ELECTROMAGNETIC WAVE SCATTERING ON A DIELECTRIC CYLINDER IN BORN APPROXIMATION

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The scattering of the plane electromagnetic wave on a spatially extended, fiber-lake target is considered. The fomula for the scattering cross section is obtained using the approximation analogous to Born one in quantum mechanics.

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INTRODUCTION

The problem of multiple scattering of the electromagnetic wave under oblique incidence on the system of parallel dielectric fibers was considered in [1]. The scattering on the single fiber had been described there in the limit of infinitely thin fiber. The wave scattering cross section in that case possesses the axial symmetry for small angle of incidence ψ<<1.

The wave scattering cross section by the cylindric fiber of finite radius is calculated in the present article using Born approximation. The substantial axial anisotropy of the scattered radiation (especially for large ψ angles) is demonstrated.

1. BORN APPROXIMATION IN RADIATION SCATTERING THEORY

The equations for the electric field of a monochromatic wave $\mathbf{E}(\mathbf{r})e^{-i\omega t}$ in non-uniform medium created in the target under passage of the particle satisfies the equations [2]

$$(\Delta + \varepsilon\, \omega^2/c^2)\mathbf{E} = \text{grad div}\, \mathbf{E}, \tag{1}$$

$$\text{div}\, (\varepsilon\mathbf{E}) = 0. \tag{2}$$

(where Δ is the Laplasian operator and $\varepsilon(\mathbf{r})$ is the dielectric permittivity of the medium for the given frequency ω) could be easily derived from Maxwell equations (see, e.g. [2, §68]). The solution of Eqs. (1), (2) for the case when our nonuniform medium has the structure of a spatially localized target in vacuum are convenient to find as the superposition

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{(0)}(\mathbf{r}) + \mathbf{E}^{(1)}(\mathbf{r}), \tag{3}$$

where $\mathbf{E}^{(0)}$ is the electric field of the incident wave that satisfies the equation

$$(\Delta + \omega^2/c^2)\mathbf{E}^{(0)} = 0. \tag{4}$$

Hence the field $\mathbf{E}^{(1)}$ in (3) has to be treated as the field of the scattered radiation.

Eq. (1) can be written using (2), (3) and (4) in the form

$$\left(\Delta + \frac{\omega^2}{c^2}\right)\mathbf{E}^{(1)} = \frac{\omega^2}{c^2}(1-\varepsilon)\mathbf{E} + \text{grad div}((1\text{-}\varepsilon)\mathbf{E})\,. \tag{5}$$

The last equation could be presented in the integral form,

$$\mathbf{E}^{(1)}(\mathbf{r}) = \int G(\mathbf{r}-\mathbf{r}')\Big\{\frac{\omega^2}{c^2}(1-\varepsilon(\mathbf{r}'))\mathbf{E}(\mathbf{r}') + \text{grad div}\big[(1-\varepsilon(\mathbf{r}'))\mathbf{E}(\mathbf{r}')\big]\Big\}d^3r'\,, \tag{6}$$

where $G(\mathbf{r}-\mathbf{r}')$ is the Green function for Eq. (5),

$$G(\mathbf{r}-\mathbf{r}') = \int \frac{e^{i\boldsymbol{\kappa}(\mathbf{r}-\mathbf{r}')}}{(\omega/c)^2 - \kappa^2 + i0}\frac{d^3\kappa}{(2\pi)^3}\,.$$

The asymptotic of that function on large distances r from the domain where $\varepsilon(\mathbf{r})$ is different from unit,

$$G(\mathbf{r}-\mathbf{r}')\big|_{r\to\infty} \to -\frac{1}{4\pi}\frac{e^{ik_f r}}{r}e^{-i\mathbf{k}_f\mathbf{r}'} \to -\frac{1}{4\pi}\frac{e^{i\omega r}}{r}e^{-i\vec{k}\vec{r}'}\,.$$

where $\mathbf{k}_f = (\omega/c)\mathbf{r}/r$ is the wave vector of the scattered wave, $|\mathbf{k}_f|=|\mathbf{k}_i|$, is needed to find the field of the scattered radiation. Substituting the last formula into (6) and integrating the second term by parts we obtain

$$\mathbf{E}^{(scattered)}(\mathbf{r}) = \mathbf{E}^{(1)}(\mathbf{r})_{r\to\infty} = -\frac{1}{4\pi}\frac{e^{ik_f r}}{r}\left(\frac{\omega^2}{c^2}\mathbf{I} - \mathbf{k}_f(\mathbf{k}_f\cdot\mathbf{I})\right), \tag{7}$$

where

$$\mathbf{I} = \int (1-\varepsilon(\mathbf{r}))\mathbf{E}(\mathbf{r})e^{-i\mathbf{k}_f\mathbf{r}}\,d^3r\,. \tag{8}$$

We see that the integrand in (8) would be nonzero only in the domain where the dielectric permittivity is not equal to unit. That illustrates the origin of the scattered radiation from the motion of the electrons in the medium excited by the incident wave.

To select the polarization of the scattered wave one have to project (7) to the chosen polarization vector $\mathbf{e}_f \perp \mathbf{k}_f$:

$$\mathbf{e}_f\cdot\mathbf{E}^{(scattered)}(\mathbf{r}) = \frac{1}{4\pi}\frac{e^{ik_f r}}{r}\frac{\omega^2}{c^2}\mathbf{e}_f\cdot\mathbf{I}\,. \tag{9}$$

The integral form (6) of the field equation (5) that leads to (7), (8) is useful for construction of various approximate solutions. The simplest case is $|1-\varepsilon| << 1$, when the difference $(1-\varepsilon)$ in (8) could be treated as a small perturbation, hence the whole electric field in the target $\mathbf{E}(\mathbf{r})$ in (8) could be replaced by the field $\mathbf{E}^{(0)}$ of the incident wave. This approximation is equivalent to Born approximation in the quantum theory of scattering (see, e.g., [3]).

Substituting into (8) the field $\mathbf{E}^{(0)} \propto \mathbf{e}_i e^{i\mathbf{k}_i\mathbf{r}}$ of the incident plane wave (where $\mathbf{k}_i$ is the incident wave vector, $\mathbf{e}_i$ is its polarization vector, and the wave amplitude has no matter for computation of the scattering cross section), calculating the average energy flux of the scattered wave (7) to the solid angle $d\Omega$ and dividing it by the average intensity of the incident wave we obtain the scattering cross section

$$\frac{d\sigma}{d\Omega}=\frac{\omega^2}{(4\pi)^2c^2}\left|\mathbf{k}_f\times\mathbf{I}^{(B)}\right|^2, \qquad (10)$$

where

$$\mathbf{I}^{(B)}=\mathbf{e}_i\int(1-\varepsilon(\mathbf{r}))e^{i(\mathbf{k}_i-\mathbf{k}_f)\mathbf{r}}\,d^3r= =\mathbf{e}_i\int(1-\varepsilon(\mathbf{r}))e^{i\mathbf{qr}}\,d^3r, \qquad (11)$$

where $\mathbf{q}=\mathbf{k}_i$-$\mathbf{k}_f$.

The scattering cross section for the radiation with the polarization $\mathbf{e}_f$ selected by the detector would be described by the formula

$$\frac{d\sigma}{d\Omega}=\frac{\omega^4}{(4\pi)^2c^4}\left|\mathbf{e}_f\cdot\mathbf{I}^{(B)}\right|^2, \qquad (12)$$

as it could be easy to see from (9).

2. THE SCATTERING ON THE DIELECTRIC CYLINDER

The formulae (10) and (12) describe the radiation scattering by the target of arbitrary structure. Consider now the simplest case of uniform cylinder of the radius a and the length $L\to\infty$ as the target (Figure), when the cylinder's axis makes the angle ψ with the direction of the wave incidence $\mathbf{k}_i$. The integrals in (11) could be easily calculated after rotation of the coordinate axes to coincidence of the new axis z' with the cylinder's axis. Then the integration over z' gives δ-function, and the integration in the transverse plane gives Bessel function, so

$$\mathbf{I}^{(B)}_{cyl}=(1-\varepsilon)\mathbf{e}_i(2\pi a)^2\delta(q_\parallel)\frac{J_1(q_\perp a)}{q_\perp a}, \qquad (13)$$

where $q_\parallel$ and $q_\perp$ are the components of $\mathbf{q}$ parallel and perpendicular to the cylinder's axis. The presence of the δ-function expresses the equality of the components of $\mathbf{k}_i$ and $\mathbf{k}_f$ that parallel to the cylinder's axis, that means the azimuthal character of the scattering: since the absolute values of the wave vectors $\mathbf{k}_i$ and $\mathbf{k}_f$ are also equal, the scattered radiation will be directed along the surface of the cone with the axis along the cylinder and the half-opening angle equal to the incidence angle ψ.

The azimuthal character of the scattering permits clear interpretation as a manifestation of Cherenkov mechanism. Indeed, the incident wave produces a perturbation in the medium that moves along the fiber with the phase velocity $v=\omega/(k_i)_\parallel=c/\cos\psi>c$. This superluminal motion generates the radiation analogous to Cherenkov one. The half-opening angle of the Cherenkov cone $\cos\theta=c/v$ is just equal to ψ.

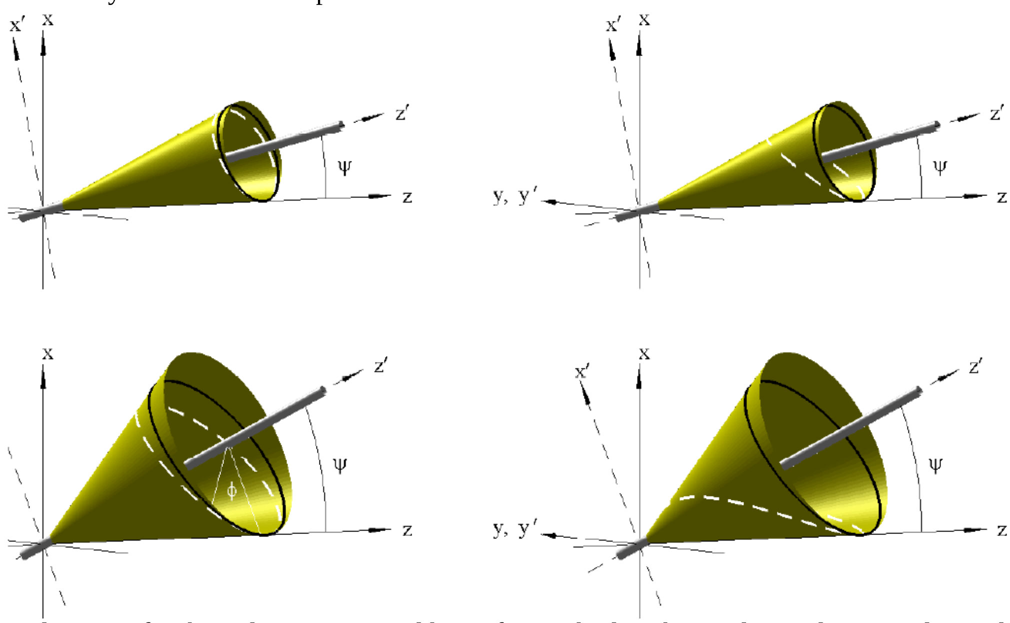
Substitution of (13) into (10) gives (using the rule of δ-function squaring, $[\delta(q_\parallel)]^2=\delta(q_\parallel)\cdot L/2\pi$) the cross section

$$\frac{d\sigma}{d\Omega}=\frac{\pi}{2}\frac{\omega^2}{c^2}\left|\mathbf{k}_f\times\mathbf{e}_i\right|^2\times \times La^4(1-\varepsilon)^2\delta(q_\parallel)\left(\frac{J_1(q_\perp a)}{q_\perp a}\right)^2. \qquad (14)$$

The cross section corresponding to the polarization $\mathbf{e}_f$ is described by the formula

$$\frac{d\sigma}{d\Omega}=\frac{\pi}{2}\frac{\omega^4}{c^4}\left|\mathbf{e}_f\cdot\mathbf{e}_i\right|^2\times \times La^4(1-\varepsilon)^2\delta(q_\parallel)\left(\frac{J_1(q_\perp a)}{q_\perp a}\right)^2. \qquad (15)$$

In the limiting case of infinitely thin fiber, $a\to 0$, Eq. (15) agrees with the corresponding result of [1] as well as the results of [4].



Direction diagrams for the radiation scattered by uniform cylinder; the incident radiation is directed along the z axis. Dashed curve represents the relative intensity of the scattered radiation according to (16), the solid curve gives the same in the limit of infinitely thin fiber when $J_1(q_{\perp'}a)/q_{\perp'}a\to 1/2$, the cone represents the azimuthally symmetric scattering. The angle of incidence is ψ = 0.2 radian (upper row) and ψ = 0.4 radian (lower row); other parameters are $a\omega/c$ = 1 (left column) and $a\omega/c$ = 3 (right column)

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The scattering cross section for unpolarized radiation results from (14) after averaging over polarizations of the incident wave:

$$\frac{d\sigma}{d\Omega}=\frac{\pi}{4}\frac{\omega^2}{c^2}\left((k_f)_z^2+\frac{\omega^2}{c^2}\right)\times$$
$$\times La^4(1-\varepsilon)^2\delta(q_\parallel)\left(\frac{J_1(q_\perp a)}{q_\perp a}\right)^2. \quad (16)$$

The directional diagram of the scattered radiation is presented on Figure the relations

$$q_\perp=2\frac{\omega}{c}\sin\psi\sin\frac{|\phi|}{2},$$
$$(k_f)_z=\frac{\omega}{c}\cos\left[2\operatorname{asin}\left(\sin\psi\sin\frac{|\phi|}{2}\right)\right],$$

(where the angle ϕ is measured from the direction $(k_i)_\perp$) had been used for plotting.

The only source of azimuthal asymmetry in (16) in the limit $a\to 0$ is the factor $(k_f)_z^2+\omega^2/c^2$ that tends to $2\omega^2/c^2$ under $\psi\to 0$. The account of the finite cylinder radius leads to the increase of the azimuthal asymmetry via the factor $J_1(q_\perp a)/q_\perp a$. The analogous behavior had been demonstrated in [5] for the transition radiation by fast charged particles on the fiber-like targets.

CONCLUSIONS

The scattering of the electromagnetic wave under its oblique incidence on the linear extended target is considered. The formulae for the scattering cross section are obtained using Born approximation. The azimuthal character of the scattering is demonstrated as well as the axial asymmetry around the target axis for the finite angle of incidence and the finite target thickness. The results could be used for improving the kinetic theory of propagation of the radiation through the system of parallel fibers [1].

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РАССЕЯНИЕ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ НА ДИЭЛЕКТРИЧЕСКОМ ЦИЛИНДРЕ В БОРНОВСКОМ ПРИБЛИЖЕНИИ

В.В. Сыщенко

Рассмотрено рассеяние плоской электромагнитной волны на пространственно протяженной нитевидной мишени. Получено выражение для сечения рассеяния с использованием приближения, аналогичного борновскому приближению в квантовой механике.

РОЗСІЯННЯ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ НА ДІЕЛЕКТРИЧНОМУ ЦИЛІНДРІ У БОРНІВСЬКОМУ НАБЛИЖЕННІ

В.В. Сищенко

Розглянуто розсіяння плоскої електромагнітної хвилі на просторово протяжної ниткуватої мішені. Отримано вираз для перерізу розсіяння з використанням наближення, що аналогічно борнівському наближенню у квантовій механіці.