FINITE LARMOR RADIUS EFFECTS ON TURBULENT TRANSPORT OF TEST-PARTICLES

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A test-particle approach to study transport processes in two-dimensional random electric field is proposed. Despite such approach is not as complete as self-consistent one it allows a better control of problem parameters and makes results more tractable. A frozen electric field is considered. Because of a strong particle trapping effect the problem is a difficult test for statistical methods. Earlier in our previous works particle transport was examined in a drift approximation; here we study finite Larmor radius effects on particle transport. Some methods to account for a finite Larmor radius are considered as generalization of our moment approximation. Results of analytical approximations and direct numerical simulation are compared, and most accurate method is found. The difference between dispersion of particle and gyrocentre displacement is discussed.

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INTRODUCTION

A drift approximation is widely used to describe transport of particles in plasmas. It is often accepted that the drift approximation gives a main contribution to transport, and the effects of finite Larmor radius can be taken into account as corrections. Along with this, account for exact motion of particles can influence the results significantly.

Recently the decorrelation trajectory method was proposed to study two-dimensional diffusion of particles in constant magnetic and random electric fields [1, 2]. In our work [3] an alternative approach to this problem in drift approximation was developed. In the paper [4] the decorrelation trajectory method was compared with our approach and some advantages of latter were shown. Here we generalize our moment approximation to take into account finite Larmor radius effects. Few different procedures of gyroaveraging are considered.

Direct numerical simulation of particle motion in constant magnetic and random electric field is performed. Statistical characteristics of particle ensemble obtained in simulation are used to check the predictions of analytical approaches. The most effective gyroaveraging procedure is found.

1. MODEL

We consider motion of test particles in a constant magnetic field perpendicular to a frozen random electric field given by the potential

$$\sigma(\boldsymbol{\chi}) = A\sum \exp\left(-\kappa_i^2\right)\cos\left(\alpha_i - 2\pi\boldsymbol{\kappa}_i\boldsymbol{\chi}\right), \quad (1)$$

where A is a normalized amplitude

$$A = \sigma_0\sqrt{\kappa_{\max} / \pi^{3/2} N_\kappa N_\theta}\ , \quad (2)$$

and set of $N_r = N_\kappa \times N_\theta$ wave vectors are

$$\boldsymbol{\kappa}_i = \left(n\kappa_{\max} / N_\kappa\right)\left\{\cos\left(2\pi m / N_\theta\right), \sin\left(2\pi m / N_\theta\right)\right\}. \quad (3)$$

Here $n = 1,\ldots, N_\kappa$, $m = 1,\ldots, N_\theta$, and $\{\alpha_i\}$ is the set of random phases that determines a realization of random potential (3).

The Eulerian correlation function of potential (1) in a laboratory frame is

$$C_{\sigma\sigma}^E(\boldsymbol{\chi}) = (2\pi)^{-1}\exp\left(-\pi^2\boldsymbol{\chi}^2/2\right) I_0\left(\pi^2\boldsymbol{\chi}^2/2\right). \quad (4)$$

Corresponding correlation function of velocity is obtained as derivative of the potential correlation function (4)

$$C_{\upsilon_d\upsilon_d}^E(\boldsymbol{\chi}) = -\left(\partial^2/\partial\chi_x{}^2 + \partial^2/\partial\chi_y{}^2\right) C_{\sigma\sigma}^E(\boldsymbol{\chi}). \quad (5)$$

Particle motion is governed by equations for coordinate of gyrocentre $\boldsymbol{\chi}_d$ and gyroradius $\boldsymbol{\rho}$

$$d\rho_i / d\tau = \varepsilon_{ij}\partial\sigma\left(\boldsymbol{\chi}_d + \boldsymbol{\rho}\right)/\partial\chi_{dj},\ i, j = x, y, \quad (6)$$

$$d\rho_i / d\tau = -\varepsilon_{ij}\left(\partial\sigma\left(\boldsymbol{\chi}_d + \boldsymbol{\rho}\right)/\partial\chi_{dj} + 2\pi\rho_j/\sigma_0\right), \quad (7)$$

where $\varepsilon_{xy} = -\varepsilon_{yx} = 1$. The solutions of $2N_r$ equations (6, 7) are found numerically by using the Runge-Kutta method of the 5-th order. Then obtained trajectories are averaged over N_r realizations, and a mean square displacement is calculated. Further it is compared with a prediction of the analytical models. Parameters of numerical models were the following: $\kappa_{max} = 2$, dimensionless amplitude of potential $\sigma_0 = 0.1$, and $N = N_\kappa \times N_\theta = 1440$ ($N_\kappa = 20$, $N_\theta = 72$).

2. ANALYTICAL APPROXIMATION

The analytical approximation is based on the Taylor relation

$$D(\tau) = \frac{1}{2}\frac{d\Delta(\tau)}{d\tau} = \int d\tau\, C_{\upsilon\upsilon}^L(\tau), \quad (8)$$

that gives a diffusion coefficient $D(\tau)$ and mean square displacement $\Delta(\tau)$ as an integral over time of the Lagrangian correlation function of velocity components along particle trajectories $C^L_{\upsilon\upsilon}(\tau)$. This correlation function is unknown and should be derived from the Eulerian correlation function (5). There is no mathematically direct way to obtain it in general case, so various approximations are used.

We start from the moment approximation for a drift motion proposed in [3]. It was analyzed and compared with the decorrelation trajectory method [1, 2] in [4]. According to the moment approximation the relation between the Eulerian and the Lagrangian correlation functions for an isotropic field is given by

$$C^L_{\upsilon_d\upsilon_d}(\tau) \approx C^E_{\upsilon_d\upsilon_d}(\mathbf{X}(\tau)),\ \mathrm{X}_i(\tau)=\sqrt{\Delta_i(\tau)}\,. \quad (9)$$

Here we generalize our previous approach [3, 4] to account for a finite Larmor radius. Two methods are considered: averaging over the Larmor gyration of the Eulerian correlation function (5)

$$C^{E\,A}_{\upsilon_d\upsilon_d}(\boldsymbol{\chi}_d)=(2\pi)^{-1}\int d\boldsymbol{\kappa}\exp(-i\boldsymbol{\kappa}\boldsymbol{\chi}_d)\,C^E_{\upsilon_d\upsilon_d}(\boldsymbol{\kappa})J_0(\kappa\rho), \quad (10)$$

and averaging of the random field (1) that gives for the Eulerian correlation function (5) the different expression

$$C^{E\,B}_{\upsilon_d\upsilon_d}(\boldsymbol{\chi}_d)=(2\pi)^{-1}\int d\boldsymbol{\kappa}\exp(-i\boldsymbol{\kappa}\boldsymbol{\chi}_d)\,C^E_{\upsilon_d\upsilon_d}(\boldsymbol{\kappa})J_0^2(\kappa\rho). \quad (11)$$

These expressions were used in the works [1, 2] in application to the decorrelation trajectories method.

Combining the assumption (9) with equations (4, 5, 8) and (10) or (11) we obtain the final equation for a mean square displacement in the moment approximation in a form

$$d^2\Delta_{\chi_d}/d\tau^2 = C^{E\,A,B}_{\upsilon_d\upsilon_d}\left(\sqrt{\Delta_{\chi_d}}\right). \quad (12)$$

The correlation functions (10, 11) are calculated by means of numerical integration. Expansion of Bessel function are taken as

$J_0(\kappa\rho) \approx 1-\kappa^2\rho^2/4+\kappa^4\rho^4/64-\kappa^6\rho^6/2304+\ldots$,

for $\kappa\rho < 1$, or asymptotic

$$J_0(\kappa\rho)\approx 1/\sqrt{\pi\kappa\rho}\,,$$

for $\kappa\rho > 1$. Results of these approximations are compared with direct numerical simulation in the next section.

3. RESULTS OF SIMULATION

The results are presented in Figs. 1-6. The mean square displacement calculated from numerical simulation (*NS*) is obtained for ensemble of $N_r = 10^4$ realizations of random potential (1). Results of calculation are given for four values of initial Larmor radius $\rho(0) = 0, 0.1, 1, 10$.

In Fig. 1 temporal evolution of mean square displacement of gyrocentres obtained from numerical simulation for a various initial Larmor radius is shown. The results for initial radius $\rho(0) = 0.1$ are found to be very close to $\rho(0) = 0$, the difference is of the order of fluctuations. For larger initial radius difference becomes significant, thus the mean square displacement for $\rho(0) = 10$ drops almost in four times in compare to $\rho(0) = 0$.

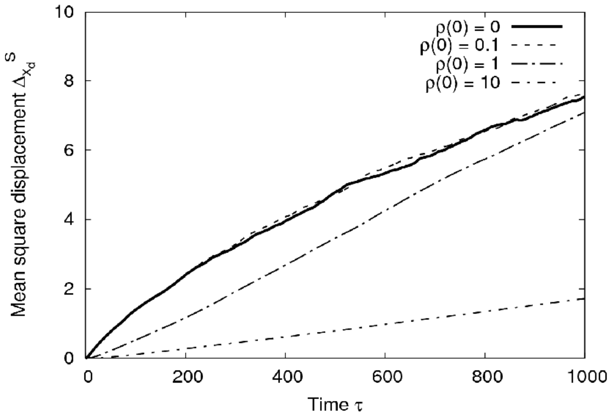

Fig. 1. Mean square displacement of gyrocentres obtained from numerical simulation (NS, $N_r = 10^4$) for initial Larmor radius $\rho(0) = 0, 0.1, 1, 10$

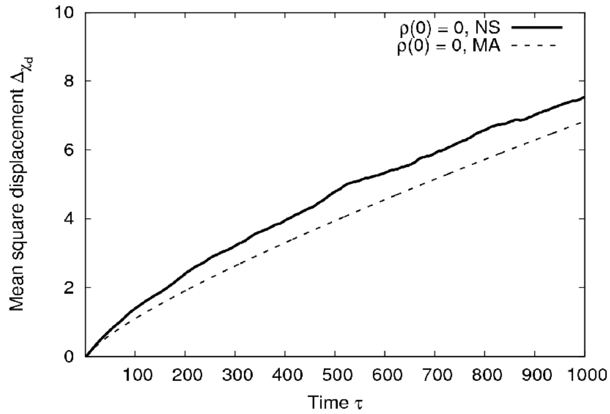

Fig. 2. Mean square displacement of particle gyrocentres for initial Larmor radius $\rho(0) = 0$. Numerical simulation (NS, $N_r = 10^4$), and the moment approximation (MA)

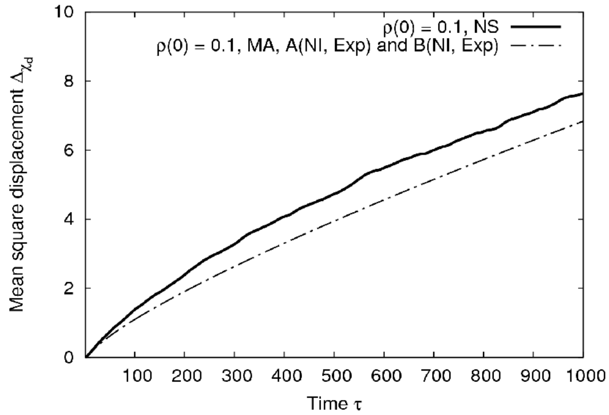

Fig. 3. Mean square displacement of gyrocentres for $\rho(0) = 0.1$. Numerical simulation (NS, $N_r = 10^4$) and analytical model (MA). Different methods of account for Larmor radius effect give the same results

The comparison of mean square displacements of gyrocentres found from numerical simulation and using the moment approximation (*MA*) for initial Larmor radius $\rho(0) = 0$ is demonstrated in Fig. 2. The moment approximation recovers the same subdiffusive behaviour as direct numerical simulation with a quantitative agreement.

The results for a small initial Larmor radius $\rho(0) = 0.1$ are shown in Fig. 3. The difference with the results for radius $\rho(0) = 0$ (see Fig. 2) is small, that is in agreement with the results given in Fig. 1. All approximations of equation (10) – A, and equation (11) – B (using series (*Exp*) and numerical integration (*NI*)) give a similar curves; difference between methods of gyroaveraging is negligible for a small gyroradii.

Mean square displacement of gyrocentres for gyroradius $\rho(0) = 10$ is shown in Fig. 4. Temporal evolution of mean square displacement obtained by the moment approximation B (both with asymptotic (*Asm*) and by numerical integration (*NI*)) is found to be in quantitative agreement with direct numerical simulation. On the contrary the results obtained with approximation A, Eqs. (10), (by numerical integration (*NI*) and with asymptotic (*Asm*)) are inconsistent with results of direct numerical simulation in a range of large gyroradii.

We may conclude that the moment approximation with gyroaveraging based on equation (11) – by means of numerical integration, series expansion and asymptotyc – quantitatively recovers the evolution of mean square displacement of gyrocentres obtained from direct numerical simulation. And thus it can be considered as the sufficiently accurate method to account for finite Larmor radius effects.

Now we switch from consideration of gyrocenters χ_d statistics to examination of exact particle trajectories $\chi = \chi_d + \rho$. In Fig. 5 the mean square displacement of particles Δ_χ obtained from direct numerical simulation for $N_r = 10^4$ realizations is given. It demonstrates a difference between gyrocentre (see Fig. 1) and particle trajectory statistics. For a small initial Larmor radius, $\rho(0) < 1$, there is no significant difference between dispersion of gyrocentres and exact particle trajectories. But with increase of the initial radius $\rho(0) > 1$ the difference becomes noticeable. For $\rho(0) = 10$ initial evolution of dispersion of guiding centre and exact particle position is completely different. The reason is a contribution from a mean square displacement of Larmor radius, its temporal evolution is shown in Fig. 6. It saturates with time: for a small initial Larmor radius a saturation value is negligible. Whether initial value is not small it grows to large magnitude. For any initial value Larmor radius reaches a saturation value within a finite time interval. Consequently a mean square displacement of particle trajectories would be shifted against a curve for gyrocentre dispersion.

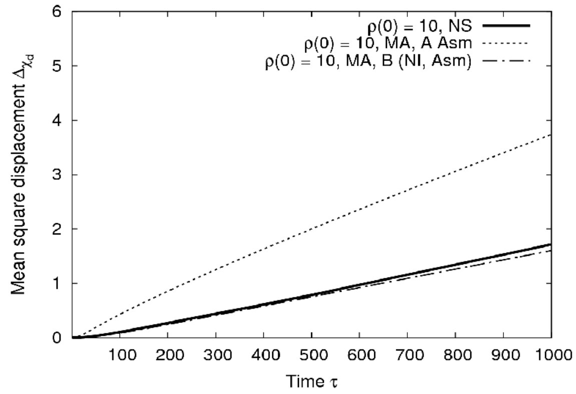

Fig. 4. Mean square displacement of gyrocentres for $\rho(0) = 10$. Numerical simulation (NS, $N_r = 10^4$) and analytical model (MA) with approximation given by Eq.(10) – (A), and Eq. (11) – (B); (Asm) – analytical asymptotic, (NI) – numerical integration

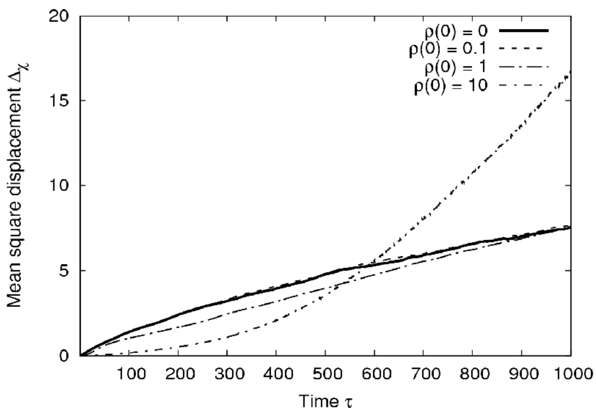

Fig. 5. Mean square displacement of particles trajectories obtained by numerical simulation (NS, $N_r = 10^4$) for initial Larmor radius $\rho(0) = 0, 0.1, 1, 10$

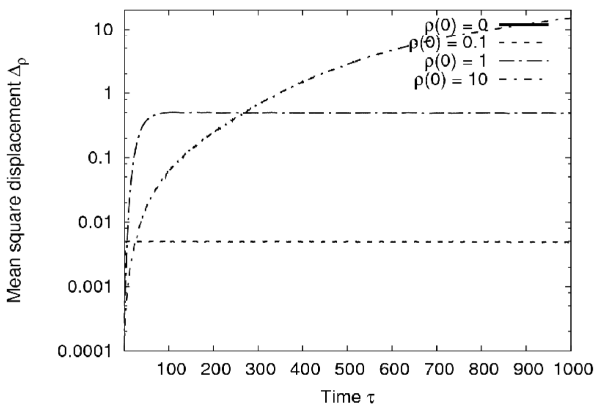

Fig. 6. Mean square displacement of Larmor radius obtained by numerical simulation (NS, $N_r = 10^4$) for initial Larmor radius $\rho(0) = 0, 0.1, 1, 10$

CONCLUSIONS

Temporal evolution of a mean square displacement of gyrocentres found from the direct numerical simulation and from the moment approximation based on a gyroveraging of random potential were compared.

For initial Larmor radius in wide range, $\rho(0) = 0$, 0.1, 1, 10, analytical method based on Eq. (11) gives a satisfactory quantitative agreement. On the contrary the moment approximation using other gyroaveraging of correlation function (10) is inconsistent with results of direct numerical simulation for large gyroradii.

The numerical simulation shows that a mean square displacement of particles trajectories is shifted from mean square displacement of gyrocentres by a value of Larmor radius dispersion. For any initial value of Larmor radius its dispersion is saturated within a finite time interval.

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ВЛИЯНИЕ КОНЕЧНОГО ЛАРМОРОВСКОГО РАДИУСА НА ТУРБУЛЕНТНЫЙ ПЕРЕНОС ПРОБНЫХ ЧАСТИЦ

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Предложен метод изучения процессов переноса пробных частиц в двухмерном случайном електрическом поле. Несмотря на его неполноту в сравнении с самосогласованным описанием он позволяет лучше управлять параметрами задачи и делает результаты исследования более объяснимыми. Рассмотрено замороженное электрическое поле. Поскольку эффекты захвата частиц сильны, эта задача служит хорошим тестом для проверки статистических методов. Ранее перенос частиц был рассмотрен нами в дрейфовом приближении; в данной работе мы исследуем влияние конечного ларморовского радиуса на этот процесс. Рассмотрено несколько способов обобщения развитого ранее дрейфового приближения. Сравнение результатов, полученных на основе аналитических приближений и прямого численного моделирования, позволило определить наиболее точный метод учёта конечного ларморовского радиуса. Уделено внимание различию в дисперсии смещения частиц и соответствующих ведущих центров.

ВПЛИВ СКІНЧЕННОГО ЛАРМОРІВСЬКОГО РАДІУСУ НА ТУРБУЛЕНТНЕ ПЕРЕНЕСЕННЯ ПРОБНИХ ЧАСТИНОК

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Запропоновано метод дослідження процесів перенесення пробних частинок у двомірному випадковому електричному полі. Попри його неповноту в порівнянні із самоузгодженим описом він дозволяє краще керувати параметрами задачі, сприяє кращому розумінню результатів дослідження. Розглянуто заморожене електричне поле. Через сильний ефект захоплення частинок ця задача є добрим тестом для перевірки статистичних методів. Раніше перенесення частинок було розглянуто нами у дрейфовому наближенні; тут ми досліджуємо вплив скінченного ларморівського радіусу на цей процес. Розглянуто декілька шляхів узагальнення розвинутого раніше дрейфового наближення. Порівняння результатів, отриманих на основі аналітичних наближень та прямого числового моделювання, дозволило визначити найбільш точний метод врахування скінченного ларморівського радіусу. Приділено увагу різниці в дисперсії зміщення частинок та відповідних ведучих центрів.