DYNAMICS OF THREE WAVE STOCHASTIC DECAYS IN NONLINEAR MATTER

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In the present work the transition of decay instability into stochastic regime in a nonlinear media is analyzed for a magnetoactive plasma and for the ferrite magnetized to the saturation level. It was shown that parameter of nonlinear interaction can be anomalously strong near the resonance frequencies: electron and ion cyclotron frequencies in the magnetoactive plasma and ferromagnetic frequency in ferrite. It was shown that the threshold value of decaying wave amplitude, when transition into stochastic regime takes place, is reducing when frequency of the wave with the lowest frequency is decreasing.

PACS: 52.35.Mw

INTRODUCTION

A weakly nonlinear approach was proposed quite long time ago for investigation of electromagnetic phenomena in plasma. With the use of this approach some important physical results were obtained (e.g., [1-3]). The simplest examples of weakly nonlinear interaction are three-wave processes. Among these are the decay and explosive instabilities. A well known regime of decay instability is characterized by periodical energy transition between three natural modes of electrodynamic system with frequency and wave number satisfying the synchronism conditions. Such regime is regular. However, our investigations showed that conditions can be realized when the decay process is chaotic [4]. This is possible in the case of modified decay. The presence of additional fourth wave is needed in similar case, with its characteristics close to the characteristics of one of the waves taking part in the decay. In such a set of waves there is a possibility to distinguish two triples of waves interacting between themselves. Every such triple is described by equation of mathematical pendulum which corresponds to the nonlinear resonance. If the resonances corresponding to different triples are overlapping, the transition to stochastic regime takes place. The overlapping is possible if the increment of decay instability exceeds the frequency shift between third and forth waves. The mathematical criterion of transition to chaos will be presented in the second part.

Theses regimes allow to transform the regular oscillations into chaotic oscillations. The theoretical conclusions are confirmed by results of numerical and experimental studies [5, 6]. In previous investigations presented, in particular, in [4-7], decays of electromagnetic wave into electromagnetic and plasma ones were considered. Meanwhile, it is interesting to investigate also the decay process of electromagnetic wave into new electromagnetic wave and plasma wave where plasma ions are taking part. Excitation of such oscillations can be an effective way to heat plasma ions.

In this work the analysis of results of decay process of high frequency waves into new high frequency waves and the low frequency ones with the frequency close to the ion cyclotron wave in magnetoactive plasma is presented.

The possibility of decay processes in other nonlinear matter, namely, in a ferrite magnetized to saturation was also investigated. It was found out that decay processes in ferrites are analogous to those inherent in plasma, when the interaction of electromagnetic and magnetostatic waves are considered.

1. IN MAGNETOACTIVE PLASMA

In the above mentioned works [4-7] mostly simple case of stochastic decay was theoretically considered, namely processes that are realized in isotropic plasma. The analysis shows that their realization in a nonisotropic (namely in giyrotropic) medium may be more complicated. The examples of such media are magnetoactive plasma and ferrites. The feature of electromagnetic system filled with similar media is the coupling of E- and H-waves. As a result, the linear transfer between these components may cause the decay process disrupted. In particular just such case was considered in [8]. In the gyrotropic medium the equations for slowly varying amplitudes are essentially complicated. They include lot of nonlinear terms and accounting of each of them requires an individual separated analysis. In this work we will not cite completely these equations but limit ourselves by their characteristic features. Namely, we will take interest in the region of low frequency range, of the order of an ion cyclotron frequency.

To investigate decay processes in magnetoactive plasma the Maxwell equations for electromagnetic field components are used and hydrodynamics equations are used for electrons and ions of plasma. It is supposed that electrodynamic system is placed into an external stationary uniform magnetic field directed along z axis. The consideration will be limited by a simple case, when magnetoactive plasma is spatially unlimited. All components of the electromagnetic field are presenting as propagating harmonic waves, which amplitudes are slowly varying along z direction:

$$E, H = E_i, H_i(\vec{r})\exp\left(i\omega_i t - i\vec{k}_i\vec{r}\right), \qquad (1)$$

$\vec{k}_i$ – wave vector. We suppose that interacting waves propagate under arbitrary angle to the external magnetic field.

ISSN 1562-6016. ВАНТ. 2016. №6(106)
PROBLEMS OF ATOMIC SCIENCE AND TECHNOLOGY. 2016, № 6. Series: Plasma Physics (22), p. 156-159.

By the use of Maxwell equations the following equations for longitudinal components of electric and magnetic fields E_{zj}, H_{zj} can be obtained:

$$-\frac{\varepsilon_{\perp i}^2-\varepsilon_{2i}^2}{\varepsilon_{\perp i}}\frac{\omega_i^2}{c^2}H_{zi}-\frac{d^2H_z}{dz^2}+k_{yi}^2H_{zi}=$$
$$=\frac{\varepsilon_{2i}\varepsilon_{\|i}}{\varepsilon_{\perp i}}\frac{\omega_i}{c}\frac{dE_{zi}}{dz}+\frac{4\pi}{c}\left(rot\,\vec{j}\right)_z,$$
$$-\varepsilon_{\|i}\frac{\omega_i^2}{c^2}E_{zi}-\frac{\varepsilon_{\|i}}{\varepsilon_{\perp}}\frac{d^2E_{zi}}{dz^2}+k_{yi}^2E_{zi}= \tag{2}$$
$$=-\frac{\varepsilon_{2i}}{\varepsilon_{\perp i}}\frac{\omega_i}{c}\frac{dH_{zi}}{dz}-i\frac{4\pi\omega}{c^2}j_z.$$

where $\varepsilon_{\perp},\varepsilon_{\|},\varepsilon_2$ are the components of permittivity tensor of magnetoactive plasma depending on frequency which has the structure:

$$\begin{pmatrix} \varepsilon_{\perp} & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\|} \end{pmatrix}, \tag{3}$$

Here j is the perturbation of current density, k_{yj} – y component of wave vector, c – velocity of light.

The last terms in the right parts of equations (2) describe nonlinear interaction of natural modes of electrodynamics system. Performing the replacement

$$E_{zi},H_{zi}\to E_{zi}(z),H_{zi}(z)\exp\left(-ik_{zi}z\right), \tag{4}$$

and taking into account synchronism conditions

$$\omega_1=\omega_2+\omega_3, k_{y1}=k_{y2}+k_{y3}, k_{z1}=k_{z2}+k_{z3}+\delta k_z, \tag{5}$$

(supposing that wave vectors of interacting waves lie in one plane) one obtains the equations for slowly varying amplitudes of interacting waves $E_{zi}(z), H_{zi}(z)$. In view of complexity of these equations they will not be presented here in fully, instead we present below their general structure and provide its analysis.

$$\frac{dH_{z1}}{dz}=\alpha_1E_{z1}+a_{h23}H_{z2}H_{z3}+b_{h23}E_{z2}H_{z3}+$$
$$c_{h23}H_{z2}E_{z3}+d_{h23}E_{z2}E_{z3},$$
$$\frac{dE_{z1}}{dz}=\beta_1H_{z1}+a_{e23}H_{z2}H_{z3}+b_{e23}E_{z2}H_{z3}+$$
$$c_{e23}H_{z2}E_{z3}+d_{e23}E_{z2}E_{z3},$$
$$\frac{dH_{z2}}{dz}=\alpha_2E_{z2}+a_{h13}H_{z1}H_{z3}^*+b_{h13}E_{z1}H_{z3}^*+$$
$$c_{h13}H_{z1}E_{z3}^*+d_{h13}E_{z1}E_{z3}^*,$$
$$\frac{dE_{z2}}{dz}=\beta_2H_{z2}+a_{e13}H_{z1}H_{z3}^*+b_{e13}E_{z1}H_{z3}^*+$$
$$c_{e13}H_{z1}E_{z3}^*+d_{e13}E_{z1}E_{z3}^*,$$
$$\frac{dH_{z3}}{dz}=\alpha_3E_{z3}+a_{h12}H_{z1}H_{z2}^*+b_{h12}E_{z1}H_{z2}^*+$$
$$c_{h12}H_{z1}E_{z2}^*+d_{h12}E_{z1}E_{z2}^*,$$
$$\frac{dE_{z3}}{dz}=\beta_3H_{z3}+a_{e12}H_{z1}H_{z2}^*+b_{e12}E_{z1}H_{z2}^*+$$
$$c_{e12}H_{z1}E_{z2}^*+d_{e12}E_{z1}E_{z2}^*. \tag{6}$$

Here the coefficients α,β,a,b,c,d with different indexes depend on parameters of the problem.

In the present work only qualitative analysis will be done of these coefficients which describe process of nonlinear interaction of different waves. For that the expression for perturbation current density is used:

$$j_l=e(n_0+\tilde{n}_i)v_i-e(n_0+\tilde{n}_e)v_e, \tag{7}$$

where e – charge of electron, n_0 – equilibrium density of plasma, $\tilde{n}_i, v_i$ – perturbations of ion density and velocity, correspondingly, $\tilde{n}_e, v_e$ – perturbations of electron density and velocity. For slowly varying amplitudes of the j-th wave the nonlinear interaction is described by terms $(\tilde{n}_iv_i)_j$ and $(\tilde{n}_ev_e)_j$ in the right parts of equations. From the linear parts of the hydrodynamic equations the expressions for perturbations of electron and ion density and velocity can be obtained:

$$\tilde{n}_{ik}, v_{ik}\approx\frac{1}{\omega_k^2-\omega_i^2},\quad \tilde{n}_{ek}, v_{ek}\approx\frac{1}{\omega_k^2-\omega_e^2}, \tag{8}$$

where ω_k is the frequency of k-th wave taking part in nonlinear interaction, ω_i,ω_e are ion and electron cyclotron frequencies correspondingly. As follows from expressions (8) the dependence of coefficients in the right parts of equations (5) on the characteristic frequency has the form:

$$a,b,c,d\approx\frac{1}{\omega_k^2-\omega_{i,e}^2}\frac{1}{\omega_l^2-\omega_{i,e}^2}, \tag{9}$$

where ω_k, ω_l are frequencies of interacting waves, satisfying synchronism condition (5). It is following from expression (9) that in the case when frequency of any of interacting wave is close to the one of the cyclotron waves, the coefficients of nonlinear interaction are anomalously large. It should be noted that condition of slowness of amplitudes change is not right when frequency of one of the interacting waves is near some of cyclotron frequencies.

The criterion for transition of decay instability into a stochastic regime was obtained in [4]:

$$\frac{VE_1}{\omega_{lf}}>1, \tag{10}$$

where V is the matrix element of nonlinear interaction (in this case it corresponds to nonlinear interaction coefficients in the set of equations (5)), E_1 – initial amplitude of decaying wave, ω_{lf} – lowest frequency of one of waves taking part in nonlinear interaction. Thus, in magnetoactive plasma there are favorable conditions for stochastic decays to be realized with participation of high frequency wave and low frequency ion wave which is of the order of the ion cyclotron frequency. This criterion is qualitative. The estimation of threshold value of the decaying wave amplitude can be found from this criterion for conditions when transition to stochastic regime is possible.

2. DECAYS IN A FERROMAGNETIC MATTER

The ferrites are an example of another matter where nonlinear interaction of natural oscillations is possible. The ferrites as well as magnetoactive plasma are gyrotropic. The gyrotropic property of ferrites becomes apparent when they are placed in a statical magnetic field. The gyrotropic properties of magnetoactive plasma are described by permittivity tensor while gyrotropy of ferrites is described by permeability tensor. However, the general regularities of nonlinear processes in plasma and in ferrites are similar. The physical processes in ferrites were minutely described, for example, in [9-11].

To obtain components of permittivity tensor for the magnetoactive plasma the hydrodynamic equations describing perturbations of electron and ion velocity and density are used. For ferromagnetic matter the Landau-Lifshitz equation describing dynamics of magnetic moment induced by external constant magnetic field is used to obtain components of magnetic permeability tensor. The high frequency tensor of magnetic permeability for the ferrite magnetized to the saturation level has the structure

$$\begin{pmatrix} \mu & -i\mu_a & 0 \\ i\mu_a & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

The nonlinear plasma properties are conditioned by the current terms in the equations (2). The nonlinear wave interaction in ferrites is conditioned by the term that takes into account ferrite magnetization arising when ferrite is placed into external magnetic field.

To obtain nonlinear equations describing the interaction of natural modes of the electrodynamics system containing ferrite, the approach analogous to the one used for magnetoactive plasma was employed. The equations for slowly varying amplitudes $E_{zi}(z), H_{zi}(z)$ of nonlinearly interacting natural waves of electrodynamic system structure obtained from Maxwell equations have the structure similar to the equations set (5). The functional frequency dependence of coefficients of the nonlinear terms is

$$a,b,c,d \approx \frac{1}{\omega_H^2 - \omega_j^2}, \tag{12}$$

where ω_j is one of frequencies of the wave taking part in nonlinear interaction, $\omega_H = \frac{eH_0}{m_e c}$ – ferromagnetic resonance frequency, e, m_e – charge and mass of electron, H_0 is external magnetizing magnetic field. As is seen, the ferromagnetic resonance frequency coincides with electron cyclotron frequency, but it also depends on the form of ferromagnetic pattern.

Thus in a ferrite placed into external constant magnetic field there is the frequency range where nonlinear interaction of parameters may be anomalously strong. As follows from criterion for arising of stochasticity (10), the most favorable conditions for chaos rising in decay instability exist in the region of small frequency values for lowest frequency wave that takes part in nonlinear interaction. In ferrite it may be magnetostatic waves.

CONCLUSIONS

In the present work the possibility of realization of stochastic regimes for decay instability in nonlinear matter is analyzed: in magnetoactive plasma and in the ferrite magnetized to saturation. It was shown that in both cases there is frequency region where nonlinear interaction coefficients are anomalously large. In the magnetoactive plasma these are two frequency regions close to electron and ion cyclotron resonances. In the magnetized ferrite this is frequency region near ferromagnetic resonance.

As follows from the criterion for the rise of stochastic regime. the less the frequency of the lowest frequency wave taking part in nonlinear interaction the lower the threshold of transition decay instability in stochastic regime.

From the above presented results it follows that the mode with chaotic dynamics is easily excited when two conditions are met;

i) the frequency of one of interacting waves is near one of the resonance frequencies, i.e., electron or ion cyclotron frequencies in plasma, and ferromagnetic resonance frequency in the ferrite magnetized to saturation;

ii) the less frequency of this resonance the lower the threshold for stochasticity rise.

In the noted above cases it may expect of effective transition of decaying wave energy into other modes with simultaneous chaotization. This fact can be used for effective heating of plasma ions.

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Article received 05.10.2016

ДИНАМИКА ТРЁХВОЛНОВЫХ СТОХАСТИЧЕСКИХ РАСПАДОВ В НЕЛИНЕЙНЫХ СРЕДАХ

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Анализируется переход распадной неустойчивости в стохастический режим в нелинейных средах: магнитоактивной плазме и намагниченном до насыщения феррите. Показано, что параметр нелинейного взаимодействия может быть аномально большим вблизи резонансных частот: электронной и ионной циклотронной в магнитоактивной плазме и частоты ферримагнитного резонанса в феррите. Отмечено, что с уменьшением частоты самой низкочастотной волны, участвующей в распаде, уменьшается пороговое значение амплитуды распадающейся волны, при котором происходит переход в стохастический режим.

ДИНАМІКА ТРИХВИЛЕВИХ СТОХАСТИЧНИХ РОЗПАДІВ У НЕЛІНІЙНИХ СЕРЕДОВИЩАХ

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Аналізується перехід розпадної нестійкості в стохастичний режим у нелінійних середовищах: магнітоактивній плазмі і намагніченому до насичення фериті. Показано, що параметр нелінійної взаємодії може бути занадто великим поблизу резонансних частот: електронної та іонної циклотронної в магнітоактивній плазмі та частоти феромагнітного резонансу у фериті. Відзначено, що зі зменшенням частоти самої низькочастотної хвилі, що бере участь у розпаді, зменшується порогове значення амплітуди хвилі, що розпадається, при якому відбувається перехід у стохастичний режим.