TRANSITION RADIATION OF THE MOVING POINT CHARGE IN PLASMA AS A RESULT OF THE BACKGROUND PLASMA ELECTRONS' ACCELERATION

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Transition radiation of the moving point charge from the border of cold isotropic plasma and vacuum is calculated directly from Maxwell equations and motion equation for plasma electrons. The calculation is carried out in the linear approximation but the proposed method can be used to study non-linear effects. Radiation pattern for the whole range of the possible frequencies and charge velocities was obtained. The data obtained coincide with the results of the traditional method of calculation.

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INTRODUCTION

Traditional method of calculation of the transition radioemission from the border of two media uses electromagnetic fields of the moving charge in these media presented in the terms of permittivity and permeability. These expressions are substituted to the boundary conditions together with the eigenmodes of the unknown magnitudes. Then these magnitudes are obtained from the boundary conditions [1]. In this interpretation the source of radioemission is the reconstruction of electromagnetic field of the moving charge at the border of two media (this field depends on the media properties, i.e. permittivity and permiability).

The alternative point of view is that the radioemission source is the accelerated motion of the media electrons caused by the electric field of the moving charge [2, 3]. For subluminal velocity of the moving charge in the homogeneous media such emission is averaged into zero. For superluminal velocity in the homogeneous media one can obtain the Cherenkov radioemission [2-5]. For subluminal velocity in the inhomogeneous media one can obtain the transition radiation [4, 5].

The aim of this work is to calculate the transition radioemission of the moving charge from the border of vacuum and cold isotropic plasma. Concepts of media pervittivity and permiability are not used in this calculation. We solve the set of Maxwell equations and motion equation for plasma electrons in order to present vector potential in the background plasma.

In this work the basic set of equations is linearized. But in principle this method gives a possibility to study the non-linear effects in the transition radioemission.

Cold plasma is treated, only the motion of electrons is taken into account. In the first approximation the charged particle's velocity is treated as the given value (i.e., it is steady and straight). Taking into account the charged particle's deceleration (caused by its radioemission) moves to the additional radiation effect, i.e. bremsstrahlung.

1. BASIC EQUATIONS

Electromagnetic field caused by the moving charge and plasma oscillations can be described by the wave equation for vector potential $\vec{A}$:

$$\frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \Delta\vec{A} = \frac{4\pi}{c}\left(\vec{\mathrm{v}}_0\rho_{\mathrm{ext}} + \vec{\mathrm{v}}\rho\right), \tag{1}$$

where $\vec{\mathrm{v}} = \vec{\mathrm{v}}(\vec{r},t)$ is the velocity of plasma electrons, $\rho < 0$ is the plasma charge density, ρ_{ext} and $\vec{\mathrm{v}}_0$ are the charge density and velocity of the external charged particle, respectively. The scalar potential can be obtained from the calibration condition:

$$\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0. \tag{2}$$

The external charged particle is treated as a point moving along the symmetry axis in cylinder coordinates with the given velocity $\vec{\mathrm{v}}_0 = \vec{e}_z \mathrm{v}_0$ and density

$$\rho_{ext} = \frac{q}{\pi r}\delta(r)\delta(z - \mathrm{v}_0 t). \tag{3}$$

Equation describing the plasma electrons' velocity $\vec{\mathrm{v}}$ caused by the electromagnetic field of the moving charged particle can be written in the usual form:

$$\frac{\partial\vec{\mathrm{v}}}{\partial t} + \left(\vec{\mathrm{v}}\cdot\nabla\right)\vec{\mathrm{v}} = \frac{e}{m}\left(\nabla\varphi + \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right), \tag{4}$$

or, taking (2) into account,

$$\frac{1}{c}\frac{\partial}{\partial t}\left[\frac{\partial\vec{\mathrm{v}}}{\partial t} + \left(\vec{\mathrm{v}}\cdot\nabla\right)\vec{\mathrm{v}}\right] = \frac{e}{m}\left[-\nabla\left(\nabla\cdot\vec{A}\right) + \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}\right]. \tag{5}$$

Equations (1), (5) are linearized after the small variables $\vec{A}$, $\vec{\mathrm{v}}$, $\tilde{\rho}$ ($\rho = \rho_0 + \tilde{\rho}$, ρ_0 is the equilibrium charge density). Then finding $\vec{\mathrm{v}}$ from (5) one can obtain the general inhomogeneous wave equation for $\vec{A}$.

The vector potential $\vec{A}$ can be described as a superposition of the external charge vector potential $\vec{A}^e$ and contribution of plasma electrons $\vec{A}^p$:

$$\vec{A} = \vec{A}^e + \vec{A}^p. \tag{6}$$

From the linearized set (1), (5)-(6) one can obtain:

$$\frac{1}{c^2}\frac{\partial^2\vec{A}^e}{\partial t^2} - \Delta\vec{A}^e = \frac{4\pi}{c}(\rho_{ext}\vec{\mathrm{v}}_0), \tag{7}$$

$$\frac{\partial^2}{\partial t^2}\left(\frac{1}{c^2}\frac{\partial^2\vec{A}^p}{\partial t^2}-\Delta\vec{A}^p\right)-\omega_p^2\left[\nabla\left(\nabla\vec{A}^p\right)-\frac{1}{c^2}\frac{\partial^2\vec{A}^p}{\partial t^2}\right]=$$
$$=\omega_p^2\left[\nabla\left(\nabla\vec{A}^e\right)-\frac{1}{c^2}\frac{\partial^2\vec{A}^e}{\partial t^2}\right], \qquad (8)$$

where $\omega_p^2=4\pi\rho_0 e/m$ is the Langmuir electron frequency. It is clear from (7), (8) that $\vec{A}^e$ is determined by the moving charge only, and $\vec{A}^p$ is determined both by $\vec{A}^e$ and by properties of the external media (i.e. by ω_p^2 for our model). In fact $\vec{A}^e$ is the well known Lienard - Wiechert potential for the case of a point charge moving with a constant velocity.

2. BACKGROUND PLASMA ELECTRONS' MOTION

In order to solve equations (7), (8), substitution $\xi=z-\mathrm{v}_0 t$ was carried out. Furthermore the Fourier transformation of $\vec{A}^e$ and $\vec{A}^p$ after ξ and Fourier-Bessel transformation after r can be represented as

$$\tilde{A}^{e,p}_{\xi,r}\left(k_r,k_\xi\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\xi\int_0^{\infty}rdr\vec{A}^{e,p}_{\xi,r}\left(r,\xi\right)J_{0,1}\left(k_r r\right)\times$$
$$\exp\left(-ik_\xi\xi\right), \qquad (9)$$

$$\vec{A}^{e,p}_{\xi,r}(r,\xi)=\int_{-\infty}^{\infty}dk_\xi\int_0^{\infty}k_r dk_r\tilde{A}^{e,p}_{\xi,r}(k_r,k_\xi)J_{0,1}\left(k_r r\right)\times$$
$$\exp\left(ik_\xi\xi\right). \qquad (10)$$

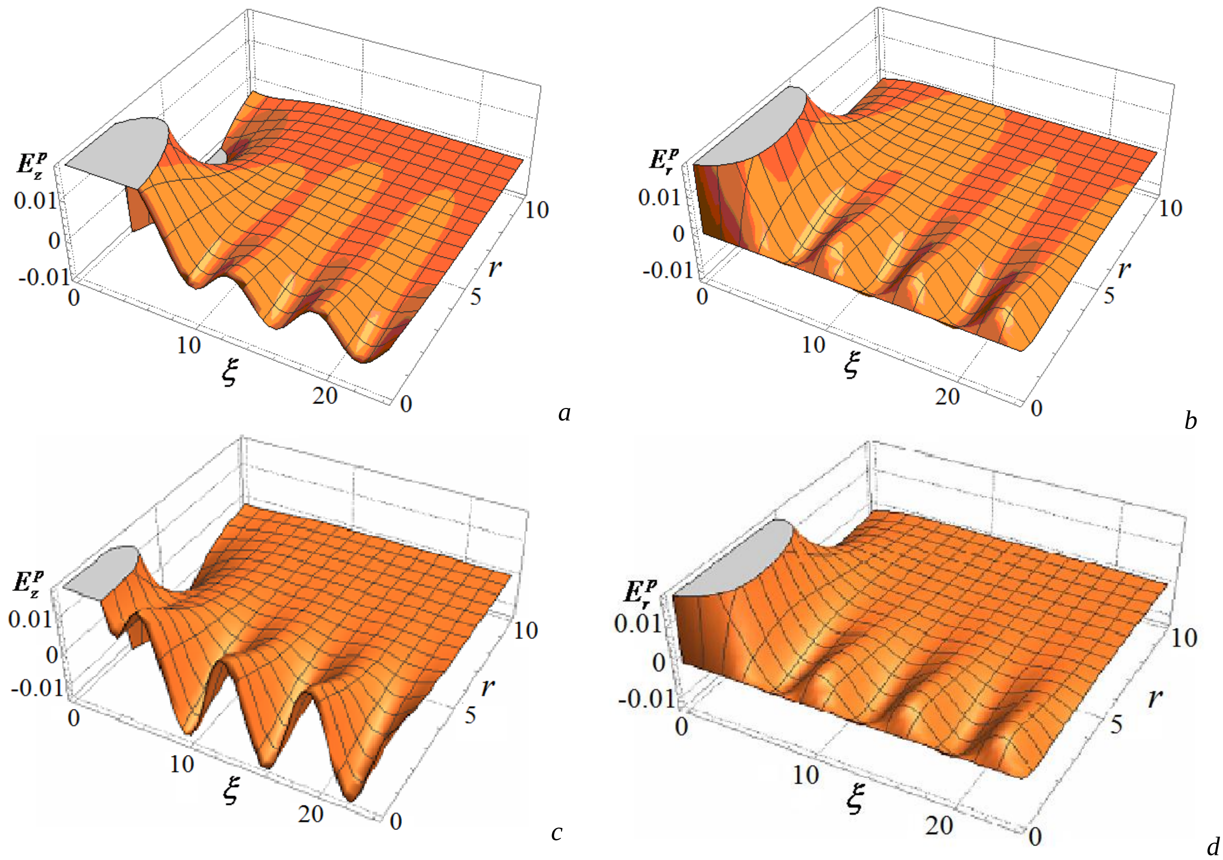

Fig. 1. Spatial distributions of the normalized axial (a, c) and radial (b, d) components of the vector potential in the background plasma caused by a point charge with a constant velocity (a, b – β=0.11; c, d – β=0.18)

As a result, the set of algebraic equations for $\tilde{A}^p_{\xi,r}$ was obtained from (7)-(8). Solution of this set gives:

$$\tilde{A}^e_\xi\left(k_\xi,k_r\right)=\frac{i\beta^2 k_\xi q}{\left[k_\xi^2+\left(1-\beta^2\right)k_r^2\right]^{3/2}}, \qquad (11)$$

$$\tilde{A}^p_r=\frac{iq\beta\omega_p^2 k_r k_\xi}{\left(\omega^2-\omega_p^2\right)\left[k^2-\left(\omega^2-\omega_p^2\right)/c^2\right]\left[k^2-\omega^2/c^2\right]}, \qquad (12)$$

$$\tilde{A}^p_\xi=\frac{q\beta\omega_p^2\left[k_\xi^2-\left(\omega^2-\omega_p^2\right)/c^2\right]}{\left(\omega^2-\omega_p^2\right)\left[k^2-\left(\omega^2-\omega_p^2\right)/c^2\right]\left[k^2-\omega^2/c^2\right]}, \qquad (13)$$

where $k^2=k_\xi^2+k_r^2$, $\omega=k_\xi \mathrm{v}_0$, $\beta=\mathrm{v}_0/c$. Inversed transformation of the spectral components $\tilde{A}^p_{r,\xi}$ gives spatial distributions of the vector potential components presented on Fig.1. Components of the plasma velocity have the similar spatial distributions in the frame connected with a moving charge. So it is clear that plasma electrons oscillate due to the external force caused by the moving charge. These oscillations (i.e. motion with the acceleration) of electrons can in principle be the source of Cherenkov radioemission (for homogeneous media; in our model it is absent) and the source of transition radioemission (for inhomogeneous media). To find out the transition radiation for the simplest case of the vacuum-homogeneous plasma border one must add the solution for the eigen waves of unknown amplitudes running away from the boundary and use the boundary conditions.

3. BOUNDARY CONDITIONS

Expressions (11), (12) were substituted to the boundary conditions together with the eigenmodes of the unknown magnitudes:

$$E_r^v\Big|_{z=0} = E_r^p\Big|_{z=0}, \qquad \varepsilon_1 E_z^v\Big|_{z=0} = \varepsilon_2 E_z^p\Big|_{z=0}, \tag{14}$$

$$E_r^{p,v} = -\frac{1}{c}\frac{\partial A_r^{p,v}}{\partial t} + c\int\left(\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} r A_r^{p,v} + \frac{\partial^2 A_z^{p,v}}{\partial r \partial z}\right)dt, \tag{15}$$

$$E_z^{p,v} = -\frac{1}{c}\frac{\partial A_z^{p,v}}{\partial t} + c\int\left(\frac{\partial}{\partial z}\frac{1}{r}\frac{\partial}{\partial r} r A_r^{p,v} + \frac{\partial^2 A_z^{p,v}}{\partial z^2}\right)dt. \tag{16}$$

The vector potential can be written in a form

$$A_{r,z}^{p,v} = \int_0^\infty k_r dk_r\left[\tilde{A}_{r,z}^{*p,v}\exp\left(\mp i k_z z\right) + \tilde{A}_{r,z}^{p,e}\exp\left(-i\frac{\omega}{v_0}z\right)\right]\times$$
$$\exp\left(i\omega t\right)J_{1,0}(k_r r). \tag{17}$$

As a result, the set of algebraic equations for $\vec{A}^{*p,v}$ was obtained and solved. Inversed integration in (9), (10) can be performed using stationary phase approximation.

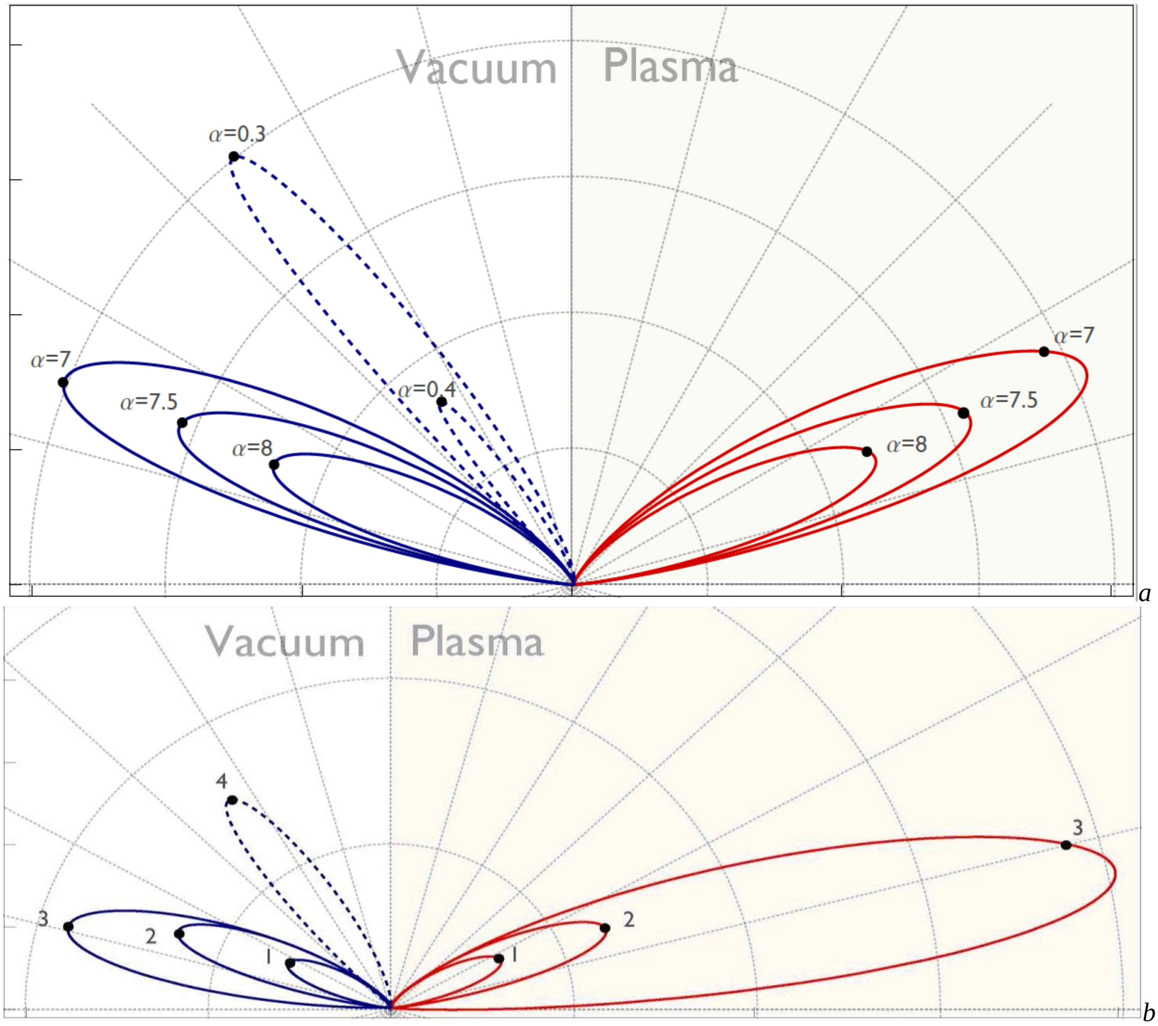

Fig. 2. Transition radiation pattern into the plasma and vacuum for the point charge moving into plasma: a – non-relativistic case β = 0.2; b: α = 14, β = 0.6 (1); α = 9, β = 0.6 (2); α = 20, β = 0.9 (3); α = 0.5, β = 0.2 (4)

4. RADIATION PATTERN

In order to analyze the eigenmodes' magnitudes $\vec{A}^{*p,v}$, Pointing vector was calculated:

$$S_R^v = \frac{\alpha^2\beta^2 q^2\left|\mathbb{P}_1\right|^2}{8\pi c\lambda^2 R^2}\left(\cot^2\Theta - 1\right)\cos^2\Theta, \tag{18}$$

$$S_R^p = \frac{q^2\beta^2(\alpha^2-1)\left|\mathbb{P}_2\right|^2}{8\pi c\lambda^2 R^2}\left(\cot^2\Theta + 1\right)\cos^2\Theta, \tag{19}$$

$$\mathbb{P}_{1,2} = \left(\sqrt{\alpha^2\beta^2 - \kappa_r^2} - \sqrt{\alpha^2\beta^2 - \beta^2 - \kappa_r^2}\right)\mathbb{D}_{1,2}, \tag{20}$$

$$\mathbb{D}_1 = \frac{k_r^2\left[2\left(\alpha^2-1\right)\left(k_r^2+\alpha^2 j+\beta^2\right)+\left(\alpha^2-k_r^2+\beta^2\right)\right]}{\beta^2\alpha\left(\alpha^2-1\right)\left(k_r^2+\alpha^2 j+\beta^2\right)\left(j\alpha^2+k_r^2\right)} +$$
$$+\frac{\left(\alpha^2 j^2 + j\beta^2 - k_r^2\right)\sqrt{\alpha^2\beta^2-\beta^2-k_r^2}}{\beta^2\left(\alpha^2-1\right)\left(k_r^2+\alpha^2 j+\beta^2\right)\left(j\alpha^2+k_r^2\right)}, \tag{21}$$

$$\mathbb{D}_2 = \frac{k_r^2\left(2\left(\alpha^2-1\right)\left(k_r^2+\alpha^2 j+\beta^2\right)+\left(\alpha^2-k_r^2+\beta^2\right)\right)}{\beta^2\alpha\left(\alpha^2-1\right)\left(k_r^2+\alpha^2 j+\beta^2\right)\left(j\alpha^2+k_r^2\right)} -$$
$$-\frac{\left(\alpha^2 j^2+j\beta^2-k_r^2\right)\sqrt{\alpha^2\beta^2-k_r^2}}{\beta^2\left(\alpha^2-1\right)\left(k_r^2+\alpha^2 j+\beta^2\right)\left(j\alpha^2+k_r^2\right)}. \qquad (22)$$

Here $j = 1-\beta^2$, $\alpha = \omega/\omega_p$, $\lambda = \mathrm{v}_0/\omega$ and $k_r = \alpha\beta\sin\Theta$. Radiation patterns calculated after (18)-(22) are presented on Fig. 2. For $\omega < \omega_p$ plasma is non-transparent for electromagnetic waves, and radioemission takes place only to vacuum (see curves for $\alpha = 0.3$ and $\alpha = 0.4$, Fig. 2,a, and curve 4, Fig. 2, b). For relativistic case the angle between the direction of maximal radioemission and charged particle's velocity tends to zero (Fig. 2, b).

The data obtained coincides with the results of the traditional method of calculation [1].

The results of this calculation are valid in the frequency range $\omega_{pi} << \omega << \mathrm{v}_0/L$, where ω_{pi} is the ion Langmuir frequency and L is the characteristic length of the vacuum-plasma border.

5. TRANSITION RADIATION FORMATION ZONE

The proposed interpretation of the transition radioemission mechanism as a result of the accelerated motion of the media electrons makes possible the natural interpretation of the transition radiation formation zone (TRFZ compare with [1]). For homogeneous media radiation of the media electrons along the trajectory of the charged particle with subluminal velocity is averaged to zero. For the sharp boundary of two media there are some areas near the boundary that give their contribution into the radioemission. These areas just form the TRFZ.

CONCLUSIONS

1. Our calculation demonstrates that accelerated electrons are the source of transition radiation for the simplest model of the vacuum - homogeneous plasma border.
2. Proposed method in principle gives the possibility to study non-linear effects in the transition radioemissioin (e.g., generation of the high order harmonics).

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ПЕРЕХОДНОЕ ИЗЛУЧЕНИЕ ЗАРЯДА НА РЕЗКОЙ ГРАНИЦЕ ПЛАЗМА-ВАКУУМ КАК РЕЗУЛЬТАТ УСКОРЕННОГО ДВИЖЕНИЯ ЭЛЕКТРОНОВ ПЛАЗМЫ

В.Я. Гафич, И.А. Анисимов

Исходя из уравнений Максвелла и уравнения движения электронов плазмы, рассчитано переходное излучение точечного заряда на резкой границе вакуума с холодной изотропной плазмой. Расчёты проведены в линейном приближении, однако данный метод позволяет учитывать нелинейные эффекты, которыми сопровождается излучение. Построены диаграммы направленности для различных частот и скоростей. Полученные результаты согласуются с полученными в рамках классического подхода.

ПЕРЕХІДНЕ ВИПРОМІНЮВАННЯ ЗАРЯДУ НА РІЗКІЙ МЕЖІ ПЛАЗМА-ВАКУУМ ЯК РЕЗУЛЬТАТ ПРИСКОРЕНОГО РУХУ ЕЛЕКТРОНІВ ПЛАЗМИ

В.Я. Гафич, І.О. Анісімов

Виходячи з рівнянь Максвелла та рівняння руху електронів плазми, розраховано перехідне випромінювання точкового заряду на різкій межі вакууму з холодною ізотропною плазмою. Розглядається лінійне наближення, однак даний метод дозволяє враховувати нелінійні ефекти, якими супроводжується випромінювання. Побудовано діаграми спрямованості для різних частот та швидкостей. Одержані результати узгоджуються з розрахованими в рамках класичного підходу.