A$ and $^3$He$-A_1$ [12,13].

There exists considerable interest in behavior of superfluid $^3$He$-A_1$ in the presence of a random disorder induced by highly open porous media (aerogel). The interest to such task is stipulated by experiments [7–9] in which are observed $A$-, $B$-, and $A_1$-like phases in superfluid $^3$He filled aerogel.

The goal of the present article is to obtain the linearized system of equations describing the propagation of sounds in an aerogel filled with a $^3$He$-A$ superfluid in the presence of finite magnetic field and calculate velocities of longitudinal waves. Such statement of a problem is considered for the first time.

## 2. Dynamic equations for system aerogel-superfluid $^3$He–*A* in magnetic field

Let make some notes for the beginning. Unit vector **l** appoints the direction of Cooper pairs' orbital angular momentum, and unit vector **s** represents the direction, on which projection of Cooper pairs' spins is equal to $s_z = \pm 1$ in $^3$He$-A$ phase but $s_z = 1$ in $^3$He$-A_1$ phase. In homogenous magnetic field located bulk phases of $^3$He$-A$ obtains homogenous structure of **l** and **s** vectors with $\mathbf{l} \perp \mathbf{s} \,||\, \mathbf{H}$. In the presence of boundaries usually arise nonuniform structures which refer as textures. We consider that aerogel skeleton can not make orientate effect on the vector **l** due to small diameters of skeleton creating threads. Besides we consider aerogel material as nonmagnetic. In such conditions we shall have above mentioned uniform structure.

It is known, that in acoustical processes vector **l** is possible to review as fixed [10,12]. Besides we shall not take in account liquid anisotropy, i.e., tensor character of some phenomenological parameters.

We shall consider low-frequency limit, when liquid normal component sticks fully aerogel skeleton and moves with it in common speed. Furthermore we shall consider ultra-aerogel approaching (porosity $\Phi \approx 1$ and tortuosity $\alpha_\infty \approx 1$).

To obtain dynamic equations in denoted conditions, we support approaches developed in works [1,10,14].

Mass saving low for aerogel gives

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \upsilon_n) = 0 . \tag{1}$$

Mass, entropy and magnetization along magnetic field saving lows for helium give [10,14]

$$\frac{\partial \rho}{\partial t} + \nabla(\rho_n \upsilon_n + \rho_s \upsilon_s + \alpha \rho_s \upsilon_{\rm sp}) = 0 , \tag{2}$$

$$\frac{\partial (\rho\sigma)}{\partial t} + \rho\sigma \nabla \upsilon_n = 0 , \tag{3}$$

$$M_s \frac{\partial \xi}{\partial t} + \nabla\left[\xi M_s \upsilon_n + \frac{M_s \rho_s}{\rho} \upsilon_{\rm sp} + \alpha \frac{M_s \rho_s}{\rho}(\upsilon_s - \upsilon_n)\right] = 0 , \tag{4}$$

where $\upsilon_n$, $\upsilon_s$, $\upsilon_{\rm sp}$, $\sigma$, $\rho_n$, $\rho_s$, $\rho$, and $\rho_a$ are normal motion (and skeleton of aerogel) velocity, superfluid motion velocity, superfluid spin velocity, entropy of unit mass of liquid, normal component, superfluid component, liquid and aerogel mass densities, respectively. The longitudinal magnetization is $M = \xi M_s$ ($\xi << 1$ for a degenerate Fermi system), $M_s$ is the magnetization of polarized liquid $^3$He ($M_s = \hbar\gamma\rho/(2m)$). The coefficient $\alpha$ is expressed in terms of the pairing amplitudes

$$\alpha = \frac{\Delta^2_{\uparrow\uparrow} - \Delta^2_{\downarrow\downarrow}}{\Delta^2_{\uparrow\uparrow} + \Delta^2_{\downarrow\downarrow}}$$

being equal to zero for the $A$ phase ($\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow}$), equal to unity for the $A_1$ phase ($\Delta_{\downarrow\downarrow} = 0$) and decreases moving away from $A_1$–$A_2$ phase transit curve.

The superfluid motion velocity and superfluid spin velocity of the liquid satisfy equations [10,14]

$$\frac{\partial \upsilon_s}{\partial t} = -\frac{1}{\rho}\nabla P + \sigma \nabla T , \tag{5}$$

$$\frac{\partial \upsilon_{\rm sp}}{\partial t} = -\frac{M_s}{\rho}\nabla h , \tag{6}$$

where $P$, $T$, $h$ are the pressure, temperature and the internal magnetic field.

We shall consider the low-frequency limit when viscous penetration depth is much larger than the pore size, so that the normal fluid and the aerogel matrix must be moved together. So, the extra inertia of the normal fluid due to the matrix is accounted by replacing normal mass density in equation of motion of normal component with $\rho_n + \rho_a$. Also, the extra restoring force due to the matrix is accounted by adding a term $-\nabla P_a$ in the equation, where $P_a$ is related to the speed of sound in dry aerogel $C_a$ with $C_a^2 = \partial P_a / \partial \rho_a$. Then for equation of joint motion of matrix and normal component of liquid we have

$$(\rho_a + \rho_n)\frac{\partial \upsilon_n}{\partial t} = -\frac{\rho_n}{\rho}\nabla P - \nabla P_a - \\ - \rho_s \sigma \nabla T - M_s\left(\xi - \alpha \frac{\rho_s}{\rho}\right)\nabla h . \tag{7}$$

Obtained seven equations describe dynamic of aerogel filled with superfluid $^3$He–$A$ liquid in low-frequency limit for zero ($\alpha = 0$ and $\xi = 0$) and for finite magnetic field conditions. The Eqs. (1)–(7) are obtained by us bring together the equations of three-speed hydrodynamics of superfluid helium [10–14] and the equations of motion of aerogel [1] with taking into account the specificity of studied system.

### 3. Sound velocities in *A*-like phase

Assuming that all alternating quantities vary according to the law $\exp i(\omega t - kx)$, we can obtain dispersion equation for longitudinal sound velocities from dynamic Eqs. (1)–(7). In zero magnetic field ($\alpha = 0$ and $\xi = 0$) we have

$$\left(C^2 - \frac{\rho_n}{\rho} C_{\rm sp}^2\right)\times$$
$$\times\left\{(C^2 - C_2^2)(C^2 - C_1^2) + \frac{\rho_a}{\rho_n}(C^2 - C_4^2)(C^2 - C_a^2)\right\} = 0 . \quad (8)$$

Here $C_1, C_2, C_4, C_{\rm sp}$ are the usual bulk velocities of first, second, fourth and spin waves, respectively:

$$C_{\rm sp}^2 = \left(\frac{\rho_s}{\rho_n}\right)\left(\frac{M_s}{\rho}\right)\left(\frac{\partial h}{\partial \xi}\right). \quad (9)$$

One solution of secular Eq. (8) is independent spin wave $C^2 = (\rho_n/\rho)C_{\rm sp}^2$ in which only longitudinal magnetization oscillates. Other two solutions in form coincide with velocities that were derived in Refs. 1, 3:

$$C_{\rm fast}^2 = \frac{C_1^2 + \frac{\rho_a}{\rho_n} C_4^2}{1 + \frac{\rho_a}{\rho_n}}, \quad C_{\rm slow}^2 = \frac{C_2^2 + \frac{\rho_a \rho_s}{\rho\rho_n} C_a^2}{1 + \frac{\rho_a \rho_s}{\rho\rho_n}} . \quad (10)$$

It is particularly interesting, that velocity of slow mode (temperature wave) in $^3$He–$A$ phase increasing when we put it in aerogel, besides as more as bigger $\rho_a/\rho$ ratio is. Most likely, this circumstance will allow discover temperature wave that was not possible till now in bulk *A* phase.

### 4. Sound velocities in $A_1$-like phase

For longitudinal sound velocities in $A_1$ phase ($\alpha = 1$, $\xi \ll 1$ and $\rho_s \ll \rho$) filled aerogel from dynamic Eqs. (1)–(7) we derive the following dispersion equation:

$$(C^2 - C_1^2)(C^2 - C_2^2 - C_{\rm sp}^2) +$$
$$+ \frac{\rho_a}{\rho}(C^2 - C_a^2)(C^2 - C_{\rm sp}^2) = 0 . \quad (11)$$

The dispersion equation (11) has two solutions. In case $\rho_a \ll \rho$ the solutions are first sound $C = C_1$ and spin-temperature wave $C = \sqrt{C_2^2 + C_{\rm sp}^2} \approx C_{\rm sp}$ that in bulk phase was observed experimentally by Corruccini and Osheroff [15]. In case $\rho_a \gg \rho$ the solutions are waves in aerogel $C = C_a$ and longitudinal spin wave $C = C_{\rm sp}$.

Commonly, for fast mode we have

$$C_{\rm fast}^2 = \frac{C_1^2 + (\rho_a/\rho)\, C_a^2}{1 + (\rho_a/\rho)} . \quad (12)$$

And for slow wave we have

$$C_{\rm slow}^2 = \frac{C_1^2 (C_2^2 + C_{\rm sp}^2) + \frac{\rho_a}{\rho} C_a^2 \left(\frac{\rho_s}{\rho} C_1^2 + \frac{\rho_n}{\rho} C_2^2 + \frac{\rho_n}{\rho} C_{\rm sp}^2\right)}{C_1^2 + \frac{\rho_a}{\rho} C_a^2} \approx$$
$$\approx C_{\rm sp}^2 + C_2^2 . \quad (13)$$

Obtained results are pointed important difference compared with He II filled aerogel. On the one hand, it is caused by that $^3$He–$A_1$ phase exists near normal-superfluid transition curve and $\rho_s/\rho \ll 1$, on the other hand, oscillations of temperature are bound with oscillations of longitudinal magnetization and propagate with the higher spin velocity in comparison of velocity of the second sound.

### 5. Summary

We obtain the linearized system of equations describing the propagation of sounds in a nonmagnetic aerogel filled with superfluid $^3$He–$A$ in magnetic field. So, the system was modeled by combining the equations of superfluid hydrodynamics of $^3$He–$A$ helium with those of elasticity of aerogel. The wave propagation velocities are calculated for a highly porous medium — nonmagnetic aerogel filled with a superfluid $^3$He–$A$ in zero and finite magnetic fields. We have been shown that sound phenomena in $^3$He–$A$ aerogel system in presence of magnetic field are quite different from ones in He II aerogel system.

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