CAVITY WITH DISTRIBUTED DIELECTRIC COATING FOR SUBTERAHERTZ SECOND-HARMONIC GYROTRON

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Metallic cavity with distributed dielectric coating for the 0.4 THz second harmonic gyrotron is proposed and studied using the generalized scattering matrix approach. It is shown that such coating provides several-fold increase in the Q-value of the operating mode with respect to that of the fundamental competing mode at a reasonable level of ohmic losses, dielectric losses and mode conversion.

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INTRODUCTION

Dielectric-coated waveguides and cavities [1] are actively studied both theoretically and experimentally for use in different electronic devices, including gyrotrons (see, for example, [2] and references therein).

The gyrotron relates to the class of oscillators based on the cyclotron maser instability of weakly relativistic electrons gyrating in an external magnetic field. Nowadays it is the most powerful radiation source in the range of millimeter waves (30…300 GHz). However, very strong magnetic field (above 12 T) and thus very cumbersome and expensive magnetic systems are required to reach higher frequencies. In this connection the higher harmonic operation capable of reducing the above-stated requirement is of prime interest for sub-THz and THz gyrotrons. As a rule, such operation, however, suffers from severe mode competition [3, 4] with the first (fundamental) cyclotron harmonic modes. Therefore, of vital importance are the possible means of alleviating the problem of harmonic mode competition.

Among them are the methods of electrodynamical selection due to appropriate profiling of the gyrotron cavity. The simplest profiled cavity is conventional one with iris loading [5]. The effect of the iris is to trap the high-harmonic operating mode more effectively than the fundamental competing modes. This allows the quality factor of the operating mode to be increased relative to those of the competitors, thus providing the inverse change in the starting currents for these modes. The improved mode selection of the gyrotron cavity loaded with iris has been demonstrated experimentally [5, 6]. However, for sub-THz and THz gyrotrons the problems of miniaturization and machining tolerances [7] make such electrodynamical selection hardly realizable.

The alternative is to use the coating of metallic cavity with dielectric layer of variable thickness. As was shown in [2], such coating acts effectively as additional wall profiling. This in its turn has effect on quality factors and oscillation threshold of the cavity modes. More importantly, such effect depends on mode frequency and is stronger for higher frequency modes. Thus the generation of the cyclotron harmonics may be favored by the proper distribution of the dielectric coating.

In the present study we consider the metallic cavity with distributed dielectric coating for the 0.4 THz second harmonic gyrotron. The new structure (Fig. 1,a) in the form of the gyrotron cavity partially coated inside with dielectric of constant thickness is proposed. Such structure is easier to fabricate than that of [2] with gradually tapered coating thickness. The study is based on scattering matrix approach [8, 9] extended to the case of dielectric-coated gyrotron cavity. This differentiates our study from that [2] performed within single-mode treatment, which ignores mode conversion induced by nonuniform dielectric coating. Besides, our approach is more comprehensive, since it also considers losses in metallic walls and dielectric coating.

1. NORMAL MODES OF DIELECTRIC-LOADED IMPEDANCE WAVEGUIDE

Consider a finite-length section of uniform circular waveguide. For generality, the waveguide is assumed to be formed by several concentric dielectric layers and bounded by conducting medium. The aim is to find wave solution $\{\mathbf{E}(\mathbf{r},t),\mathbf{B}(\mathbf{r},t)\}=\{\mathbf{E}(r,z),\mathbf{B}(r,z)\}\times \times\exp(-i\omega t+il\varphi)$ to Maxwell's equations subject to the continuity conditions for the axial and the azimuth field components inside the waveguide and the impedance boundary conditions at the conducting surface $r=R$

$$E_\varphi/B_z=Z\,,\ E_z/B_\varphi=-Z\,, \tag{1}$$

where $Z=Z(\omega)=k\delta(1-i)/2$ is the surface impedance, $k=\omega/c$, $\delta=\sqrt{2/(\omega\mu_0\sigma)}$, σ is the conductivity.

For each dielectric layer we may write solution as:

$$\begin{aligned} B_z &= f(z)h_z(r)=f(z)\sqrt{\varepsilon}k_\perp^2\psi(r),\\ E_z &= y(z)e_z(r)=-iy(z)k_\perp^2\varphi(r), \end{aligned} \tag{2}$$

where $\varepsilon=\varepsilon(\omega)$ is the complex permittivity of the dielectric layer, $k_\perp^2=\varepsilon k^2-k_z^2$, k_z is the axial wavenumber, $f(z)=a(z)+b(z)$, $y(z)=a(z)-b(z)$, $a(z)=a_0\exp(ik_zz)$, $b(z)=b_0\exp(-ik_zz)$, a_0 and b_0 are the amplitudes of forward and backward waves, respectively, $\psi(r)$ and $\varphi(r)$ are the general solutions to Bessel differential equations

$$\left(\Delta_\perp+k_\perp^2\right)\psi=0\,,\ \left(\Delta_\perp+k_\perp^2\right)\varphi=0\,, \tag{3}$$

with some integration constants.

These solutions define the radial structure for both the axial (2) and the transverse ($\mathbf{E}_\perp=f(z)\mathbf{e}_\perp(r)$, $\mathbf{B}_\perp=y(z)\mathbf{h}_\perp(r)$)

$$\begin{aligned} \mathbf{E}_\perp &= f(z)\left(i\sqrt{\varepsilon}k\left[\nabla_\perp\psi\times\mathbf{z}\right]+ik_z\nabla_\perp\varphi\right),\\ \mathbf{B}_\perp &= \sqrt{\varepsilon}y(z)\left(ik_z\nabla_\perp\psi-\sqrt{\varepsilon}k\left[\nabla_\perp\varphi\times\mathbf{z}\right]\right) \end{aligned} \tag{4}$$

components of the field. Here $\nabla_\perp = \mathbf{i_r}\partial/\partial r + \mathbf{i}_\varphi\, il/r$.

Inside dielectric layer the field (2) and (4) must be finite and satisfies either or both continuity and boundary conditions. Together, these conditions imposed for all dielectric layers form a system of linear algebraic equations with unknown integration constants. The system of equations has nontrivial solution if its determinant $D(\omega,k_z)$ vanishes. The resultant dispersion equation yields the axial wavenumbers $k_z(\omega)$ for TE-like (HE) and TM-like (EH) hybrid modes of a uniform multilayer dielectric waveguide.

For any hybrid mode with $k_z = k_{zs}(\omega)$ the radial structure of the field $\{\mathbf{e}_s,\mathbf{h}_s\}$ is found from (2)-(4). Together, such normal modes form an infinite orthogonal set. To prove this statement, we first evaluate the following indefinite integral:

$$\left(k_{\perp s}^2 - k_{\perp n}^2\right)\int rdr\left\langle \mathbf{e}_{\perp s},\mathbf{h}_{\perp n}\right\rangle = ir\left\{k_{zs}\left[e_{zs}h_{\varphi n} - e_{zn}h_{\varphi s}\right] - \right.$$
$$\left. -k_{zn}\left[e_{\varphi s}h_{zn} - e_{\varphi n}h_{zs}\right]\right\} + ir\left(\varepsilon_1-\varepsilon_2\right)ke_{rs}e_{zn}, \qquad (5)$$

where $\left\langle \mathbf{e}_{\perp s},\mathbf{h}_{\perp n}\right\rangle = (e_{rs}h_{\varphi n} + e_{\varphi s}h_{rn})$ will be called the inner product of $\mathbf{e}_{\perp s}$ and $\mathbf{h}_{\perp n}$, $\mathbf{e}_s$ and $\mathbf{h}_n$ are found from (2)-(4) with $\varepsilon = \varepsilon_1$, $k_\perp^2 = k_{\perp s}^2 = \varepsilon_1 k^2 - k_{zs}^2$ and $\varepsilon = \varepsilon_2$, $k_\perp^2 = k_{\perp n}^2 = \varepsilon_2 k^2 - k_{zn}^2$, respectively.

If $\{\mathbf{e}_s,\mathbf{h}_s\}$ and $\{\mathbf{e}_n,\mathbf{h}_n\}$ are the eigenfields of the same ($\varepsilon_1(r) = \varepsilon_2(r)$) dielectric waveguide and have continuous variation of φ and z components, then from (5) it immediately follows that modes of the multilayer dielectric waveguide are orthogonal in the extreme case of perfectly conducting ($\sigma = \infty$) outer medium [10]. In this case φ and z components of $\mathbf{e}_s$ and $\mathbf{e}_n$ vanish at $r = R$ and integral $P_{sn} = \int_0^R rdr\left\langle \mathbf{e}_{\perp s},\mathbf{h}_{\perp n}\right\rangle$ goes to zero, provided that $k_{zs}^2 \neq k_{zn}^2$.

Under impedance boundary conditions (1), this integral takes the form:

$$P_{sn} = \frac{-iR\left(Z_s - Z_n\right)}{k_{zn}^2 - k_{zs}^2}\left[k_{zs}h_{\varphi s}h_{\varphi n} + k_{zn}h_{zs}h_{zn}\right]_{r=R}, \qquad (6)$$

where Z_s and Z_n are the surface impedances for modes with axial wavenumbers $k_z = k_{zs}$ and $k_z = k_{zn}$, respectively. If the surface impedance in (1) is independent of k_z and thus $Z_s = Z_n$, then $P_{sn} = 0$. Exception is the case $n = s$, when integral (6) takes the form of nonzero normalization factor $P_s = P_{ss}$ [10] of s-th mode. The above results clearly demonstrate mode orthogonality for the multilayer dielectric waveguide embedded in conductor. Evidently, the same is also true for a circular waveguide filled with radially inhomogeneous dielectric, which can always be treated as a finite or infinite number of concentric dielectric layers.

The amplitudes $f(z)$ and $y(z)$ of the normal modes are still to be evaluated. They are determined by the boundary conditions at the ends of the waveguide section. Only a limited number of such conditions admit single-mode solution (2)-(4) to the eigenvalue problem for the multilayer dielectric waveguide. In the general case, however, solution is sought as expansion in terms of the normal modes with unknown amplitudes.

2. GYROTRON CAVITY WITH DISTRIBUTED DIELECTRIC COATING

Consider conventional gyrotron cavity in the form of a hollow circular metallic waveguide with slowly varying radius $R(z)$ and finite length. The cavity (see Fig. 1,a) is partially coated inside with a single dielectric layer of constant thickness d. Copper with conductivity $\sigma_{Cu} = 5.8\times10^7$ s/m, and PTFE (Teflon) with permittivity $\varepsilon = 2.1\times(1 + i\tan\delta)$, $\tan\delta = 10^{-3}$ were adopted as wall and coating materials, respectively.

In our study we use a stepwise approximation [8, 9] to model the cavity radius $R(z)$. As a result, the gyrotron cavity is represented as a finite number ($j = 1, 2...N$) of the jointed uniform cylindrical sections loaded with dielectrics. For each section of the radius $R = R_j$, the field is expanded in terms of the eigenfields (4) of the normal modes

$$\mathbf{E}_\perp^{(j)} = \sum_s\left[a_s^{(j)}(z) + b_s^{(j)}(z)\right]\mathbf{e}_{\perp s}^{(j)}(r),$$
$$\mathbf{B}_\perp^{(j)} = \sum_s\left[a_s^{(j)}(z) - b_s^{(j)}(z)\right]\mathbf{h}_{\perp s}^{(j)}(r) \qquad (7)$$

and satisfies continuity conditions $\mathbf{E}_\perp^{(j)} = \mathbf{E}_\perp^{(j+1)}$ and $\mathbf{B}_\perp^{(j)} = \mathbf{B}_\perp^{(j+1)}$ at the interface $z = z_j$ between neighboring sections. Taking the inner product of the first condition with $\mathbf{h}_{\perp n}^{(j)}$ and that of $\mathbf{e}_{\perp n}^{(j)}$ with the second condition, and then integrating over the radius $R = R_j$, we obtain

$$\left(a_i(z_j) + b_i(z_j)\right)P_i = \sum_j T_{ki}\left(a_k(z_j) + b_k(z_j)\right),$$
$$\left(a_i(z_j) - b_i(z_j)\right)P_i = \sum_j T_{ik}\left(a_k(z_j) - b_k(z_j)\right), \qquad (8)$$

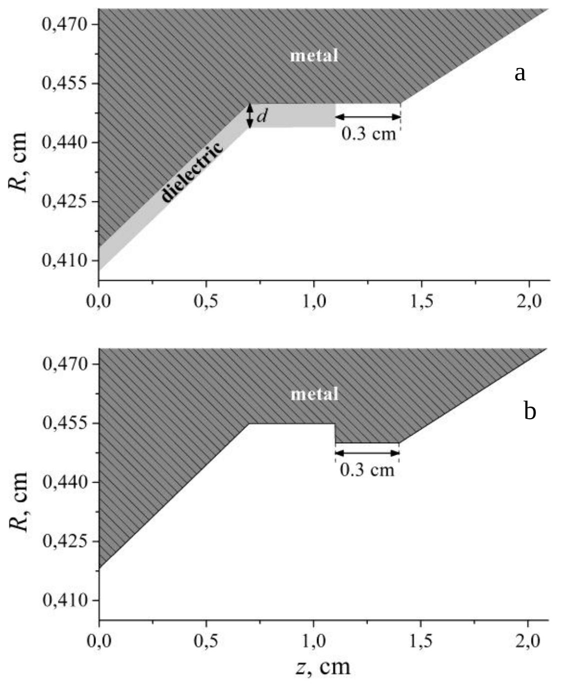

Fig. 1. Cavity with distributed dielectric coating (a) and equivalent uncoated cavity with iris (b)

where $i=\{s,j\}$ and $k=\{n,j+1\}$ are the pairs of the lower and upper indices used in (7) for designation of the normal modes, $T_{ik}=\int_0^{R_j} rdr\langle \mathbf{e}_{\perp i},\mathbf{h}_{\perp k}\rangle$ are the coefficients of mode coupling due to the structural nonuniformity, which initiates energy transformation from the incident mode with amplitude a_i to a number of the transmitted modes with amplitudes a_k ($k\neq i$). Such effect is known as mode conversion (MC).

Coupling coefficients T_{ik} are evaluated using indefinite integral (5). To perform this evaluation, both neighboring sections are divided into the same concentric regions, each filled with uniform dielectrics ε_1 for $z<z_j$ and ε_2 for $z>z_j$. Note that the number of such regions is generally larger than a given number of dielectric layers in either of two jointed sections. Coefficients T_{ik} are obtained as a sum of integrals (5) over all selected regions.

For $j=1,2...N$, relations (8) supplemented with outgoing wave boundary conditions at the cavity ends constitute a system of linear algebraic equations with unknown amplitudes of forward and backward waves. Cavity eigenfrequencies are obtained by nullifying the determinant of the system matrix. To avoid problems associated with large-size matrices, we have implemented the scattering matrix formalism [8, 9].

We consider two possible [3, 4] harmonic competitors with closely spaced caustic radiuses $R_c\approx l/k$. The first is the operating second cyclotron-harmonic $TE_{8,9}$ mode. The second is the fundamental competing $TE_{4,5}$ mode. In the case of uncoated cavity ($d=0$), frequency f, diffractive Q_d and heat (ohmic) Q_h quality factors of $TE_{8,9}$ and $TE_{4,5}$ modes equal 399.23 GHz, 2142 and 41575, and 204.05 GHz, 511 and 28502, respectively.

Fig. 2,a shows the effect of dielectric coating on the frequencies of $TE_{8,9}$ and $TE_{4,5}$ modes. Both frequencies decrease with increasing d. This is because the dielectric coating lowers the eigenvalues of TE-like modes near cutoff [1]. Specifically [2], the eigenvalue $\chi=(k^2-k_z^2)^{1/2}R$ of $TE_{l,s}$ mode tends to $(s-m)$-th zero of the Bessel function $J_l(x)$ or its derivative as $\sqrt{\varepsilon}kd$ approaches $\pi/2+\pi(m-1)$ or πm, respectively. That is why the effect of dielectric coating is mode-dependent and shows up more dramatically for higher frequency operating mode (see Fig. 2,a).

Thus, TE modes have reduced eigenvalues inside the dielectric-coated part of the gyrotron cavity shown in Fig. 1,a. Evidently, these modes are identical to those of equivalent uncoated cavity (Fig. 1,b) having an iris with mode-dependent depth. It is well known [5, 6, 9] that the iris loading enlarges the diffractive Q-values of the cavity modes. More importantly, for higher frequency modes the iris of equivalent cavity has the larger depth and thus induces the higher increments of Q_d. This explains the results shown in Fig. 2,b.

However, in addition to the diffractive losses, there are other dissipative effects in the gyrotron cavity with distributed dielectric coating. They are attributed to ohmic losses in conducting wall, dielectric losses and mode conversion (MC). These effects are negligible in the case of uncoated cavity ($d=0$). For $d\neq 0$, however, all of them - individually and in combination- tend to decrease the total quality factor $Q_{8,9}$ of operating $TE_{8,9}$

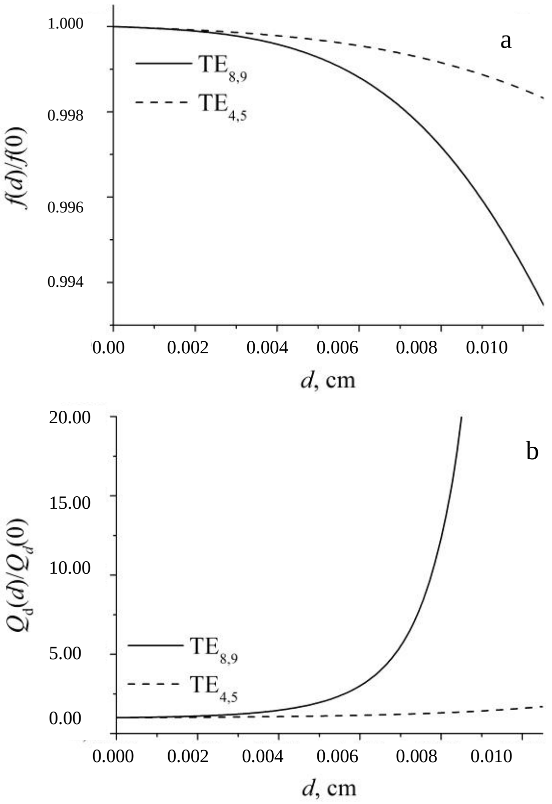

Fig. 2. Normalized frequencies (a) and diffractive Q-values of TE modes versus d (b)

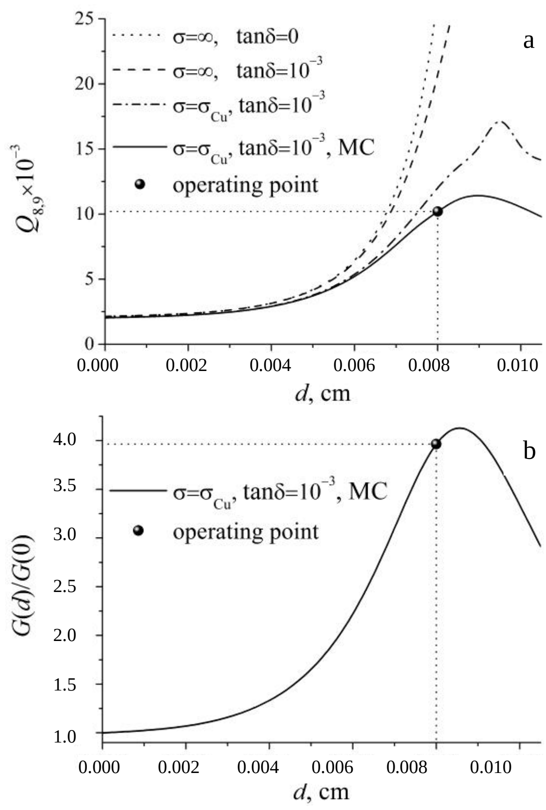

Fig. 3. Loss-induced change of $Q_{8,9}$ (a) and normalized value of $G=Q_{8,9}/Q_{4,5}$ versus d (b)

mode. Such a tendency is evident from Fig.3,a, which also depicts the final operating point. For sub-terahertz second-harmonic gyrotron, this point ensures a reasonable level of mode conversion (82 % of mode purity) and heat losses in both copper and PTFE ($Q_d \approx Q_h$). The operating point corresponds to 0.008 cm of dielectric thickness, which is achievable [11] with present-day technology. It is also depicted in Fig. 3,b, which shows the effect of coating thickness d on the normalized ratio between the total quality factors of operating $TE_{8,9}$ and competing $TE_{4,5}$ modes. As is easy to see, this ratio increases with increasing d. Since the starting current $I^{st}_{l,s}$ of $TE_{l,s}$ mode is inversely proportional to the total quality factor, such a change in Q-values implies a decrease of the ratio $I^{st}_{8,9}/I^{st}_{4,5}$. Thus, for the gyrotron under investigation, the distributed dielectric coating of the cavity provides almost four-fold decrease in the starting current of the operating mode with respect to that of the competing mode at a reasonable level of heat losses and mode conversion.

CONCLUSIONS

Copper cavity partially coated inside with PTFE (Teflon) of constant thickness has been proposed for the 0.4 THz second harmonic gyrotron. Electromagnetic properties of the cavity have been studied numerically. The study is based on the stepwise approximation of the cavity radius and the field expansion in terms of the normal hybrid modes of a uniform dielectric-loaded waveguide with impedance walls. These modes were shown to be orthogonal. It has been demonstrated that distributed Teflon coating may provide several-fold decrease in the starting current of the operating mode with respect to that of the competing fundamental cyclotron mode at a reasonable level of copper losses, Teflon losses and mode conversion.

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РЕЗОНАТОР С РАСПРЕДЕЛЁННЫМ ДИЭЛЕКТРИЧЕСКИМ ПОКРЫТИЕМ ДЛЯ СУБТЕРАГЕРЦОВОГО ГИРОТРОНА НА ВТОРОЙ ГАРМОНИКЕ ГИРОЧАСТОТЫ

В.И. Щербинин, Г.И. Загинайлов, В.И. Ткаченко

Металлический резонатор с распределённым диэлектрическим покрытием предложен для использования в 0,4 ТГц гиротроне на второй гармонике гирочастоты и исследован с помощью обобщённого метода матрицы рассеяния. Показано, что такое покрытие позволяет в несколько раз увеличить добротность рабочей моды гиротрона по сравнению с добротностью конкурирующей фундаментальной циклотронной моды при приемлемом уровне омических потерь, потерь в диэлектрике и конверсии мод.

РЕЗОНАТОР З РОЗПОДІЛЕНИМ ДІЕЛЕКТРИЧНИМ ПОКРИТТЯМ ДЛЯ СУБТЕРАГЕРЦОВОГО ГІРОТРОНА НА ДРУГІЙ ГАРМОНІЦІ ГІРОЧАСТОТИ

В.І. Щербінін, Г.І. Загинайлов, В.І. Ткаченко

Металевий резонатор з розподіленим діелектричним покриттям запропоновано для використання в 0,4 ТГц гіротроні на другій гармоніці гірочастоти та досліджено за допомогою узагальненого методу матриці розсіювання. Показано, що таке покриття дозволяє в кілька разів збільшити добротність робочої моди у порівнянні з добротністю конкуруючої фундаментальної циклотронної моди при прийнятному рівні омічних втрат, втрат у діелектрику та конверсії мод.