NUMERICAL SIMULATION OF WAKEFIELDS EXCITATION IN THE DIELECTRIC STRUCTURE FILLED WITH INHOMOGENEOUS PLASMA

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Results of numerical simulation of wakefield excitation in the dielectric structure filled with plasma created in capillary discharge are given. Three different models of dependence of plasma density on radius were investigated: 1) homogeneous, 2) parabolic and 3) the dependence obtained numerically by N.A. Bobrova [9]. The obtained results have shown that an amplitude of the accelerating field weakly depends on the model of plasma density dependence. At the same time the transverse wakefield force significantly changes in an amplitude: for the model 3) amplitudes of transverse force is twice higher than in the case of homogeneous plasma.

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INTRODUCTION

One of shortcomings of the dielectric wakefield accelerators (DWA) limiting the effective length of acceleration is susceptibility of the leading bunches to beam breakup (BBU) instability [1]. With the purpose to suppress a development of this instability recently it has been proposed to fill the drift channel of dielectric structure with plasma of certain density [2]. The suggested new accelerating structure has been called plasma-dielectric wakefield accelerator (PDWA). Analytical and numerical researches have shown [2-5] that PDWA keeps the high accelerating gradients which are available in DWA and also stabilizes transverse dynamics of driver and the accelerated bunches. The reason of improvement of transverse stability is an excitation of the plasma wave possessing the focusing properties in the drift channel.

The researches [2-5] were made for dielectric wakefield structures with homogeneous plasma filling the cross-section of the drift channel. Such consideration is right if plasma comes to the drift channel from external source. Other, widely way of plasma creation is capillary discharge. This method is used in researches on laser wakefield acceleration (LWFA) by transmission of currents through dielectric tube. Plasma is created by ablation of atoms and ions from inner surface of dielectric tube, or by ionization of atoms and molecules if the dielectric tube channel is filled with neutral gas [6-10]. In the latter case on big times from the beginning of discharge (it is more than a half of pulse width of discharge current) steady distribution of plasma density with inhomogeneous distribution of discharge density forms [9]. At short distances from the axis this distribution is well described by parabolic dependence [9, 10]

$$n(r) \approx n_0[1+0.33(r/a)^2], \qquad (1)$$

where a is the inner radius of dielectric tube.

Investigations on excitation of wakefield in PDWA with and inhomogeneous transverse distribution of plasma density were not carried out so far. For practical application of the proposed accelerating structure it is important to know how the accelerating gradient and focusing forces of test bunches will change in case when plasma in the drift channel is created as a result of the capillary discharge. Results of such researches are given below.

It should be noted that on considerable initial time interval of capillary discharge (40 of 200 ns [9]) there is quasihomogeneous distribution of plasma density. I.e. PDWA operating mode with homogeneous distribution of plasma density is possible also in case when plasma is created as a result of the capillary discharge.

STATEMENT OF THE PROBLEM

In our researches the dielectric tube with inner radius a and outer radius b, surrounded with cylindrical metal waveguide was used. The channel of dielectric tube was filled by plasma.

The drive electron bunch passes through the slowing-down structure and excites wakefield in it. After a certain delay time t_{del}, folowing the driver bunch, in system the witness bunch with charge by 10 times smaller, than the driver charge is injected and gets under influence of the drive wakefield. We supposed that the initial sizes of the drive and witness bunches are identical. The schematic view of cylindrical dielectric structure is shown in Fig. 1.

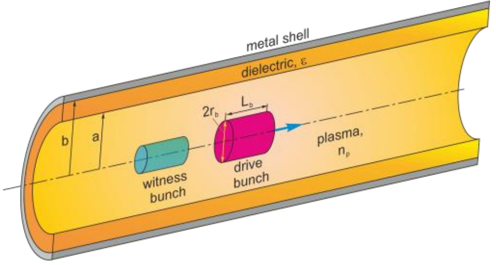

Fig. 1. Schematic view of a cylindrical dielectric waveguide. Magenta cylinder shows driver electron bunch, blue cylinder – witness

In Table the parameters of waveguide, driver and witness bunches used in calculation are given.

Parameters used in calculation

Inner radius of dielectric tube a	0.5 mm
Outer radius of dielectric tube b	0.6 mm
Waveguide length L	8 mm
Relative dielectric constant ε	3.8 (quartz)
Bunch energy E_0	5 GeV
Total driver bunch charge	3 nC
Total witness bunch charge	0.3 nC
Bunch axial RMS dimension 2σ (Gaussian charge distribution)	0.1 mm
Full bunch length used in PIC simulation	0.2 mm
Bunch diameter $2r_b$	0.9 mm
Axial plasma density	4.41×10^{14}cm^{-3}

Three different models of dependence of plasma density on radius were investigated: 1) homogeneous, 2) parabolic with square dependence on radius (1) and 3) the dependence obtained numerically by N.A.Bobrova [9] when modeling capillary discharge (Fig. 2).

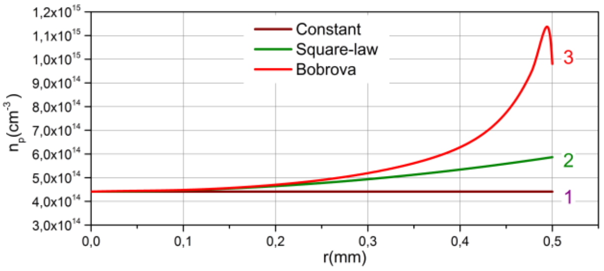

Fig. 2. Models of dependence of plasma density n_p *on radius r*

RESULTS OF 2.5D-PIC CODE SIMULATION

At numerical simulation by means of the 2.5D-PIC code created by us we examined a structure of wakefield and dynamics of electron bunches at their motion in the drift camera for three different models of dependence of plasma density on radius (see above).

In Fig. 3 comparative snapshots of the Lorentz force components affecting on test electron in dielectric waveguide with plasma filling for the time $t = 26.69\ ps$ from the beginning of injection of driver bunch for different dependences of plasma density on radius are shown: a) homogeneous, b) parabolic(1) and c) the dependence obtained by N.A. Bobrova [9].

A possible locations of accelerated bunches combined with dependences of longitudinal $F_z(z)$ and transverse forces $F_r(z)$ at $r = 0.45\,\text{mm}$ for the same time as in Fig. 3 for different dependences of density of plasma on radius are shown in Fig. 4.

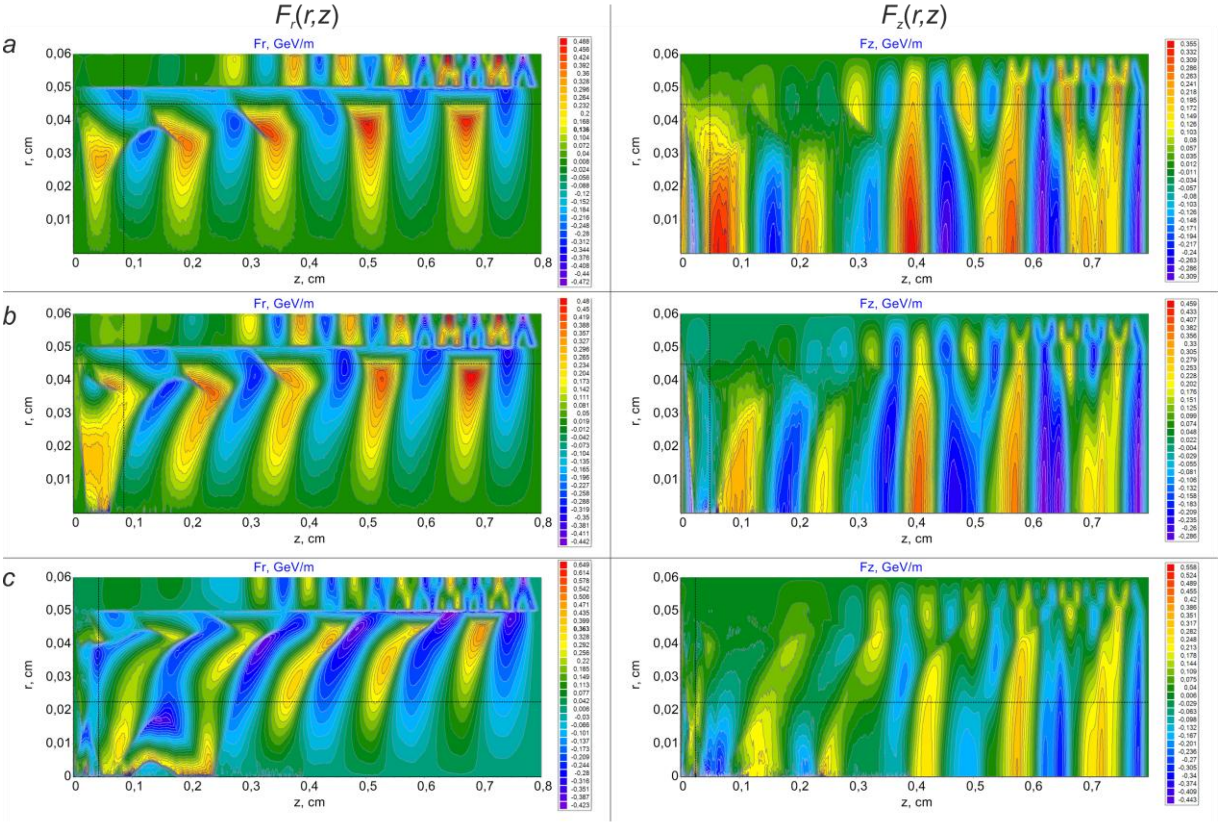

Fig. 3. Color maps and level lines for transverse $F_r(r,z)$ *(at the left) and longitudinal* $F_z(r,z)$ *(on the right) components of Lorentz force, affecting on test electron for time point* $t = 26.69\ ps$ *from the beginning of injection of drive bunch for different dependences of plasma density on radius: a – homogeneous, b – parabolic, c – the dependence obtained by N.A. Bobrova et al. [9]*

The magenta rectangle near output end on the configuration space of Fig. 4 shows the position of drive bunch. Cyan vertical segments have noted the possible placements of the witness bunch providing not only its acceleration in a local maximum of the longitudinal force F_z, but also the transverse focusing caused by negative value of transverse force F_r in these locations.

As appears from Fig. 4 amplitude (to 200 MeV/m) and frequency (372.2 GHz) characteristics of the longitudinal (accelerating) wakefield component weakly depend on what model of dependence of density of plasma on radius is used in calculations. At the same time the transverse (focusing) wakefield component significantly changes both amplitude, and frequency characteristics. Thus for model proposed by N.A. Bobrova (see Fig. 4,c) amplitudes of transverse field is approximately twice more, and the frequency by 1.4 times higher, than at homogeneous distribution (see Fig. 4,a). It should be taken into account at elaborating the two-beam plasma-dielectric wakefield accelerator [2-5].

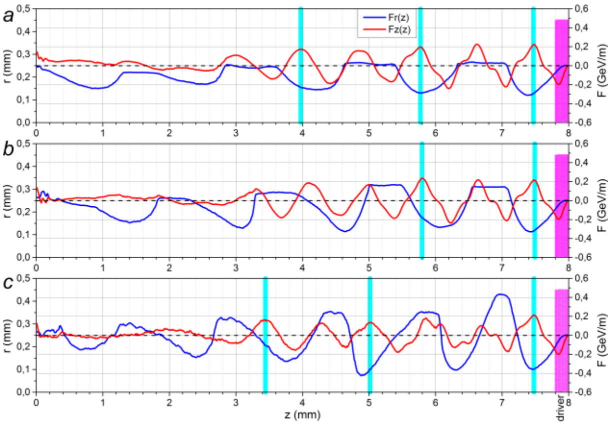

Fig. 4 Longitudinal $F_z(z)$ and transverse forces $F_r(z)$ at $r = 0.45$ mm for the same time as in Fig. 3: a – homogeneous plasma, b – parabolic plasma density (1), c – the plasma density dependence obtained by N.A. Bobrova. Cyan rectangles show possible locations of accelerated bunch and magenta rectangle is location of drive bunch

To provide the required shift of the witness bunch relative to driver it is necessary to carry out the injection of the accelerated bunch with certain delay t_{del}.

Let's consider case of homogeneous plasma. Then to provide the required placements of the witness bunch shown on Fig. 4,a, delay of injection of the witness bunch must be $t_{del} = 1.651$ ps, $t_{del} = 7.273$ ps or $t_{del} = 13.114$ ps.

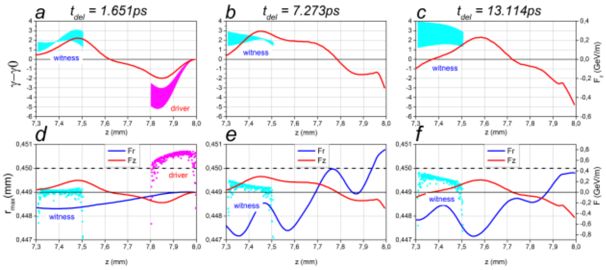

Fig. 5. a, b, c – the phase plane energy – longitudinal coordinate combined with dependence of longitudinal force $F_z(z)$; d, e, f – the configuration space displaying the position of peripheral electrons of bunches combined with dependences of longitudinal $F_z(z)$ (red curve) and transverse forces $F_r(z)$ (blue curve). By magenta color the points relating to drive bunch, cyan – to witness are shown. a, d – $t_{del} = 1.651$ ps, $t = 26.69$ ps; b, e – $t_{del} = 7.273$ ps, $t = 32.312$ ps; c, f – $t_{del} = 13.114$ ps, $t = 38.153$ ps. The plasma is homogeneous

To make sure that at the chosen delays the witness bunch will be accelerated and at the same time will be focused, let's pay attention to Fig. 5. The upper set of patterns shows the phase planes energy – longitudinal coordinate, combined with dependence of longitudinal force $F_z(z)$ for the delays stated above. In the lower series the configuration space displaying the position of peripheral electrons of bunches combined with dependences of longitudinal $F_z(z)$ and transverse forces $F_r(z)$ for the same delays is shown. To show the witness bunch at the identical longitudinal position we have chosen the following times: Snapshots in Fig. 5,a, d correspond to time $t = 26.69$ ps when the driver bunch is near output end of structure. Snapshots in Fig. 5,b, e correspond to time $t = 32.312$ ps, and Fig. 5,c, f – to time $t = 38.153$ ps. At that the driver bunch has left the structure and therefore is not shown in snapshots.

More detailed information on energy transmission from the driver bunch to the witness is given by Fig. 6 in which distribution functions on energy of electrons of driver and witness for the different delay of witness bunch injection are given. Distribution functions evaluation was carried out at the times corresponding to snapshots in Fig. 5.

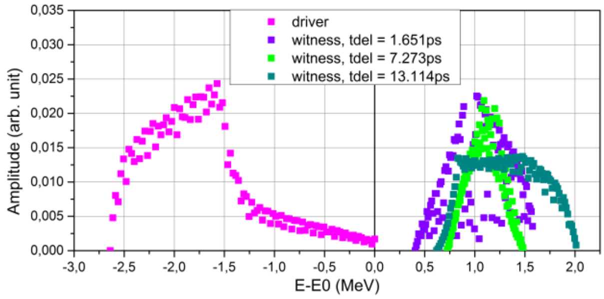

Fig. 6. Distribution functions on energy of electrons of the driver and witness bunches for different delay of injection of the witness bunch. The time moments are the same in Fig. 5, $E_0 = 5$ GeV. The plasma density across radius is constant

As appears from Figs. 5, 6, for all chosen values of delay it is observed both acceleration of the witness bunch, and its focusing. At that the driver bunch effectively transfers energy, exciting wakefield in the studied system. The greatest acceleration of the witness bunch will be at $t_{del} = 13.114$ ps, and its best focusing at $t_{del} = 1.651$ ps.

Let's pass to research of the bunch dynamics in plasma with the parabolic law of changing of plasma density on radius (1). In this case to provide the required positions of the witness bunch shown on Fig. 4,b, delay of injection of the witness bunch must be $t_{del} = 1.651$ ps or $t_{del} = 7.006$ ps. In Fig. 7 the phase planes energy – longitudinal coordinate, the configuration space displaying the positions of peripheral electrons of bunches, and also dependences of longitudinal $F_z(z)$ and transverse $F_r(z)$ forces are represented.

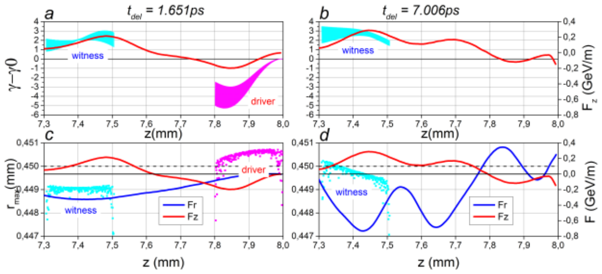

Fig. 7. The same, as in Fig. 5 for parabolic dependence of plasma density on radius (1), a, c – $t_{del} = 1.651$ ps, $t = 26.69$ ps; b, d – $t_{del} = 7.006$ ps, $t = 32.045$ ps

Energy distribution functions of electrons of the driver and witness bunches for the specified values of delay of injection of the witness bunch are given in Fig. 8.

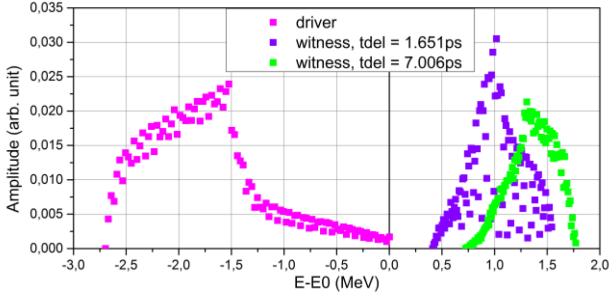

Fig. 8. The same, as in Fig. 6 for the parabolic dependence of plasma density on radius

As well as in the previous case for the chosen values of delay it is observed both acceleration of the witness bunch, and its focusing. The greatest acceleration of the witness bunch will be at $t_{del} = 7.006$ ps, and its best focusing at $t_{del} = 1.651$ ps.

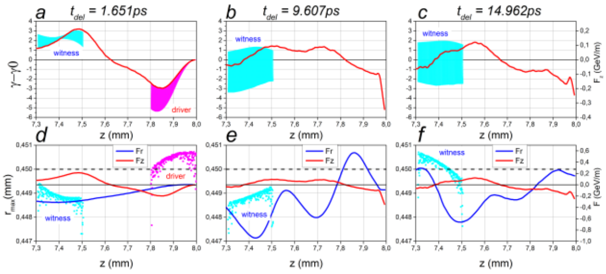

Fig. 9. The same, as in Fig. 5 for dependence of plasma density on radius obtained by N.A. Bobrova, a, d – $t_{del} = 1.651$ *ps*, $t = 26.69$ *ps; b, e –* $t_{del} = 9.607$ *ps*, $t = 34.646$ *ps; c, f –* $t_{del} = 14.962$ *ps*, $t = 40.001$ *ps*

And let's finally turn to research the dynamics of bunches in plasma with the law of density change on radius obtained in numerical simulation by N.A. Bobrova et al. [9]. Then for ensuring the required positions of the witness bunch shown on Fig. 4,c, delay of injection of witness bunch must be $t_{del} = 1.651$ ps, $t_{del} = 9.607$ ps or $t_{del} = 14.962$ ps. In Fig. 9 the phase planes energy-longitudinal coordinate, the configuration space displaying the positions of peripheral electrons of bunches, and also dependences of longitudinal $F_z(z)$ and transverse $F_r(z)$ forces for the examined case are represented.

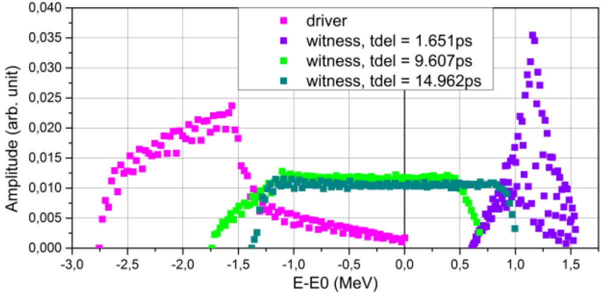

Fig. 10. The same, as in Fig. 6 for dependence of plasma density on radius obtained by N.A. Bobrova et al. [9]

Distribution functions of electrons of the driver and witness bunches on energy for the specified sizes of delay of injection of the witness bunch are given in Fig. 10.

As appears from Figs. 9 and 10, effective acceleration of the witness bunch together with its focusing in the case under consideration of dependence of plasma density on radius is observed only at delay $t_{del} = 1.651$ ps. At delay $t_{del} = 9.607$ ps only the small part of electrons of the witness bunch is accelerated, though good focusing of bunch is observed. If to choose delay $t_{del} = 14.962$ ps we will receive both bad acceleration, and poor focusing of the witness bunch.

As at delay $t_{del} = 1.651$ ps it is observed both acceleration, and focusing of the witness bunch for all studied dependences of plasma density on radius, comparison of these parameters is of interest. The phase planes energy – longitudinal coordinate and the positions of peripheral electrons combined with dependences of longitudinal $F_z(z)$ and transverse $F_r(z)$ forces for different dependences of plasma density on radius are given in Fig. 11: a) homogeneous, b) square, c) the dependence obtained by N.A. Bobrova et al. [9].

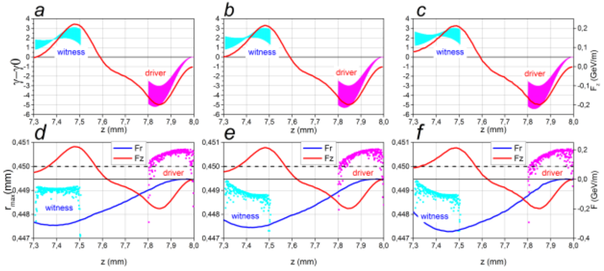

Fig. 11. a, b, c – the phase plane energy – longitudinal coordinate combined with dependence of longitudinal force $F_z(z)$*; d, e, f – the configuration space displaying the positions of peripheral electrons of bunches combined with dependences longitudinal* $F_z(z)$ *(red curve) and transverse forces* $F_r(z)$ *(blue curve) for different dependences of plasma density on radius: a, c – homogeneous; b, e – square; c, f – the dependence obtained by N.A. Bobrova et al. [9]. The delay is* $t_{del} = 1.651$ *ps*

In Fig. 12 distribution functions of electrons of the driver and witness bunches on energy for different dependences of plasma density on radius at delay $t_{del} = 1.651$ ps are shown.

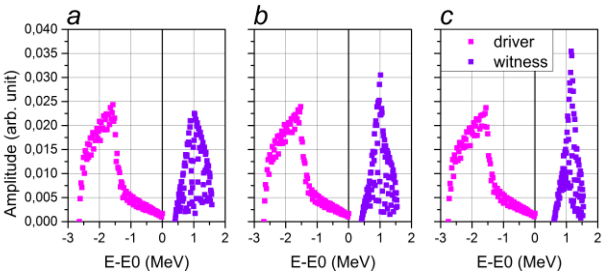

Fig. 12. Distribution functions of electrons of the driver and witness bunches on energy for different dependences of plasma density on radius: a) homogeneous, b) parabpolic (1), c) the dependence obtained by N.A. Bobrova et al. [9], $E_0 = 5$ *GeV. The delay is* $t_{del} = 1.651$ *ps*

Comparing Figs. 11 and 12 it is possible to conclude that the greatest acceleration and focusing of test bunch are observed for the dependence of plasma density obtained numerically by N.A. Bobrova et al.[9].

CONCLUSIONS

The results of numerical researches of excitation of wakefields and dynamics of the charged particles in the plasma-dielectric wakefield accelerator with transversally inhomogeneous are presented.

The analytical researches carried out earlier in linear approach for homogeneous plasma have shown that PDWA allows to accelerate the test bunch and at the same time to focus it. The results of numerical simulation by "particle in cell" method, given above, testify that inhomogeneity of plasma in transverse cross-section is noncritical and PDWA advantages with homogeneous plasma remain. I.e. even if plasma in PDWA is created by capillary discharge [9], opportunity to accelerate and focus test bunch remains. Moreover, the greatest acceleration and the best focusing of test bunch are observed for the dependence obtained numerically in work [9].

It should be note that the witness bunch is focused both in homogeneous, and in inhomogeneous plasma.

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ВОЗБУЖДЕНИЯ КИЛЬВАТЕРНЫХ ПОЛЕЙ В ДИЭЛЕКТРИЧЕСКОЙ СТРУКТУРЕ, ЗАПОЛНЕННОЙ НЕОДНОРОДНОЙ ПЛАЗМОЙ

П.И. Марков, И.Н. Онищенко, Г.В. Сотников

Приведены результаты численного моделирования возбуждения кильватерных полей в диэлектрической структуре, заполненной плазмой, создаваемой в капиллярном разряде. Были исследованы три различных модели зависимости плотности плазмы от радиуса: 1) однородная, 2) квадратная и 3) зависимость, полученная численно Н.А. Бобровой [9]. Полученные результаты показали, что амплитуда ускоряющегося поля слабо зависит от модели зависимости плотности плазмы от радиуса. Одновременно поперечная кильватерная сила значительно изменяется по амплитуде: для модели 3, амплитуда поперечной силы вдвое выше, чем в случае однородной плазмы (модель 1).

ЧИСЕЛЬНЕ МОДЕЛЮВАННЯ ЗБУДЖЕННЯ КІЛЬВАТЕРНИХ ПОЛІВ У ДІЕЛЕКТРИЧНІЙ СТРУКТУРІ, ЯКА ЗАПОВНЕНА НЕОДНОРІДНОЮ ПЛАЗМОЮ

П.І. Марков, І.М. Оніщенко, Г.В. Сотніков

Наведені результати чисельного моделювання збудження кільватерних полів у діелектричній структурі, заповненої плазмою, створеною у капілярному розряді. Були досліджені три різні моделі залежності щільності плазми від радіуса: 1) однорідна, 2) квадратична й 3) залежність, отримана чисельно Н.А. Бобровою [9]. Отримані результати показали, що амплітуда прискорюваного поля слабко залежить від моделі залежності щільності плазми від радіуса. Одночасно поперечна кільватерна сила значно змінюється за амплітудою: для моделі 3 амплітуда поперечної сили вдвічі вище, чим у випадку однорідної плазми (модель 1).