# DISTRIBUTION FUNCTION OF PLASMA PARTICLES IN AXIAL MAGNETIC AND RADIAL ELECTRIC FIELDSFOR THE TRANSVERSE INJECTION OF NEUTRAL GAS

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The distribution function of particles in plasma which is created in crossed axial magnetic and radial electric fields by ionization of gas is obtained. It is assumed that the neutral gas before its ionization rotates with a constant angular velocity and the gas particle velocity distribution function in rotating frame is Maxwellian. Produced plasma particles move in crossed fields without collisions. The obtained distribution function is written in the coordinates of the guiding center. The expressions for the distribution function in the various special cases are also obtained.

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#### INTRODUCTION

Particle distribution function of cylindrically symmetric plasma in crossed axial magnetic and radial electric fields depends on the conditions of its creation. In [1,2] the distribution function of plasma created as a result of ionization of a cold gas was considered:

 $F(\varepsilon_{\perp}, \mu_{\phi}, \nu_{z}) \propto Y(e\Phi_{0} - \varepsilon_{\perp})\delta(\varepsilon_{\perp} - \omega_{e}\mu_{\phi})\delta(\nu_{z}), (1)$ where Y is the Heaviside step function,  $\delta$  is the Dirac delta function,  $\Phi_0$  is the potential on the external electrode (anode),  $\omega_e = -cE_r / Br$ . Variables  $\varepsilon_{\perp \alpha}$ ,  $\mu_{\varphi \alpha}$ and  $v_z$  are the energy of the transverse motion, the generalized angular momentum and velocity along the magnetic field respectively. The radial profile of the electric field potential was assumed parabolic  $\Phi(r) = \Phi_0(r/a)^2$ , which ensured the independence of the rate of angular rotation of the particle on the radius. A distinctive feature of that plasma was a strong radial electric field produced by uncompensated charge of electrons, so that the ions move radially in very elongated orbits and azimuthal precession with frequency  $-\omega_{ci}/2$  where  $\omega_{ci}$  is the ion cyclotron frequency. That model was applied in a Penning discharge with a lowdensity plasma ( $n_0 \sim 10^9 \text{ cm}^{-3}$ ) in the analysis of plasma stability in particular in the study of low frequency oscillations.

In [3] the expression for the plasma particle distribution functions in crossed axial magnetic and radial electric fields, where the thermal motion of atoms before their ionization taken into account, was obtained:

$$f_{0\alpha}(\rho_{\alpha}, R_{\alpha}, V_{z}) = \frac{n_{0}}{(2\pi)^{1/2} v_{T0}^{3}} I_{0} \left( \frac{\Omega_{\alpha}(\Omega_{\alpha} + \omega_{c\alpha})}{v_{T0}^{2}} R_{\alpha} \rho_{\alpha} \right) \times \exp \left( -\frac{\Omega_{\alpha}^{2} R_{\alpha}^{2}}{2v_{T0}^{2}} - \frac{(\Omega_{\alpha} + \omega_{c\alpha})^{2} \rho_{\alpha}^{2}}{2v_{T0}^{2}} - \frac{v_{z}^{2}}{2v_{T0}^{2}} \right) \times Y(R_{\alpha} + \rho_{\alpha} - a), \tag{2}$$

where  $\rho_{\alpha}$  and  $R_{\alpha}$  are variables of the guiding center of particle  $\alpha$  species respectively Larmor radius and radial coordinate of the center of the Larmor circle,  $n_0$  and  $v_{T0}$  are the density and the thermal velocity of the particles before to their appearance in crossed fields.

 $I_0(x)$  is the Bessel function of imaginary argument,  $\Omega_{\alpha}$  is the angular velocity of the drift motion of particles in crossed fields:

$$\Omega_{\alpha} = \frac{\omega_{c\alpha}}{2} \left( -1 + \sqrt{1 + \frac{8e\Phi_0}{m_{\alpha}\omega_{c\alpha}^2 a^2}} \right). \tag{3}$$

This distribution function can be used to analyze the stability of plasma created in crossed fields of vapors of substance that have high boiling point. The application of that plasma, created by the reflective discharge in order to separate elements and isotopes was discussed in [4-6]. The usual analysis of the stability of plasma with a known function of the distribution can also be supplemented by searching the optimum particle distribution function of vapor of working substance, whereby the efficiency of the separation of elements would be enhanced. In this paper, the function of the plasma particle distribution was obtained under conditions when a neutral gas (or vapor of working substance) before ionization rotates with a certain angular velocity, taking into account the thermal spread of velocities of the gas molecules in the rotating frame of reference. Choosing an appropriate gas angular rotation velocity depending on the ion cyclotron frequency of a particular element, or the velocity of the drift motion of plasma particles in crossed fields makes it possible to control the particle distribution function of plasma, thereby regulate the plasma wave processes, as well as affect on the process of separation elements.

### 1. GENERAL EXPRESSION FOR THE DISTRIBUTION FUNCTION

Now we obtain the particle distribution function of plasma in the crossed fields assuming that the gas rotates at a constant angular velocity before its ionization. Here we use the same considerations as in [3] where gas initially was in rest.

Suppose that cylindrically symmetric collisionless plasma is placed in crossed longitudinal magnetic and radial electric fields. In the radial direction plasma is limited by metal electrode (anode) with the radius a and which has a positive potential relative to the axis, so that the electric field is directed into the plasma. We assume that the potential in plasma has a quadratic

dependence on the radius  $\Phi(r) = \Phi_0(r/a)^2$  so that the rotational velocity of the particles is not depends on the radius. This potential distribution occurs in negatively charged plasma with uniform radial distribution of the electron density. Along the magnetic field the plasma is considered to be unlimited. The equilibrium distribution function of plasma particles of a species  $\alpha$  (ion or electron) should depends on variables  $\varepsilon_{\perp\alpha}$ ,  $\mu_{\varphi\alpha}$ ,  $v_z$ , where

$$\varepsilon_{\perp\alpha} = \frac{m_{\alpha}v_{\perp}^2}{2} + e\Phi(r), \, \mu_{\phi\alpha} = \frac{m_{\alpha}r^2\omega_{c\alpha}}{2} + m_{\alpha}v_{\phi}r, \quad (4)$$

 $v_{\perp}^2 = v_r^2 + v_{\varphi}^2$ . Suppose that the particle of  $\alpha$  species appeared in the crossed fields with the initial values of the radial coordinate  $r_0$  and velocity components  $v_{r0}, v_{\varphi 0}, v_{z0}$ . The probability that, moving in crossed fields, the particle will be in the phase volume  $d\varepsilon_{\perp\alpha}d\mu_{\varphi\alpha}dv_z$  is equal to:

$$dp = \delta(\varepsilon_{\perp\alpha} - \frac{m_{\alpha}v_{\perp0}^2}{2} - e\Phi(r_0))\delta(\mu_{\phi\alpha} - \frac{m_{\alpha}r_0^2\omega_{c\alpha}}{2} - m_{\alpha}v_{\phi0}r_0)\delta(v_z - v_{z0})d\varepsilon_{\perp\alpha}d\mu_{\phi\alpha}dv_z.$$
 (5)

This form of dependence of the probability density is a consequence of the laws of conservation of energy and angular momentum of a particle. If, before the appearance of the particles in crossed fields, they have a certain distribution of the coordinates and velocities  $f_{\alpha}(r_0, v_{r0}, v_{\varphi 0}, v_{z0})$ , then the probability that the particle will be found in the phase volume  $d\varepsilon_{\perp\alpha}d\mu_{\varphi\alpha}dv_z r_0dr_0 dv_{r0} dv_{\varphi 0} dv_{z0}$  is equal to:

$$dP = f(r_0, v_{\perp 0}, v_{z0}) dp \, r_0 dr_0 \, dv_{r0} \, dv_{\varphi 0} \, dv_{z0}.$$
 (6)

In order to obtain the particle distribution function, the expression (6) should be integrated over the variables  $r_0, v_{r0}, v_{\omega 0}, v_{z0}$ :

$$F(\varepsilon_{\perp\alpha}, \mu_{\varphi\alpha}) = \int f_{\alpha}(r_0, v_{r0}, v_{\varphi0}, v_{z0}) \delta(\varepsilon_{\perp\alpha} - \frac{m_{\alpha}v_{\perp 0}^2}{2} - e\Phi(r_0))$$

$$\times \delta(\mu_{\varphi\alpha} - \frac{m_{\alpha}r_{0}^{2}\omega_{c\alpha}}{2} - m_{\alpha}v_{\varphi0}r_{0})\delta(v_{z} - v_{z0})r_{0}dr_{0}dv_{r0}dv_{\varphi0}dv_{z0}.$$
 (7)

Integration over variables  $v_{r0}, v_{\varphi 0}, v_{z0}$  gives

$$F(\varepsilon_{\perp \alpha}, \mu_{\varphi \alpha}) = \int_{0}^{\infty} f(r_0, v_{r0}, v_{\varphi 0}, v_{z0}) \frac{1}{m_{\alpha}^2 v_{r0}} dr_0, \qquad (8)$$

where

$$\begin{split} v_{\varphi 0} &= \frac{1}{2m_{\alpha}r_0} (2\mu_{\varphi\alpha} - m_{\alpha}r_0^2\omega_{c\alpha})\,, \\ v_{r 0} &= \frac{1}{2m_{\alpha}\omega_{c\alpha}*r_0} \bigg[ \Big(4\varepsilon_{\perp\alpha} + 2\mu_{\varphi\alpha}\omega_{c\alpha}\Big)^2 - 4\omega_{c\alpha}^2*\mu_{\varphi\alpha}^2\, \bigg]^{1/2} \\ &\times \Bigg[ 1 - \frac{\Big(m\omega_{c\alpha}^2*r_0^2 - 4\varepsilon_{\perp\alpha} + 2\mu_{\varphi\alpha}\omega_{c\alpha}\Big)^2}{\Big(4\varepsilon_{\perp\alpha} + 2\mu_{\varphi\alpha}\omega_{c\alpha}\Big)^2 - 4\omega_{c\alpha}^2*\mu_{\varphi\alpha}^2} \Bigg]^{1/2}\,, \end{split}$$

 $\omega_{c\alpha^*} = (2\Omega_{\alpha} + \omega_{c\alpha})$  is the modified cyclotron frequency in the crossed fields. Now we replace in integral (8) the variable  $r_0$  by  $\varphi$  as

$$\cos \varphi = \left(r_0^2 - \frac{4\varepsilon_{\perp\alpha} + 2\mu_{\varphi\alpha}\omega_{c\alpha}}{m\omega_{c\alpha^*}^2}\right) \times \left[\left(\frac{4\varepsilon_{\perp\alpha} + 2\mu_{\varphi\alpha}\omega_{c\alpha}}{m_{\alpha}\omega_{c\alpha^*}^2}\right)^2 - \frac{4\mu_{\varphi\alpha}^2}{m_{\alpha}^2\omega_{c\alpha^*}^2}\right]^{-1/2} . (9)$$

It gives

$$F(\varepsilon_{\perp\alpha}, \mu_{\varphi\alpha}) = \frac{1}{m_{\alpha}^2 \omega_{c\alpha^*}} \int_0^{\pi} f(r_0, v_{r0}, v_{\varphi 0}, v_{z0})$$

$$\times Y(r_{\alpha} - r_{0}(\varphi)) d\varphi. \tag{10}$$

Suppose that the distribution function  $f_{\alpha}(r_0, v_{r0}, v_{\varphi_0}, v_{z0})$  is given by:

$$f_{\alpha}(r_{0}, v_{r0}, v_{\varphi 0}, v_{z0}) = \frac{n_{0}}{(2\pi)^{3/2} v_{T0}^{3}} \exp\left(-\frac{v_{r0}^{2}}{2v_{T0}^{2}} - \frac{\left(v_{\varphi 0} - \Omega_{0} r_{0}\right)^{2}}{2v_{T0}^{2}} - \frac{v_{z0}^{2}}{2v_{T0}^{2}}\right) \times Y(r_{0} - r_{a}),$$
(11)

that corresponds initially rotating gas with angular velocity  $\Omega_0$ . Then, taken into account (9), we obtain

$$F(\varepsilon_{\perp\alpha}, \mu_{\varphi\alpha}) = \frac{1}{m_{\alpha}^{2} \omega_{c\alpha*}} \frac{n_{0}}{(2\pi)^{3/2}} \int_{0}^{\pi} \exp\left(-\frac{\varepsilon_{\perp\alpha}}{m_{\alpha} v_{T0}^{2}} + \frac{\Omega_{0} \mu_{\varphi\alpha}}{m_{\alpha} v_{T0}^{2}}\right) - \frac{v_{z0}^{2}}{2v_{T0}^{2}} + \frac{1}{2v_{T0}^{2}} \left(\Omega_{\alpha} \left(\Omega_{\alpha} + \omega_{c\alpha}\right) - \Omega_{0} \left(\Omega_{0} + \omega_{c\alpha}\right)\right)$$

$$\times \left| \frac{4\varepsilon_{\perp\alpha} + 2\mu_{\varphi\alpha}\omega_{c\alpha}}{m_{\alpha}\omega_{c\alpha^*}^2} + \cos\varphi \left[ \left( \frac{4\varepsilon_{\perp\alpha} + 2\mu_{\varphi\alpha}\omega_{c\alpha}}{m_{\alpha}\omega_{c\alpha^*}^2} \right)^2 - \frac{4\mu_{\varphi\alpha}^2}{m^2\omega_{c\alpha^*}^2} \right]^{1/2} \right| \right]$$

$$\times Y(r_a - r_0(\phi))d\phi. \tag{12}$$

In the frame of reference rotating with the angular velocity  $\Omega_{\alpha}$  particle of species  $\alpha$  to perform a circular motion like Larmor rotation in the magnetic field. The role of the cyclotron frequency in this case has a modified cyclotron frequency  $\omega_{c\alpha*} = \left(2\Omega_{\alpha} + \omega_{c\alpha}\right)$  and the Larmor radius  $\rho_{\alpha}$  as well as the radial coordinate of the center of the Larmor circle  $R_{\alpha}$  (variables of the guiding center of the particle) related to the variables  $\varepsilon_{\perp\alpha}$  and  $\mu_{\alpha\alpha}$  by the following relations [7]

$$\rho_{\alpha} = \frac{1}{\omega_{c\alpha^*}^2} \sqrt{\frac{2}{m_{\alpha}} \left(\varepsilon_{\perp} - \Omega_{\alpha} \mu_{\varphi}\right)}, \qquad (13)$$

$$R_{\alpha} = \frac{1}{\omega_{c\alpha^*}^2} \sqrt{\frac{2}{m_{\alpha}} \left(\varepsilon_{\perp} + \left(\Omega_{\alpha} + \omega_{c\alpha}\right) \mu_{\varphi}\right)}. \tag{14}$$

Note that  $\rho_{\alpha}$  and  $R_{\alpha}$  are also integrals of motion. Distribution function in the variables of the guiding center will have the form

$$\begin{split} F(\varepsilon_{\perp\alpha}, \mu_{\varphi\alpha}) &= \frac{1}{m_{\alpha}^{2} \omega_{c\alpha*}} \frac{n_{0}}{(2\pi)^{3/2} v_{T0}^{3}} \\ &\times \int_{0}^{\pi} \exp \left( -\frac{1}{2v_{T0}^{2}} \rho_{\alpha}^{2} \left( \Omega_{\alpha} + \omega_{c\alpha} + \Omega_{0} \right)^{2} - \frac{1}{2v_{T0}^{2}} R_{\alpha}^{2} \left( \Omega_{\alpha} - \Omega_{0} \right)^{2} \right) \end{split}$$

$$+R_{\alpha}\rho_{\alpha}\left(\Omega_{\alpha}+\omega_{c\alpha}+\Omega_{0}\right)\left(\Omega_{\alpha}-\Omega_{0}\right)\cos\varphi-\frac{v_{z0}^{2}}{2v_{T0}^{2}}$$

$$\times Y\left(R_{\alpha}+\rho_{\alpha}-r_{0}\right). \tag{15}$$

Integrating (15) over  $\varphi$ , we obtain

$$\begin{split} f_{0\alpha}\left(\rho_{\alpha},R_{\alpha},v_{z}\right) &= \frac{n_{0}}{\sqrt{2\pi}v_{T0}^{3}} \\ \times I_{0}\left(\left(\Omega_{\alpha}+\omega_{c\alpha}+\Omega_{0}\right)\left(\Omega_{\alpha}-\Omega_{0}\right)\frac{R_{\alpha}\rho_{\alpha}}{v_{T0}^{2}}\right) \\ \times \exp\left(-\frac{1}{2v_{T0}^{2}}\rho_{\alpha}^{2}\left(\Omega_{\alpha}+\omega_{c\alpha}+\Omega_{0}\right)^{2} \\ &-\frac{1}{2v_{T0}^{2}}R_{\alpha}^{2}\left(\Omega_{\alpha}-\Omega_{0}\right)^{2}-\frac{v_{z0}^{2}}{2v_{T0}^{2}}\right)Y\left(R_{\alpha}+\rho_{\alpha}-r_{0}\right). \end{split} \tag{16}$$

Expression (16) is the desired distribution function of particles in crossed fields for initially rotated gas with an arbitrary relationship between the energy of motion of particles in an electric field and their energy of thermal motion. Note, that in the limiting case  $\Omega_0 = 0$  expression (16) reduces to (2).

## 2. SPECIAL CASES FOR THE DISTRIBUTION FUNCTION

Now we consider the limiting cases of the distribution function for the different limit ratios of the thermal velocity  $v_{T0}$  and the  $R_{\alpha}\rho_{\alpha}\left(\Omega_{\alpha}+\omega_{c\alpha}+\Omega_{0}\right)\left(\Omega_{\alpha}-\Omega_{0}\right)$  value

Assume first that  $v_{T0} \sim 0$ , while the anode potential  $\Phi_0$  is high enough so that most of the volume of plasma (except the paraxial region) the inequality

 $R_{\alpha}\rho_{\alpha}(\Omega_{\alpha} + \omega_{c\alpha} + \Omega_{0})(\Omega_{\alpha} - \Omega_{0}) \gg v_{T0}^{2}$  (17) holds. Then, for the Bessel functions, we can use the asymptotic form for large values of the argument  $I_{0}(x) \sim e^{x}/\sqrt{2\pi x}$ . In this case the distribution function (10) has the form

$$f_{0\alpha}\left(\rho_{\alpha}, R_{\alpha}, v_{z}\right) = \frac{n_{0}}{2\pi v_{T0}^{2}}$$

$$\times \frac{1}{\sqrt{\left(\Omega_{\alpha} + \omega_{c\alpha} + \Omega_{0}\right)\left(\Omega_{\alpha} - \Omega_{0}\right)R_{\alpha}\rho_{\alpha}}}$$

$$\times \exp\left(-\frac{\left(\left(\Omega_{\alpha} - \Omega_{0}\right)R_{\alpha} - \left(\Omega_{\alpha} + \omega_{c\alpha} + \Omega_{0}\right)\rho_{\alpha}\right)^{2}}{2v_{T0}^{2}} - \frac{v_{z}^{2}}{2v_{T0}^{2}}\right)$$

$$\times Y\left(R_{\alpha} + \rho_{\alpha} - r_{0}\right). \tag{18}$$

It follows from Eq. (18), that the main contribution to the equilibrium distribution function gives the particles, which satisfy to condition:

$$(\Omega_{\alpha} - \Omega_0)R_{\alpha} \approx (\Omega_{\alpha} + \omega_{c\alpha} + \Omega_0)\rho_{\alpha} \gg v_{T0}$$
. (19) Inequality (19) determines the approximation of a strong radial electric field when the thermal velocity of the particles before their appearance in crossed fields is much less than the velocity of the particle drift in crossed fields  $V_{T0} \ll (\Omega_{\alpha} - \Omega_0)R_{\alpha}$ . In the limiting case  $v_{T0} = 0$  the distribution function (18) reduces into  $\delta$ -function:

 $f_{0\alpha}(\rho_{\alpha},R_{\alpha}) \propto \delta((\Omega_{\alpha}-\Omega_{0})R_{\alpha}-(\Omega_{\alpha}+\omega_{c\alpha}+\Omega_{0})\rho_{\alpha})$ . (20) Similar ion distribution function in the form of  $\delta$ -function (1) was considered in [1,2], where the problem of the excitation of the ion cyclotron instability of the plasma in crossed B and  $E_{r}$  fields was solved.

Assume now that the inequality opposite to (17)

$$R_{\alpha}\rho_{\alpha}(\Omega_{\alpha} + \omega_{c\alpha} + \Omega_{0})(\Omega_{\alpha} - \Omega_{0}) \ll v_{T0}^{2}$$
 (21) holds. This inequality corresponds to the case of a weak electric field. In this case, the distribution function (16) takes the form:

$$f_{0\alpha}(\rho_{\alpha}, R_{\alpha}, v_{z}) = \frac{n_{0}}{(2\pi)^{1/2} v_{T0}^{3}}$$

$$\times \exp\left(-\frac{(\Omega_{\alpha} - \Omega_{0})^{2} R_{\alpha}^{2}}{2v_{T0}^{2}} - \frac{(\Omega_{\alpha} + \omega_{c\alpha} + \Omega_{0})^{2} \rho_{\alpha}^{2}}{2v_{T0}^{2}} - \frac{v_{z}^{2}}{2v_{T0}^{2}}\right)$$

$$\times Y(R_{\alpha} + \rho_{\alpha} - a). \tag{22}$$

Note that despite the uniform ionization along the radius, the distribution function (22) is Gaussian on radial coordinate of the guiding center.

Now we will obtain the distribution function for the particular values of angular velocity  $\Omega_0$ . Suppose in (16)  $\Omega_0 = \Omega_\alpha$ , i.e. angular velocity rotation of the gas coincides with the angular velocity of the drift motion of particles in crossed fields. Then the distribution function has the form

$$f_{0\alpha}(\rho_{\alpha}, R_{\alpha}, v_{z}) = \frac{n_{0}}{(2\pi)^{1/2} v_{T0}^{3}} \exp\left(-\frac{\omega_{c\alpha^{*}}^{2} \rho_{\alpha}^{2}}{2v_{T0}^{2}} - \frac{v_{z0}^{2}}{2v_{T0}^{2}}\right) \times Y(R_{\alpha} + \rho_{\alpha} - a). \tag{23}$$

It is obvious that distribution function (23) not depends on  $R_{\alpha}$  and plasma is homogeneous. This is true for arbitrary ratios of the thermal velocity and the velocity of the drift motion of the guiding center.

Assume now, that  $\Omega_0 = -\omega_{c\alpha}$ . Then the distribution function (16) equals

$$f_{0\alpha}\left(\rho_{\alpha}, R_{\alpha}, v_{z}\right) = \frac{n_{0}}{\sqrt{2\pi}v_{T0}^{3}} I_{0}\left(\frac{R_{\alpha}\rho_{\alpha}}{v_{T0}^{2}}\Omega_{\alpha}\left(\Omega_{\alpha} + \omega_{c\alpha}\right)\right)$$

$$\times \exp\left(-\frac{1}{2v_{T0}^{2}}\rho_{\alpha}^{2}\Omega_{\alpha}^{2} - \frac{1}{2v_{T0}^{2}}R_{\alpha}^{2}\left(\Omega_{\alpha} + \omega_{c\alpha}\right)^{2} - \frac{v_{z0}^{2}}{2v_{T0}^{2}}\right)$$

$$Y\left(R_{\alpha} + \rho_{\alpha} - r_{0}\right). \tag{24}$$

As can be seen the expression (24) is obtained from (16) by interchange of variables  $R_{\alpha} \leftrightarrow \rho_{\alpha}$ . The distribution function (24) corresponds to the azimuthal flow component  $\alpha$ , rotating with angular velocity  $\Omega_{\alpha} = -\omega_{c\alpha}$ , however written in the laboratory frame. As shown in [8], the transition to a frame of reference rotating with angular velocity  $\Omega = -\omega_{c\alpha}$  leads to interchange of variables  $R_{\alpha} \leftrightarrow \rho_{\alpha}$  in distribution function and thus in a new frame of reference, we again obtain the expression (16).

Now we consider the case  $\Omega_0 = -(\Omega_\alpha + \omega_{c\alpha})$ . Then the distribution function takes the form

$$f_{0\alpha}\left(\rho_{\alpha}, R_{\alpha}, v_{z}\right) = \frac{n_{0}}{\left(2\pi\right)^{1/2} v_{T0}^{3}} \exp\left(-\frac{\omega_{c\alpha^{*}}^{2} R_{\alpha}^{2}}{2 v_{T0}^{2}} - \frac{v_{z0}^{2}}{2 v_{T0}^{2}}\right) \times Y\left(R_{\alpha} + \rho_{\alpha} - a\right). \tag{25}$$

This expression coincides with (23) up to the replacement  $R_{\alpha} \leftrightarrow \rho_{\alpha}$ , and corresponds to particles encircling plasma axis.

### **CONCLUSIONS**

The expression for the distribution function of plasma particles in crossed longitudinal magnetic and radial electric field using the probabilistic approach is obtained. It is assumed that gas up to the ionization rotates with angular velocity  $\Omega_0$ . This expression takes into account non-zero initial velocity of the atoms in rotating frame. The distribution function includes the product of the modified Bessel function and exponential (16) whose arguments are the coordinates of the guiding center

From the general expression for the distribution function the limiting expressions in the cases of strong (18) and weak (22) radial electric field was obtained. These expressions are consistent with the previously obtained expressions.

The expressions for the distribution function for particular values of the angular velocity  $\Omega_0$  where obtained, in particular:

- 1) When the equality  $\Omega_0 = \Omega_\alpha$  satisfied the distribution function (23) not depends on  $R_\alpha$  and plasma is homogeneous;
- 2) When the equality  $\Omega_0 = -\omega_{c\alpha}$  satisfied the distribution function (24) corresponds to (16) with interchange of variables  $R_{\alpha} \leftrightarrow \rho_{\alpha}$ .
- 3) When the equality  $\Omega_0 = -(\Omega_\alpha + \omega_{c\alpha})$  satisfied the distribution function (25) corresponds to (23) with interchange of variables  $R_\alpha \leftrightarrow \rho_\alpha$ .

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### ФУНКЦИЯ РАСПРЕДЕЛЕНИЯ ЧАСТИЦ ПЛАЗМЫ В ОСЕВОМ МАГНИТНОМ И РАДИАЛЬНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЯХ ПРИ ПОПЕРЕЧНОЙ ИНЖЕКЦИИ НЕЙТРАЛЬНОГО ГАЗА

### Д.В. Чибисов

Получена функция распределения частиц в плазме, которая создаётся в скрещённых осевом магнитном и радиальном электрическом полях. Предполагается, что нейтральный газ перед ионизацией вращается с постоянной угловой скоростью, а функция распределения частиц газа по скоростям во вращающейся системе отсчёта является максвелловской. Образовавшиеся частицы плазмы движутся в скрещённых полях без столкновений. Полученная функция распределения записана в координатах ведущего центра. Получены также выражения для функции распределения в различных частных случаях.

### ФУНКЦІЯ РОЗПОДІЛУ ЧАСТИНОК ПЛАЗМИ В ОСЬОВОМУ МАГНІТНОМУ І РАДІАЛЬНОМУ ЕЛЕКТРИЧНОМУ ПОЛЯХ ПРИ ПОПЕРЕЧНІЙ ІНЖЕКЦІЇ НЕЙТРАЛЬНОГО ГАЗУ

### Д.В. Чібісов

Отримано функцію розподілу частинок у плазмі, яка утворюється в схрещених осьовому магнітному і радіальному електричному полях. Передбачається, що нейтральний газ перед іонізацією обертається зі сталою кутовою швидкістю, а функція розподілу частинок газу за швидкостями в обертовій системі відліку є максвеллівською. Утворині частинки плазми рухаються в схрещених полях без зіткнень. Отримана функція розподілу записана в координатах ведучого центру. Отримано також вирази для функції розподілу в різних окремих випадках.