

Kinetic equations in the theory of normal Fermi liquid

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On the bases of the improved approximation for the spectral function of one-particle states the Landau–Silin kinetic equations for the normal Fermi liquids of neutral and electrically charged particles are shown to be valid at finite temperature above the temperature of superfluid transition.

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71.10.Ay Fermi-liquid theory and other phenomenological models;
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1. Introduction

The exclusive successfulness of the phenomenological Landau normal Fermi liquid theory [1] in predicting and describing a set of new phenomena, among them the zero sound in liquid He³ and spin waves in nonferromagnetic metals, made this theory a subject of investigation on the basis of strict microscopic theory. Most attention was devoted to the derivation of the kinetic equation for the quasiparticle distribution function. The initial derivation of this equation was produced by Kadanoff and Byam [2] on the basis of the quasiparticle (QP) approximation for the spectral function and was continued by some followers who used so called extended quasiparticle (EQP) approximation [3–7]. However, in all these cases the second Poisson bracket in the left side of Kadanoff–Baym (KB) generalized kinetic equation could not be eliminated in a lawful mathematical way (see below). This fact made the temperature range of validity of the kinetic equation for quasiparticle distribution very narrow, strictly speaking the theory was proved to be valid only in the vicinity of absolute zero.

Experimental discovery of the superfluidity of He³ at the temperature lower than the temperature at which the zero-sound in the normal Fermi liquid was discovered, and theoretical works devoted to the description of the superfluid state [8,9], left no room for the temperature range of validity of the equation for normal Fermi liquid. At the same time theoretical predictions of this theory

turned to be in a perfect numerical agreement with the experimental data. This fact stimulated a second wave of attempts to deriving the kinetic equation for normal Fermi liquid, but as it was mentioned above, the result was not achieved to a satisfactory extent. Further development of KB theory went in the direction of the constructing of a nonlocal quasiparticle kinetic equations [10,11], a development of a detailed selfconsistent microscopic treatment of arbitrary initial correlations in the system [12], etc., but the question of the temperature range of validity of the Landau’s kinetic equation remained open.

2. Spectral function and kinetic equation for normal Fermi liquid

The problem consisted in a mathematically lawful elimination of the second generalized Poisson bracket (a «puzzling term» in the terminology accepted in [5]) in the generalized Kadanoff–Byam kinetic equation written in the collisionless approximation in the case of slowly varying in space and time disturbances when only the first derivatives with respect to T and \mathbf{R} are taken into account [2]:

$$[\omega - e(\mathbf{p}\omega; \mathbf{R}T), g^<(\mathbf{p}\omega; \mathbf{R}T)] + \\ + [\text{Re } g(\mathbf{p}\omega; \mathbf{R}T), \sigma^<(\mathbf{p}\omega; \mathbf{R}T)] = 0, \quad (1)$$

here $[A, B]$ — the generalized Poisson bracket, defined by the expression:

$$[A, B] = \frac{\partial A}{\partial \omega} \frac{\partial B}{\partial T} - \frac{\partial A}{\partial T} \frac{\partial B}{\partial \omega} - \nabla_{\mathbf{p}} A \cdot \nabla_{\mathbf{R}} B + \nabla_{\mathbf{R}} A \cdot \nabla_{\mathbf{p}} B, \quad (2)$$

$e(\mathbf{p}\omega; \mathbf{R}T)$ — the energy of a particle, defined by the equality:

$$e(\mathbf{p}\omega; \mathbf{R}T) = E^{HF}(\mathbf{p}; \mathbf{R}T) + \text{Re} \sigma(\mathbf{p}\omega; \mathbf{R}T), \quad (3)$$

E^{HF} is a one-particle energy in the Hartree–Fock approximation, $\text{Re} \sigma$ is a real part of the correlation energy of a particle related to the imaginary part through the Hilbert transform,

$$\text{Re} \sigma(\mathbf{p}\omega; \mathbf{R}T) = P \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\Gamma(\mathbf{p}\omega'; \mathbf{R}T)}{\omega - \omega'}. \quad (4)$$

Here, P refers to a principle value integration.

It was shown in [2] that the function g entering the Eq. (1) can be taken in the form

$$g(\mathbf{p}Z; \mathbf{R}T) = [Z - E^{HF}(\mathbf{p}; \mathbf{R}T) - \text{Re} \sigma(\mathbf{p}Z; \mathbf{R}T)]^{-1}. \quad (5)$$

Equations (4) and (5) lead to the following general form for a spectral function of one-particle states in the system:

$$a(\mathbf{p}\omega; \mathbf{R}T) = \frac{\Gamma(\mathbf{p}\omega; \mathbf{R}T)}{[\omega - e(\mathbf{p}\omega; \mathbf{R}T)]^2 + \Gamma^2(\mathbf{p}\omega; \mathbf{R}T)/4}. \quad (6)$$

The spectral function satisfies the exact sum rule:

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} a(\mathbf{p}\omega; \mathbf{R}T) = 1. \quad (7)$$

Correlation function $g^<$ is related to a spectral function by the equality [2]:

$$g^<(\mathbf{p}\omega; \mathbf{R}T) = a(\mathbf{p}\omega; \mathbf{R}T) f(\mathbf{p}\omega; \mathbf{R}T). \quad (8)$$

The complexity of the expression (6) makes to search for a certain approximations for the spectral function which could be successfully used in calculations.

Quasiparticle approximation for the spectral function (6) corresponds to the case $\Gamma \rightarrow 0$, when the quasiparticles are stable:

$$a_{QP} = 2\pi Z(\mathbf{p}; \mathbf{R}T) \delta[\omega - E(\mathbf{p}; \mathbf{R}T)], \quad (9)$$

where $E(\mathbf{p}; \mathbf{R}T)$ is the solution of the equation:

$$E(\mathbf{p}; \mathbf{R}T) = E^{HF}(\mathbf{p}; \mathbf{R}T) + \text{Re} \sigma(\mathbf{p}\omega; \mathbf{R}T)|_{\omega=E(\mathbf{p}; \mathbf{R}T)}, \quad (10)$$

and the renormalizing factor $Z(\mathbf{p}; \mathbf{R}T)$ is given by the expression:

$$Z^{-1}(\mathbf{p}; \mathbf{R}T) = 1 - \frac{\partial \text{Re} \sigma(\mathbf{p}\omega; \mathbf{R}T)}{\partial \omega} \Big|_{\omega=E(\mathbf{p}; \mathbf{R}T)}. \quad (11)$$

Using the approximation (9) one follows the KB way to deriving the Landau's kinetic equation when the second Poisson bracket in Eq. (1) is dropped considering $\sigma^< = f \Gamma$ to

be negligibly small what is valid only at $\omega = \mu$ at zero temperature.

The extended quasiparticle (EQP) [3–7] and improved extended quasiparticle (iEQP) [4] approximations for the spectral function are written in the form:

$$a_{EQP} = 2\pi Z \delta(\omega - E) + P \frac{\Gamma}{(\omega - E)^2}, \quad (12)$$

$$a_{iEQP} = 2\pi Z \delta(\omega - E) + ZP \frac{\Gamma}{(\omega - E)^2}, \quad (13)$$

where E and Z are determined by the Eqs. (10) and (11) correspondingly.

It is easy to verify that the approximations (12) and (13) do not satisfy the KB equation for a nonequilibrium spectral function:

$$[\omega - e(\mathbf{p}\omega; \mathbf{R}T), a(\mathbf{p}\omega; \mathbf{R}T)] + [\text{Re} g(\mathbf{p}\omega; \mathbf{R}T), \Gamma(\mathbf{p}\omega; \mathbf{R}T)] = 0, \quad (14)$$

and do not lead to the elimination of the second Poisson bracket in Eq. (1) in the case of finite Γ . Only such a mathematically lawful elimination of this term for finite values of Γ would testify the validity of the Landau's equation at a finite range of temperature.

The expressions (12) and (13) both were obtained on the basis of the general expression (6) by means of the Taylor expansion in powers of Γ in the frame of different approximations [3–7]. As a result, some important factors were lost. In reality there does not exist a mathematically strict correct form for the expansion of (6) in power series of Γ which starts with the delta function when $\Gamma = 0$.

Another expression for the spectral function can be offered on the basis of the following consideration. We utilize a well known relation of the Fourier transform in the case of a constant value of c :

$$\int_{-\infty}^{\infty} e^{-|t|c} e^{itx} dt = \frac{2c}{c^2 + x^2}, \quad c > 0. \quad (15)$$

Expanding the first exponent in the left side of (15) in Taylor series, we get:

$$\frac{2c}{c^2 + x^2} = \int_{-\infty}^{\infty} (1 - c|t| + \frac{c^2 t^2}{2!} - \dots) e^{itx} dt. \quad (16)$$

Now we use the formulas equivalent to those represented in [13,14]:

$$\int_{-\infty}^{\infty} t^{(2n)} e^{itx} dt = 2\pi (-i)^{2n} \delta^{(2n)}(x), \quad n = 0, 1, 2, \dots, \quad (17)$$

$$\int_{-\infty}^{\infty} |t|^{(2n+1)} e^{itx} dt = -2 \sin\left((2n+1) \frac{\pi}{2}\right) (2n+1)! \frac{1}{|x|^{2n+2}},$$

$$n=0, 1, 2, \dots \quad (18)$$

If the quantity Γ in Eq. (6) was not a function of ω , then the expressions (17), (18) would lead to a strict correct expansion of the spectral function (6) in terms of the power series of Γ . In the case of Γ depending on ω , one can rely only on the first two terms of the expansion: the delta function independent of Γ and the term proportional to Γ . Then, taking into account that

$$\omega - e(\mathbf{p}\omega; \mathbf{R}T) = Z^{-1}(\mathbf{p}; \mathbf{R}T)[\omega - E(\mathbf{p}; \mathbf{R}T)], \quad (19)$$

we come to the following approximation for the spectral function (6)

$$a = 2\pi Z \delta(\omega - E) + Z^2 P \frac{\Gamma}{(\omega - E)^2}. \quad (20)$$

It is not difficult to verify that the approximation (20) satisfies the sum rule (7), with the same precision as approximations (12) and (13) do, but contrary to them the approximation (20) satisfies the Eq. (14) for the spectral function and eliminates the second Poisson bracket in the left side of Eq. (1). Indeed, when we substitute Eq. (20) into Eq. (14), the first term in the right of this expression gives:

$$[\omega - e, 2\pi Z \delta(\omega - E)] = 2\pi Z [(\omega - E) Z^{-1}, \delta(\omega - E)] = 0, \quad (21)$$

due to the property of the generalized Poisson bracket

$$[A, f(A)] = 0.$$

The second term in the right side of Eq. (20) leads to the expression:

$$\left[\omega - e, Z^2 \frac{\Gamma}{(\omega - E)^2} \right] = Z \frac{1}{(\omega - E)^2} [\omega - E, \Gamma], \quad (22)$$

the second poisson bracket in Eq. (14) due to Eq. (5) gives the expression:

$$[\text{Re } g, \Gamma] = Z \left[\frac{1}{(\omega - E)}, \Gamma \right] = -Z \frac{1}{(\omega - E)^2} [\omega - E, \Gamma]. \quad (23)$$

Thus, the *ansatz* (20) satisfies the Eq. (14) exactly. Now we consider the Eq. (1). Using Eq. (8), we get:

$$[\omega - e, af] + [\text{Re } g, f\Gamma] = 0. \quad (24)$$

The substitution of the first term in Eq. (20) into Eq. (24) leads directly to the kinetic equation for the quasiparticle distribution function $n(\mathbf{p}; \mathbf{R}T)$ [2]:

$$\frac{\partial n}{\partial T} + \nabla_{\mathbf{p}} E \cdot \nabla_{\mathbf{R}} n - \nabla_{\mathbf{R}} E \cdot \nabla_{\mathbf{p}} n = 0, \quad (25)$$

$$n(\mathbf{p}; \mathbf{R}T) = f(\mathbf{p}\omega; \mathbf{R}T)|_{\omega=E(\mathbf{p}; \mathbf{R}T)}.$$

The second term in Eq. (20), being substituted to Eq. (24), gives the expression:

$$Z^2 \left[\omega - e, \frac{\Gamma f}{(\omega - E)^2} \right] = Z [\omega - E, \Gamma f] \frac{1}{(\omega - E)^2}, \quad (26)$$

which is eliminated by the second poisson bracket in Eq (24):

$$\begin{aligned} [\text{Re } g, \sigma^<] &= \left[\frac{1}{\omega - e}, \Gamma f \right] = Z \left[\frac{1}{\omega - E}, \Gamma f \right] = \\ &= Z [\Gamma f, \omega - E] \frac{1}{(\omega - E)^2}. \end{aligned} \quad (27)$$

Thus, the kinetic Eq. (25) is valid for finite values of Γ and correspondingly, for nonzero temperatures. For qualitative estimation of the precision, we can use the third term in the expansion (16) which is proportional to Γ^2 . Substituting this term into Eq. (24), it is not difficult to show that this equation is valid up to the terms of order Γ^2 .

3. Kinetic equation for normal Fermi liquid in a magnetic field

The kinetic equation for the normal Fermi liquid consisting of charged particles in the presence of compensating background was considered in [15] in the quasiparticle approximation for the spectral function. It was shown that the spin splitting of the energy levels being neglected, the spectral function for a system in a non-quantizing magnetic field can be written in the form:

$$a(\mathbf{p}\omega; \mathbf{R}T) = \frac{\Gamma(\mathbf{p}\omega; \mathbf{R}T)}{\left[\omega - \frac{(\mathbf{p} - e \mathbf{A}(\mathbf{R}T)/c)^2}{2m} - u(\mathbf{R}T) - \sigma^{HF}(\mathbf{p}; \mathbf{R}T) - \text{Re } \sigma(\mathbf{p}\omega; \mathbf{R}T) \right]^2 + \frac{\Gamma^2(\mathbf{p}\omega; \mathbf{R}T)}{4}}, \quad (28)$$

where \mathbf{p} is the canonical momentum and $u(\mathbf{R}T)$, $\mathbf{A}(\mathbf{R}T)$ are scalar and vector potentials of the electromagnetic field, $\mathbf{A}(\mathbf{R}T)$ being chosen in the Coloumb gauge $\text{div } \mathbf{A}(\mathbf{R}T) = 0$.

The kinetic equation for quasiparticle distribution is written in the form (25) [15] and acquires a usual form after the transition to kinetic momentum $\mathbf{P} = \mathbf{p} - (e/c)\mathbf{A}(\mathbf{R}T)$ [1].

We would like to stress that the kinetic equations for normal Fermi liquid were written in [16] right in the form, corresponding the expression (28) for the spectral function, concerning the dependence of all the quantities on vector potential \mathbf{A} and canonical momentum \mathbf{p} and then were transformed into gauge-invariant form [17]. More details about the development of transport theory of interacting fermions in an electromagnetic field can be found in [18–20].

The derivation of the phenomenological Landau–Silin kinetic equations in the case when spin splitting of energy levels is taken into account was produced in [21] also in the quasiparticle approximation. In this case all quantities become matrices in spin space, in particular the quantity $e(\mathbf{p}\omega; \mathbf{RT})$ has the form:

$$e(\mathbf{p}\omega; \mathbf{RT}) = \left[\frac{(\mathbf{p} - e\mathbf{A}(\mathbf{RT})/c)^2}{2m} + u(\mathbf{RT}) \right] \mathbf{I} - \frac{e}{mc} \hat{\mathbf{S}} \cdot \text{curl} \mathbf{A}(\mathbf{RT}) + \tilde{\sigma}(\mathbf{p}\omega; \mathbf{RT}) \quad (29)$$

where \mathbf{I} and $\hat{\mathbf{S}}$ are the unit matrix and the set of three spin matrices [1], $\tilde{\sigma}$ is a Hermitian part of the self-energy matrix σ . The spectral function is still given by Eq. (28). The spectral function a and the inverse life-time of a particle's state Γ are Hermitian matrices in the case under consideration.

When Γ is considered to be finite (nonzero), the next possibilities can occur. If the magnetic field is not strong, so that Γ exceeds the spin splitting of the energy levels, the last is not essential and $e(\mathbf{p}\omega; \mathbf{RT})$ can be written in the form:

$$e(\mathbf{p}\omega; \mathbf{RT}) = \frac{(\mathbf{p} - e\mathbf{A}(\mathbf{RT})/c)^2}{2m} + u(\mathbf{RT}) + \sigma^{HF}(\mathbf{p}; \mathbf{RT}) + \text{Re} \sigma(\mathbf{p}\omega; \mathbf{RT}). \quad (30)$$

The approximation for the spectral function in this case can be written in a way analogous to Eq. (20):

$$a(\mathbf{p}\omega; \mathbf{RT}) = 2\pi Z(\mathbf{p}; \mathbf{RT}) \delta(\omega - E(\mathbf{p}; \mathbf{RT})) + Z^2 P \frac{\Gamma}{(\omega - E)^2}, \quad (31)$$

where E is a solution of Eq. (10) with e given by Eq. (30).

The situation turns to be more complicated when spin splitting of the energy levels should be taken into account. More convenient form of the theory suitable for the generalization of the quasiparticle approximation for the spectral function, than that presented in [21], was developed in [22]. First of all, we should stress, that even in the quasiparticle approximation for the spectral function the kinetic equation of the phenomenological theory are valid only with the precision to the squared ratio of spin splitting of the energy levels to the chemical potential of the

system [22]. Thus, producing the derivation of the kinetic equation with the improved approximation for the spectral function we should take into account only the terms that do not exceed this precision.

We start with the expansion of the matrices $g^<$ and e over the full set of matrices in spin space,

$$g^< = \frac{1}{2} g_0 \mathbf{I} + \hat{\mathbf{S}} \cdot \hat{\mathbf{g}}, \quad (32)$$

$$e = e_1 \mathbf{I} + 2\hat{\mathbf{S}} \cdot \hat{\mathbf{e}}_2. \quad (33)$$

These expansions should be substituted into the generalized KB kinetic equations which under consideration are written in the form [22]:

$$(\omega \mathbf{I} - e + \frac{1}{2} \Gamma) g^< + \frac{1}{2} \left[\omega \mathbf{I} - e + \frac{1}{2} \Gamma, g^< \right] - \sigma^< \left(\tilde{\mathbf{g}} + \frac{1}{2} a \right) - \frac{1}{2} \left[\sigma^<, \tilde{\mathbf{g}} + \frac{1}{2} a \right] = 0, \quad (34)$$

$$g^< (\omega \mathbf{I} - e - \frac{1}{2} \Gamma) + \frac{1}{2} \left[g^<, \omega \mathbf{I} - e - \frac{1}{2} \Gamma \right] - \left(\tilde{\mathbf{g}} - \frac{1}{2} a \right) \sigma^< - \frac{1}{2} \left[\tilde{\mathbf{g}} - \frac{1}{2} a, \sigma^< \right] = 0, \quad (35)$$

where $\tilde{\mathbf{g}}$ is a Hermitian part of the matrix $g(\mathbf{pZ}; \mathbf{RT})$. Separating in Eqs. (34) and (35) the terms corresponding to the collisionless case, we get in the quasiparticle approximation ($\Gamma = 0$) the equations:

$$[\omega - e_1, g_0] - [e_{2i}, g_i] = 0, \quad (36)$$

$$[\omega - e_1, \mathbf{g}] - [e_2, g_0] - 2\mathbf{e}_2 \times \mathbf{g} = 0, \quad (37)$$

$$(\omega - e_1) g_0 - \mathbf{e}_2 \cdot \mathbf{g} = 0, \quad (38)$$

$$(\omega - e_1) g_i - e_{2i} g_0 = 0. \quad (39)$$

Here, the quantities with Latin indexes are the cartesian projections of the corresponding vectors: Summation over the repeated indexes is supposed to be done. The Eqs. (36)–(39) lead to the kinetic equations of the phenomenological Landau–Silin theory [22] in the limit $\Gamma \rightarrow 0$: the Eqs. (38) and (39) give the expressions for the spectral functions, while the Eqs. (36) and (37) give kinetic equations on the basis of the determined spectral functions.

With the help of Eqs. (38) and (39) we get the following expressions for the functions g_0 and \mathbf{g} in the quasiparticle approximations [22]:

$$g_0 = f_\uparrow \delta(\omega - e_\uparrow) + f_\downarrow \delta(\omega - e_\downarrow), \quad (40)$$

$$\mathbf{g} = \mathbf{f}_\uparrow \delta(\omega - e_\uparrow) + \mathbf{f}_\downarrow \delta(\omega - e_\downarrow) + \mathbf{f} \delta(\omega - e_1), \quad (41)$$

where

$$e_{\uparrow} = e_1 - |\mathbf{e}_2|; \quad e_{\downarrow} = e_1 + |\mathbf{e}_2|. \quad (42)$$

Vector \mathbf{f}_{\uparrow} is antiparallel to \mathbf{e}_2 , \mathbf{f}_{\downarrow} is parallel to \mathbf{e}_2 , and \mathbf{f} is perpendicular to \mathbf{e}_2 .

The argument of all the delta functions in (40), (41) except one of them, are not equal to $\omega - e_1$. Thus, the substitution of expressions (40), (41) into the Eqs. (36) and (37) will generate the terms with the derivatives of the δ -functions. As a result, after rather tedious transformations of Eq. (36) we get the Eq. (19.41) from [22]. The analysis of the distinction between different renormalizing factors entering this equation, leads to the mentioned above conclusion about the precision of the validity of the phenomenological kinetic equation in the case $\Gamma = 0$, i.e. at zero temperature. In the case of finite Γ , (but Γ being less than the spin splitting) it is necessary to put down the system of Eqs. (36)–(39) in the collisionless approximation, but saving the terms with Γ , which do not enter the collision integrals. It can be done, tracing carefully what terms with Γ in the equations in the absence of magnetic field do not enter collision integrals. Thus, we come to the system of equations:

$$(\omega I - e)g^{\lessgtr} + \frac{1}{2}[\omega I - e, g^{\lessgtr}] - \sigma^{\lessgtr} \tilde{g} - \frac{1}{2}[\sigma^{\lessgtr}, \tilde{g}] = 0, \quad (43)$$

$$g^{\lessgtr}(\omega I - e) + \frac{1}{2}[g^{\lessgtr}, \omega I - e] - \tilde{g}\sigma^{\lessgtr} - \frac{1}{2}[\tilde{g}, \sigma^{\lessgtr}] = 0, \quad (44)$$

where the expansions of the type (32), (33) should be done for the functions \tilde{g} and $\sigma^{\lessgtr} = \Gamma f$:

$$\tilde{g} = \frac{1}{2}\tilde{g}_0 I + \hat{\mathbf{S}}\tilde{\mathbf{g}}, \quad (45)$$

$$\sigma^{\lessgtr} = \sigma_1^{\lessgtr} I + 2\hat{\mathbf{S}} \cdot \sigma_2^{\lessgtr}. \quad (46)$$

Finally, taking into account the comment about the terms, exceeding the precision of the equation's validity, we get the system of equations that will lead to the kinetic equations:

$$[\omega - e_1, g_0] - [e_{2i}, g_i] + [\tilde{g}_0, \sigma_1^{\lessgtr}] = 0, \quad (47)$$

$$[\omega - e_1, \mathbf{g}] - [\mathbf{e}_2, g_0] - 2\mathbf{e}_2 \times \mathbf{g} + [\tilde{g}_0, \sigma_2^{\lessgtr}] = 0, \quad (48)$$

and the system of equations that will give the expressions for spectral functions:

$$(\omega - e_1)g_0 - \mathbf{e}_2 \cdot \mathbf{g} - \tilde{g}_0\sigma_1^{\lessgtr} = 0, \quad (49)$$

$$(\omega - e_1)g_i - e_{2i}g_0 - \tilde{g}_0\sigma_{2i}^{\lessgtr} = 0. \quad (50)$$

Now we consider the case $e_{\uparrow} = e_1 - |\mathbf{e}_2|$ and choose the approximations for g_0 and \mathbf{g} in the form:

$$g_0 = Z_{\uparrow} f_{\uparrow} \delta(\omega - E_{\uparrow}) + Z^2 \frac{\sigma_1^{\lessgtr}}{(\omega - E)^2}, \quad (51)$$

$$\mathbf{g} = Z_{\uparrow} \mathbf{f}_{\uparrow} \delta(\omega - E_{\uparrow}) + Z^2 \frac{\sigma_2^{\lessgtr}}{(\omega - E)^2}, \quad (52)$$

where E_{\uparrow} and E are the solutions of the equations:

$$E(\mathbf{p}; \mathbf{R}T) = e_1(\mathbf{p}; \mathbf{R}T)|_{\omega=E}, \quad (53)$$

$$E_{\uparrow}(\mathbf{p}; \mathbf{R}T) = [e_1(\mathbf{p}\omega; \mathbf{R}T) - e_2(\mathbf{p}; \mathbf{R}T)]|_{\omega=E}, \quad (54)$$

and renormalizing factors Z_{\uparrow} and Z are defined by the expressions:

$$Z_{\uparrow}^{-1}(\mathbf{p}; \mathbf{R}T) = 1 - \left. \frac{\partial e_{\uparrow}(\mathbf{p}\omega; \mathbf{R}T)}{\partial \omega} \right|_{\omega=E_{\uparrow}}, \quad (55)$$

$$Z^{-1}(\mathbf{p}; \mathbf{R}T) = 1 - \left. \frac{\partial e_1(\mathbf{p}\omega; \mathbf{R}T)}{\partial \omega} \right|_{\omega=E}. \quad (56)$$

If we substitute the expressions (51), (52) into Eqs. (49) and (50), these equations would be satisfied with the precision to the terms of the order Γ^2 , $|\mathbf{e}_2|^2$ and $\Gamma|\mathbf{e}_2|$. The substitution of expressions (51), (52) into Eqs. (47) and (48) lead to the Landau–Silin kinetic equations with the precision to the terms of the indicated order. The same is valid, if we consider the case $e_{\downarrow} = e_1 + |\mathbf{e}_2|$. Thus, the kinetic equations of the phenomenological theory, (the Eqs. (7.21) and (7.23) in [22]) are valid in the case of finite temperature up to the terms linear in Γ , provided Γ does not exceed the spin splitting of the energy levels.

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