

# CYCLOTRON WAVE ABSORPTION IN D-SHAPED TOKAMAKS

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Transverse dielectric susceptibility elements are derived for radio-frequency waves in a large aspect ratio toroidal plasma with D-shaped magnetic surfaces by solving the Vlasov equation for untrapped and usual  $t$ -trapped particles under moderate elongation and small triangularity in the case, when the so-called  $d$ -trapped particles are absent. These dielectric characteristics are suitable for estimating the wave dissipation by the fundamental cyclotron resonance damping in the frequency range of ion-cyclotron and electron cyclotron resonances.

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## INTRODUCTION

To achieve the fusion conditions in tokamaks an additional plasma heating must be employed. Effective schemes of the heating and current drive in these devices can be realized by the wave dissipation in the frequency range of ion-cyclotron (ICR) and/or electron-cyclotron (ECR) resonances. As is well known, kinetic wave theory of high-temperature plasmas should be developed by solving the Vlasov-Maxwell's equations. However, this problem is not simple even in the scope of the linear theory since to solve the wave equations we should use the suitable dielectric tensor valid in the given frequency range for realistic plasma models. In this paper the transverse susceptibility elements are derived for radio-frequency waves in a two-dimensional (2D) axisymmetric tokamak with D-shaped magnetic surfaces under moderate elongation and small triangularity using an approach developed in [1-3].

## 1. REDUCED VLASOV EQUATION

To describe an axisymmetric D-shaped tokamak we use the quasi-toroidal coordinates  $(r, \theta, \phi)$  connected with the cylindrical ones  $(R, \phi, Z)$  as (see Fig.1)

$$R = R_0 + r \cos \theta - \frac{dr^2}{a^2} \sin^2 \theta, \quad \phi = \phi, \quad Z = -\frac{b}{a} r \sin \theta,$$

where  $R_0$  is the radius of the magnetic axis;  $a$  and  $b$  are, respectively, the minor and major semiaxes of the cross-section of the external magnetic surface. In this model, all magnetic surfaces have the same elongation equal to  $b/a$ ; their triangularity is small  $d/a \ll 1$ , the Shafranov shift is not accounted; the cylindrical components of an equilibrium magnetic field  $\mathbf{H}_0$  are

$$H_{0R} = H_{\theta 0} \frac{R_0}{R} \sin \theta \left( 1 + 2 \frac{dr}{a^2} \cos \theta \right), \quad (1)$$

$$H_{0\phi} = H_{\phi 0} \frac{R_0}{R}, \quad H_{0Z} = H_{\theta 0} \frac{b}{a} \frac{R_0}{R} \cos \theta.$$

Here  $H_{\phi 0}$  and  $H_{\theta 0}$  are, respectively, the toroidal and poloidal magnetic field maximums at a given (by  $r$ ) magnetic surface. Thus, the module  $H_0 = |\mathbf{H}_0|$  of an equilibrium magnetic field is

$$H_0(r, \theta) = \sqrt{H_{\phi 0}^2 + H_{\theta 0}^2} g(\theta), \quad (2)$$

$$g(\theta) = \frac{\sqrt{1 + \lambda \cos^2 \theta + \kappa \cos \theta \sin^2 \theta}}{1 + \varepsilon \cos \theta - \delta \varepsilon \sin^2 \theta},$$

where

$$\varepsilon = \frac{r}{R_0}, \quad \delta = \frac{dr}{a^2}, \quad \lambda = h_\theta^2 \left( \frac{b^2}{a^2} - 1 \right), \quad \kappa = 4\delta h_\theta^2, \\ h_\theta = \frac{H_{\theta 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}, \quad h_\phi = \frac{H_{\phi 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}. \quad (3)$$

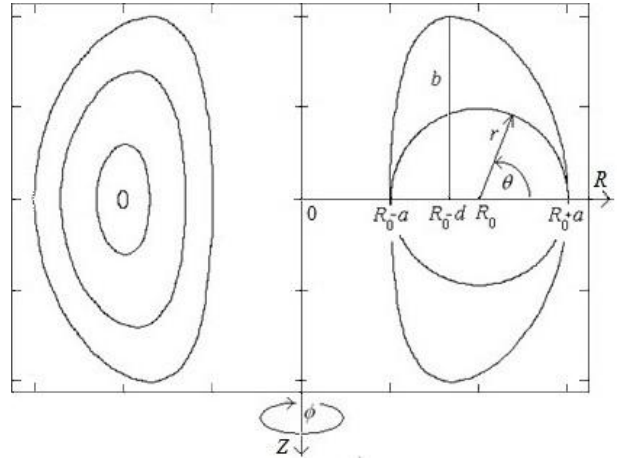


Fig. 1. Cylindrical  $(R, \phi, Z)$  and quasi-toroidal  $(r, \theta, \phi)$  coordinates to describe D-shaped tokamak

To evaluate the transverse susceptibility elements for cyclotron waves in such plasma we should resolve the Vlasov equation for the first,  $l = \pm 1$ , harmonics of the perturbed distribution functions of ions and electrons:

$$f(t, \mathbf{r}, \mathbf{v}) = \sum_s^{\pm 1} \sum_l^{\pm \infty} f_l^s(r, \theta, v, \mu) \exp(-i\omega t + in\phi - il\sigma), \quad (4)$$

we use the standard method of switching to new variables associated with conservation integrals of energy,  $v_{\parallel}^2 + v_{\perp}^2 = \text{const}$ , and magnetic moment,  $v_{\perp}^2 / 2H_0 = \text{const}$ . Introducing the variables  $v$  and  $\mu$  in velocity space instead of parallel,  $v_{\parallel}$ , and perpendicular,  $v_{\perp}$ , components of the particle velocity:

$$v^2 = v_{\parallel}^2 + v_{\perp}^2, \quad \mu = \frac{v_{\perp}^2}{v_{\parallel}^2 + v_{\perp}^2} \frac{1}{g(\theta)}, \quad (5)$$

the kinetic equation for harmonics  $f_l^s(r, \theta, v, \mu)$ , in the zeroth order over the magnetization parameter, after averaging over the gyrophase angle in velocity space

can be reduced to the first order differential equation with respect to the poloidal angle  $\theta$ :

$$\begin{aligned} & \sqrt{\frac{1-\mu g(\theta)}{1+\lambda \cos^2 \theta + \kappa \cos \theta \sin^2 \theta}} \times \\ & \times \left( \frac{\partial f_l^s}{\partial \theta} + \frac{in q f_l^s}{1+\varepsilon \cos \theta - \varepsilon \delta \sin^2 \theta} \right) - \quad (6) \\ & -i \frac{sr}{h_\theta v} [\omega - l \Omega_{c0} g(\theta)] f_l^s + il \sqrt{1-\mu g(\theta)} \gamma(\theta) f_l^s = \\ & = -\frac{serF_0}{Mh_\theta v_T^2} \sqrt{\mu g(\theta)} E_l, \quad l = \pm 1, \end{aligned}$$

where  $q = \varepsilon h_\phi / h_\theta$ , and the variables  $r$ ,  $v$ ,  $\mu$  (as well as  $R$ ,  $a$ ,  $b$ ,  $q$ ,  $N$ ,  $T$ ) appear as the parameters. Here  $E_l = E_n + ilE_b$  is the combination of the normal and binormal (respectively to  $\mathbf{H}_0$ ) electric field projections, equilibrium distribution function  $F_0$  is maxwellian

$$F_0 = \frac{N_0}{\pi^{1.5} v_T^3} \exp\left(-\frac{v^2}{v_T^2}\right), \quad v_T^2 = \frac{2T_0}{M}, \quad (7)$$

with the particle density  $N_0$ , temperature  $T_0$ , charge  $e$ , mass  $M$ . The cyclotron frequency of plasma particles is

$$\Omega_{c0} = \frac{e\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}{Mc}. \quad (8)$$

Account of centrifugal forces in Eq.(1) is reduced to

$$\begin{aligned} \gamma(\theta) = & \frac{3\frac{a}{b}h_\phi}{\cos^2 \theta + \frac{b^2}{a^2}\sin^2 \theta(1+2\delta\cos\theta)} + \\ & + \frac{3\frac{b}{a}h_\phi\varepsilon\cos\theta}{1+\varepsilon\cos\theta-\varepsilon\delta\sin^2\theta} + \quad (9) \\ & + \frac{a}{b}h_\phi\left(\frac{b^2}{a^2}-1\right)\frac{\cos^2\theta\sin^2\theta(1+2\delta\cos\theta)}{\cos^2\theta+\frac{b^2}{a^2}\sin^2\theta(1+2\delta\cos\theta)} + \\ & + \frac{b}{a}h_\phi\left(1-\frac{r}{q}\frac{dq}{dr}\right)\left[\cos^2\theta+\frac{b^2}{a^2}\sin^2\theta(1+2\delta\cos\theta)\right]. \end{aligned}$$

By  $s = \pm 1$  we distinguish the perturbed distribution functions,  $f_l^s$ , of particles with positive and negative values of the parallel velocity

$$v_{||} = sv\sqrt{1-\mu g(\theta)} \quad (10)$$

relative to  $\mathbf{H}_0$ .

Describing the wave-particle interaction in elongated tokamaks we should separate all particles (in the general case, if  $\lambda > \varepsilon$ , Fig.2,a) on the three groups of untrapped,  $t$ -trapped and  $d$ -trapped particles. Such separation can be done by inequalities for  $\mu$  and  $\theta$ :

$$\begin{aligned} 0 \leq \mu \leq \mu_u & \quad -\pi \leq \theta \leq \pi & \quad \text{untrapped particles,} \\ \mu_u \leq \mu \leq \mu_t & \quad -\theta_t \leq \theta \leq \theta_t & \quad \text{t-trapped particles,} \\ \mu_t \leq \mu \leq \mu_d & \quad -\theta_t \leq \theta \leq -\theta_d & \quad \text{d-trapped particles,} \\ \mu_t \leq \mu \leq \mu_d & \quad \theta_d \leq \theta \leq \theta_t & \quad \text{d-trapped particles,} \end{aligned}$$

analyzing the condition  $v_{||}(\mu, \theta) = 0$ . Here

$$\mu_u = 1 - \varepsilon - \frac{\lambda}{2}, \quad \mu_t = 1 + \varepsilon - \frac{\lambda}{2}, \quad \mu_d = 1 + \frac{\varepsilon^2}{2\lambda}, \quad (11)$$

and the angels  $\pm\theta_t$  and  $\pm\theta_d$  are the stop points of  $t$ - and  $d$ -trapped particles, respectively, on the considered magnetic surface.

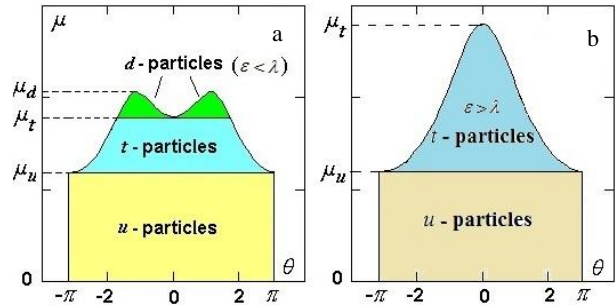


Fig. 2. Untrapped,  $t$ - and  $d$ -trapped particles in the D-shaped tokamaks

However, if  $\lambda < \varepsilon$ , the  $d$ -trapped particles are absent in the D-shaped tokamaks, Fig. 2,b. In this more realistic case, when tokamak has the large aspect ratio,  $\varepsilon \ll 1$ ; the moderate elongation,  $\lambda \ll 1$  or roughly  $b/a < 2$ ; the small triangularity  $\delta \ll 1$ , the stop points for the  $t$ -trapped particles are:

$$\begin{aligned} \pm\theta_t \approx & \pm \arccos \frac{\mu-1}{\varepsilon} \mp \frac{1}{\left(\varepsilon - \frac{\kappa}{2}\right)\sqrt{1-\left(\frac{\mu-1}{\varepsilon}\right)^2}} \times \\ & \times \left[ \varepsilon\delta - \left(\varepsilon\delta - \frac{\lambda}{2}\right)\left(\frac{\mu-1}{\varepsilon}\right)^2 + \frac{\kappa}{2}\frac{\mu-1}{\varepsilon} - \frac{\kappa}{2}\left(\frac{\mu-1}{\varepsilon}\right)^3 \right]. \quad (12) \end{aligned}$$

To find the perturbed distribution functions of untrapped  $f_{l,u}^s$  and  $t$ -trapped particles we should resolve Eq. (6) using the corresponding boundary conditions: the periodicity of  $f_{l,u}^s$  on  $\theta$ , and continuity of  $f_{l,t}^s$  at the stop points  $\pm\theta_t$ ; introducing the new time-like variable instead of poloidal angle  $\theta$  as

$$\tau(\theta) = \int_0^\theta \frac{1 + \lambda \cos^2 \eta + \kappa \cos \eta \sin^2 \eta}{1 - \mu \cdot g(\eta)} d\eta. \quad (13)$$

In this case, the transit-time of  $u$ -particles and the bounce-period of  $t$ -trapped particles are proportional to  $T_u = 2\tau(\pi)$  and  $T_t = 4\tau(\theta_t)$ , respectively.

## 2. TRANSVERSE SUSCEPTIBILITY

Knowing  $f_{l,u}^s$  and  $f_{l,t}^s$ , we can calculate the contribution of  $u$ - and  $t$ -particles to the 2D transverse current density components by

$$\begin{aligned} j_l(r, \theta) = & \frac{\pi e}{2} g(\theta)^{3/2} \sum_s^{\pm 1} \int_0^\infty v^3 dv \times \\ & \times \left\{ \int_0^{\mu_t} \frac{f_{l,u}^s \sqrt{\mu} d\mu}{\sqrt{1-\mu g(\theta)}} + \int_{\mu_u}^{1/g(\theta)} \frac{f_{l,t}^s \sqrt{\mu} d\mu}{\sqrt{1-\mu g(\theta)}} \right\}. \quad (14) \end{aligned}$$

To evaluate the transverse susceptibility elements we use the Fourier-expansions of the perturbed current density and electric field components on angle  $\bar{\theta}$ :

$$j_l(\theta) \frac{(1 + \varepsilon \cos \theta - \varepsilon \delta \sin^2 \theta)^{5/2}}{(1 + \lambda \cos^2 \theta + \kappa \cos \theta \sin^2 \theta)^{3/4}} = \sum_m^{\pm\infty} j_l^{(m)} e^{im\bar{\theta}}, \quad (15)$$

$$E_l(\theta) \frac{(1 + \lambda \cos^2 \theta + \kappa \cos \theta \sin^2 \theta)^{3/4}}{(1 + \varepsilon \cos \theta - \varepsilon \delta \sin^2 \theta)^{1/2}} = \sum_{m'}^{\pm\infty} E_l^{(m')} e^{im'\bar{\theta}}. \quad (16)$$

Here

$$\bar{\theta}(\theta) = \frac{2(\varepsilon + \delta)}{\varepsilon + \delta(1 - \sqrt{1 - \varepsilon^2})} \arctg \left( \sqrt{\frac{1 - \varepsilon}{1 + \varepsilon}} \operatorname{tg} \frac{\theta}{2} \right) - \frac{\delta \sqrt{1 - \varepsilon^2}}{\varepsilon + \delta(1 - \sqrt{1 - \varepsilon^2})} \left( \theta - \frac{\varepsilon \sin \theta}{1 + \varepsilon \cos \theta} \right), \quad (17)$$

is the new poloidal angle in the coordinate system, where the equilibrium magnetic field lines are straight.

As a result, the  $m$ -th harmonic  $j_l^{(m)}$  of the transverse current density can be calculated by

$$\frac{4\pi i}{\omega} j_l^{(m)} = \sum_m^{\pm\infty} \chi_{l,u}^{m,m'} E_l^{(m')} = \sum_{m'}^{\pm\infty} (\chi_{l,u}^{m,m'} + \chi_{l,t}^{m,m'}) E_l^{(m')}, \quad (18)$$

where  $\chi_{l,u}^{m,m'}$  and  $\chi_{l,t}^{m,m'}$  denote the independent contribution of the untrapped and  $t$ -trapped particles of any kind (electrons or ions) to the transverse susceptibility elements  $\chi_{l,u}^{m,m'}$ :

$$\chi_{l,u}^{m,m'} = \frac{\omega_p^2 r \rho}{8\omega h_\theta \nu_T \pi^{5/2}} \sum_{p=-\infty}^{\infty} \int_0^{\mu_2} \mu d\mu \int_{-\infty}^{\infty} \frac{v^4 e^{-v^2} A_l^{m,p} A_l^{m',p} dv}{Z_{l,u}^p(v, \mu)}, \quad (19)$$

$$\chi_{l,t}^{m,m'} = \frac{\omega_p^2 r \rho}{8\omega h_\theta \nu_T \pi^{5/2}} \sum_{p=-\infty}^{\infty} \int_{\mu_u}^{\mu_2} \mu d\mu \int_{-\infty}^{\infty} \frac{v^4 e^{-v^2} B_l^{m,p} \tilde{B}_l^{m',p} dv}{Z_{l,t}^p(v, \mu)}. \quad (20)$$

Here

$$\rho = \frac{\sqrt{1 - \varepsilon^2}}{1 + \frac{\delta}{\varepsilon} (1 - \sqrt{1 - \varepsilon^2})},$$

$$Z_{l,u}^p = \frac{r T_u}{2\pi \nu_T h_\theta} (l \Omega_{c0} \bar{g}_u - \omega) - \left[ p - n q_t + l \frac{I_\gamma(\pi)}{\pi} \right] \nu,$$

$$A_l^{m,p}(v, \mu) = \int_{-\pi}^{\pi} \frac{\exp[i\Phi_{l,u}^{p,m}(\theta, v, \mu)]}{\sqrt{1 - \mu g(\theta)}} d\theta,$$

$$\Phi_{l,u}^{p,m}(\theta, v, \mu) = 2\pi(p - n q_t) \frac{\tau(\theta)}{T_u} + (m - n q_t) \bar{\theta}(\theta) -$$

$$- l \frac{r \Omega_{c0}}{\nu_T \nu h_\theta} \left[ \bar{g}_u \tau(\theta) - I_g(\theta) \right] - l I_\gamma(\theta) + l \frac{2\tau(\theta)}{T_u} I_\gamma(\pi),$$

$$\bar{g}_u = \frac{2}{T_u} \int_0^\pi g(\theta) \frac{\sqrt{1 + \lambda \cos^2 \theta + \kappa \cos \theta \sin^2 \theta} d\theta}{\sqrt{1 - \mu g(\theta)}},$$

$$Z_{l,t}^p = \frac{r T_t}{2\pi \nu_T h_\theta} (l \Omega_{c0} \bar{g}_t - \omega) - p \nu,$$

$$B_l^{m,p}(v, \mu) = \int_{-\theta_1}^{\theta_2} \frac{\exp[i\Phi_{l,t}^{p,m}(\theta, v, \mu)]}{\sqrt{1 - \mu g(\theta)}} d\theta,$$

$$\tilde{B}_l^{m,p}(u, \mu) = B_l^{m,p}(u, \mu) + (-1)^p B_l^{m,-p}(-u, \mu),$$

$$\Phi_{l,t}^{p,m}(\theta, v, \mu) = 2\pi p \frac{\tau(\theta)}{T_t} - (m - n q_t) \bar{\theta}(\theta) -$$

$$- l \frac{r \Omega_{c0}}{\nu_T \nu h_\theta} \left[ \bar{g}_t \tau(\theta) - I_g(\theta) \right] - l I_\gamma(\theta),$$

$$\bar{g}_t = \frac{2}{T_t} \int_0^\pi g(\theta) \frac{\sqrt{1 + \lambda \cos^2 \theta + \kappa \cos \theta \sin^2 \theta} d\theta}{\sqrt{1 - \mu g(\theta)}},$$

$$I_g(\theta) = \int_0^\theta g(\eta) \frac{\sqrt{1 + \lambda \cos^2 \eta + \kappa \cos \eta \sin^2 \eta} d\eta}{\sqrt{1 - \mu g(\eta)}},$$

$$I_\gamma(\theta) = \int_0^\theta \frac{\gamma(\eta) \cdot d\eta}{\sqrt{1 + \lambda \cos^2 \eta + \kappa \cos \eta \sin^2 \eta}},$$

$$q_t = \frac{q}{\sqrt{1 - \varepsilon^2}} \left( 1 + \frac{\delta}{\varepsilon} (1 - \sqrt{1 - \varepsilon^2}) \right). \quad (21)$$

The safety factor  $q_t$ , introduced in Eqs.(21), for the D-shaped tokamaks with large aspect ratio and small ellipticity and triangularity (e.i. when  $\varepsilon \ll 1$ ,  $\lambda \ll 1$  and  $\delta \ll 1$ ) can be simplified to

$$q_t \approx q \left( 1 + \frac{\varepsilon(\varepsilon + \delta)}{2} \right). \quad (22)$$

As was mentioned above these equations describe the contribution of any kind particles to the transverse susceptibility elements. The corresponding expressions for plasma electrons and ions can be received, as usual, by changing  $T_0$ ,  $N_0$ ,  $M$ ,  $e$  on the parameters of electrons  $T_{0e}$ ,  $N_{0e}$ ,  $m_e$ ,  $e_e < 0$  and ions  $T_{0i}$ ,  $N_{0i}$ ,  $M_i$ ,  $e_i$ . In order to obtain the general expressions for  $\chi_{l,u}^{m,m'}$  and  $\chi_{l,t}^{m,m'}$  we should sum over all kinds of plasma particles.

The expressions (19) and (20) for the transverse susceptibility elements are written by the summation of bounce-resonant terms including the double integration in velocity space, the phase coefficients  $A_l^{m,p}$ ,  $B_l^{m,p}$  and the resonant denominators:

$$\frac{r T_u}{2\pi \nu_T h_\theta} (l \Omega_{c0} \bar{g}_u - \omega) - \left[ p - n q_t + l \frac{I_\gamma(\pi)}{\pi} \right] \nu = 0 \quad (23)$$

for the untrapped particles; and

$$\frac{r T_t}{2\pi \nu_T h_\theta} (l \Omega_{c0} \bar{g}_t - \omega) - p \nu = 0 \quad (24)$$

for  $t$ -trapped particles.

These wave-particle resonance conditions in axisymmetric D-shaped tokamaks involve the energetic characteristics of particles (by the non-dimensional parameters  $\nu = v/\nu_T$  and  $\mu$ ), the wave frequency  $\omega$ , the integer numbers of cyclotron (by  $l$ ) and bounce (by  $p$ ) resonances.

As for the cyclotron harmonics with high numbers  $|l| \geq 2$ , the resonance conditions in this case will be coinciding with Eq.(23) and Eq.(24) for the untrapped and trapped particles, respectively. For the low  $l$ , as usual, we have the conditions of the:

- Cherenkov resonance, if  $l = 0$ ;
- normal ion-cyclotron resonance, if  $l = 1$ ;
- normal electron-cyclotron resonance, if  $l = -1$ ;

for both the untrapped and  $t$ -trapped particles.

Of course, analyzing the wave-particle resonance conditions in toroidal geometry we should take into account the phase coefficients  $A_l^{m,p}$  and  $B_l^{m,p}$  for the untrapped and  $t$ -trapped particles, respectively.

The resonance conditions (23) and (24) are written for unspecified plasma particles. The corresponding

resonance conditions for electrons and ions can be obtained from Eq. (23) and Eq. (24) by the change of mass  $M$  and charge  $e$  on  $M_e, e_e$  and  $M_i, e_i$ , respectively.

## CONCLUSIONS

In conclusion, let us summarize the main results of the paper.

As is well known, the collisionless wave dissipation in the frequency range of ICR and ECR can be realized under the conditions if the plasma particles interact effectively with the transverse electric field components,  $E_n \pm iE_b$ . The bounce-averaged wave-particle resonance conditions in the frequency range of the fundamental cyclotron resonances ( $l = \pm 1$ ) are presented in Eq. (23) and Eq. (24) for untrapped and  $t$ -trapped particles, respectively.

The specific features of the wave-particle interactions in the D-shaped tokamaks are due to that i) the resonance conditions for untrapped and  $t$ -trapped particles are different, and ii) all the harmonics of  $E_{\pm 1} = E_n \pm iE_b$  contribute into the  $m$ -th harmonic of the transverse current density component,  $j_{\pm 1}^{(m)}$ .

The absorbed wave power under the high frequency plasma heating on the fundamental cyclotron harmonic,

$$P_{C,l} = 0.5 \operatorname{Re}(E_l j_{(l)}^*), \quad (25)$$

can be estimated by the expression

$$P_{C,l} = \frac{\omega}{8\pi} \sum_m^{\pm\infty} \sum_{m'}^{\pm\infty} (\operatorname{Im} \chi_{l,u}^{m,m'} + \operatorname{Im} \chi_{l,t}^{m,m'}) \times \quad (26)$$

$$\times [\operatorname{Re} E_l^{(m)} \operatorname{Re} E_l^{(m')} + \operatorname{Im} E_l^{(m)} \operatorname{Im} E_l^{(m')}].$$

As was mentioned above,  $l=1$  corresponds to wave power absorbed under the ICR plasma heating, when

$\omega \sim \Omega_{c,i}$  and the left-hand polarized waves ( $E_n + iE_b$ ) interact effectively with the resonant ions. The case  $l=-1$  should be considered under the ECR plasma heating when  $\omega \sim |\Omega_{c,e}|$  and the right-hand polarized waves ( $E_n - iE_b$ ) interact with the electrons.

Contribution of untrapped and  $t$ -trapped particles to the imaginary parts of the transverse susceptibility elements,  $\operatorname{Im} \chi_{l,u}^{m,m'}$  and  $\operatorname{Im} \chi_{l,t}^{m,m'}$ , can be estimated by Eq. (19) and Eq. (20) using the well known Landau residues method.

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## ПОГЛОЩЕНИЕ ЦИКЛОТРОННЫХ ВОЛН В D-ОБРАЗНЫХ ТОКАМАКАХ

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Поперечные элементы диэлектрической восприимчивости для радиочастотных волн в плазме аксиально-симметричных токамаков с большим аспектным отношением, умеренной эллиптичностью и малой треугольностью магнитных поверхностей D-образного сечения получены на основании решения уравнений Власова для пролётных и  $t$ -запертых частиц в случае, когда так называемые  $d$ -запертые частицы отсутствуют. Эти диэлектрические характеристики применимы для оценки циклотронного поглощения электромагнитных волн (например, во время нагрева плазмы) в диапазоне частот ионно-циклотронного или электронно-циклотронного резонансов.

## ПОГЛИНАННЯ ЦИКЛОТРОННИХ ХВИЛЬ В D-ПОДІБНИХ ТОКАМАКАХ

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Поперечні елементи діелектричної сприйнятливості для радіочастотних хвиль в аксіально-симетричних токамаках з великим аспектним співвідношенням, помірної еліптичністю та малої трикутністю магнітних поверхонь D-подібного перерізу отримані через розв'язок рівнянь Власова для пролітних та  $t$ -запертих частинок в умовах коли так звані  $d$ -заперті частинки відсутні. Ці діелектричні характеристики мають бути застосовані для оцінки циклотронного поглинання електромагнітних хвиль (наприклад, під час нагріву плазми) у діапазоні частот іонно-циклотронного або електронно-циклотронного резонансів.