ДИНАМИКА ПУЧКОВ

THE PECULIARITIES OF PARTICLE DYNAMICS IN THE FERMI ACCELERATION SCHEME

V.A. Buts

National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

E-mail: vbuts@kipt.kharkov.ua

With examples of discrete and distributed mathematical models of the Fermi acceleration mechanism, a usefulness, or even necessity, of taking into account of singular solutions is demonstrated. Also the role is shown of those parts of phase space where the uniqueness theorem conditions to form the dynamics of physical systems are broken. It was found that the dynamics of particles in discrete and distributed mathematical schemes of Fermi acceleration can be significantly different. The difference is due to the fact that the distributed model takes into account the effects of phase space where conditions do not correspond to those necessary for application of the uniqueness theorem. The role of singular solutions is under discussion as well.

PACS: 05.45.-a

INTRODUCTION

It was shown in [1 - 4] that, when analyzing the dynamics of physical systems, it is necessary to take into account the influence of the phase space where the uniqueness theorem conditions are not fulfilled. In particular, consideration must be given to the singular solutions of ordinary differential equations (ODE). Note, that singular solutions are those where the conditions of uniqueness theorem are not met. Consideration of such solutions, as was found out in the mentioned works, allows one to expand significantly the range of parameters of physical systems under study, in which the regimes with chaotic behavior can be realized. In particular, with taking into account singular solutions, the regimes with chaotic behavior can be realized in systems with one degree of freedom, and even in systems that are fully integrable. The singular solutions have to be taken into consideration not only in the existing models of the physical processes, but also at mathematical modeling of those physical processes. This does mostly occur in cases when the discrete mathematical models are used as models of real physical processes. If in the course of modeling the information about the areas of phase space, where the uniqueness theorem gets broken is lost then the results of the analysis of such models can not reflect the real dynamics of the studied systems. In the present paper, these peculiarities of influence of the singular solutions on the dynamics of the studied systems will be illustrated.

The scheme of acceleration of charged particles by colliding with magnetic clouds in space, proposed by Fermi [5, 6], is usually being modeled by particle oscillations between two reflecting walls, with position of one of them (or both) are oscillating. The reflection off the walls is an instantaneous process of elastic reflection. In some cases, an energy loss during reflection process is taken into account. Such a model of Fermi acceleration will be called below in the text as the *discrete* model. To date, discrete schemes with regularly moving walls have been studied in detail, and various regimes of particle dynamics in similar schemes were found. The review of the results of analysis of such schemes can be found, for example, in [7 - 9].

In this paper the scheme of interaction of particles with reflecting walls is modified to be closer to the natural scheme. Namely, it is assumed that the particles move in a certain potential, which has several maxima and each of them can reflect the particle. The position of maxima may vary on regular periodic law. In the simplest case, it looks like the movement of particles between two reflective peaks of potential whose positions are changing periodically. Such a scheme of acceleration can be called a *distributed* scheme. The dynamics of particles in such a scheme differs from that of the particles in the discrete scheme with most significant difference consisting in the fact that chaotic regimes have a much greater volume in parameter space than the volume in the case of a discrete pattern. In most cases, as in the discrete model, there is a maximum energy gained by particles. The main reason is an appearance in a distributed model of the areas where the conditions of uniqueness theorem are not hold. The discussion about possible reasons of difference in the particle dynamics in the discrete and the distributed model ends the paper.

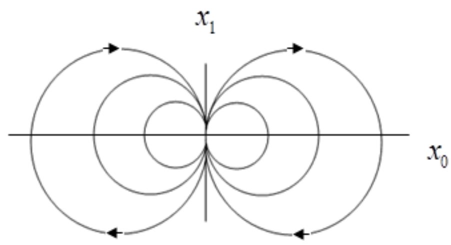

Fig. 1. Phase portrait of the system (1) ($x_1 = \dot{x}$)

1. SINGULAR SOLUTIONS OF ODE

The author does not know the examples with singular solution in use when analyzing the dynamics of physical systems. Let us briefly describe an example when such solutions are necessary to be taken into account. This model is a model of a nonlinear oscillator of the type:

$$\ddot{x} - \left(\frac{\dot{x}}{2x}\right)\dot{x} + 0.5 \cdot x = 0 . \qquad (1)$$

Equation (1) has the following integral: $\varphi = (x-R)^2 + \dot{x}^2 - R^2 = 0$. Moreover, this equation has a general solution, expressed by elementary functions: $x = R \cdot \left[1 \pm \sin\left(t + C\right)\right]$. Looking at this general solution and at the integral, it is difficult to imagine that the dynamics of a nonlinear oscillator (1) can be complex, chaotic. Indeed, the considered oscillator has only one degree of freedom, has an integral and even has a general solution. Such systems should not have complex irregular dynamics. In reality, however, this dynamic is chaotic. The reason for this is the presence of singular solutions. The phase portrait of the equation (1) is shown in Fig. 1. The phase trajectories are circles whose centers are located on the axis $\dot{x} \equiv x_1 = 0$. The radius of these circles is the distance from the center point to the point of singular solution. Moving along any of these circles (depending on the choice of initial conditions), the phase trajectory gets to the point of singular solution. At this point, the trajectory can randomly pass to any circle which passes through this point. More details about analysis of this dynamics one can found in [1 - 4]. Note that the transitions from one circle to another circle are completely randomly. This example shows that the presences of singular solutions can quality change of the known dynamics, even in such simple systems. Thus, the presence of singular solutions creates an additional mechanism for the appearance of unpredictability and irreversibility.

2. DISCRETE AND CONTINUOUS SCHEME FERMI ACCELERATION

The simplest discrete scheme Fermi acceleration is shown in Fig. 2. In this scheme, the particle moves between the two elastically reflecting walls, one of which (lower) varies periodically. This and similar schemes are studied in detail (see, for example, [6 - 9]).

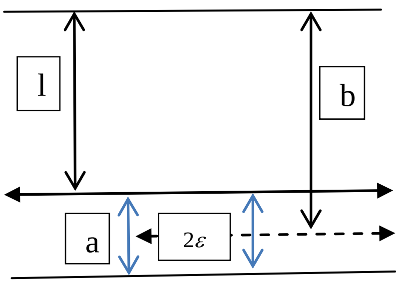

Fig. 2. Discrete scheme of Fermi acceleration

The most important fact is availability of the analytical criterion of the appearance of stochastic acceleration. Let the parameters acceleration schemes satisfy the following inequalities: $a/l << 1$; $a = 2\varepsilon$; $u = v/V_0 >> 1$; $\Delta u << u$. Here b – distance between fixed walls; V_0 – velocity of the moveable wall; v – velocity of the particle.

Then the criterion of occurrence of chaotic dynamics is the inequality:

$$K = 2l/au^2 > 1. \qquad (2)$$

Instead of the discrete model, we introduce continuous scheme Fermi acceleration. To do this, we assume that the particle moves in the potential well, one of the walls which periodically vibrate (Fig. 3).

The equations of motion of a particle in this will have the form:

$$\dot{x}_0 = x_1, \qquad (3)$$
$$\dot{x}_1 = F(x_0),$$

where $F(x_0) = -\left[\frac{1}{\left(x_0 - (b + \varepsilon \cdot \cos \omega t)\right)^2} - \frac{1}{(x_0 + b)^2}\right]$.

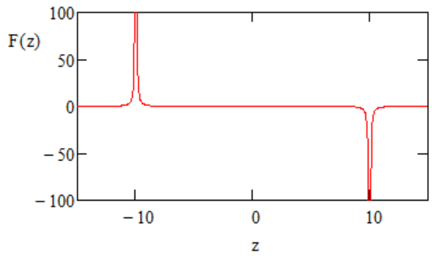

Fig. 3. The force, which are acting on the particle in a distributed scheme of Fermi acceleration

Here, as in the discrete model, $2b$ – the distance between the peaks of the stationary potential. The analogy between discrete and continuous models is visible. Therefore, one would expect that if the condition $K = 2l/au^2 > 1$ is not fulfilled, the dynamics of a distributed scheme, as well as the dynamics of the discrete scheme, will be regular. However, the dynamics of a distributed system does not obey this criterion. Indeed, let choose the following parameters of the distribution scheme: $b = 21$; $\varepsilon = 1$; $\omega = 10$; $v = 100$; $x_0(0) = 1$; $x_1(0) = 100$. Then the parameter K is small: $K = 4 \cdot 10^{-3} << 1$. A typical dependence of the particle position on the time, its speed, spectrum of motion of the particle and its correlation function are shown in Figs. 4-7. At the numerical calculations we use: $TOL = CTOL = 10^{-8}$; $m = 18 \Rightarrow N = 2.622 \cdot 10^5$; length of realization was 100.

It is clear from these figures that the dynamics of the particles is irregular. Such a difference in the dynamics of seemingly similar schemes, is due to the fact that in a distributed system on the dynamics of the particles affects the region of the phase space ($x_0 = \pm b$; $x_1 = 0$), in the vicinity of which violated the conditions of uniqueness theorem.

Thus at modeling of physical systems it is necessary to take into account the presence of the phase areas, in which the uniqueness theorem is violated. The ignoring these areas can lead to incorrect results. Most often this happens when execute the transition from distributed models to discrete models.

To illustrate this feature, consider as the simplest example of such task. Suppose we want to determine the dynamics of a particle in a potential well whose walls are fixed ($\varepsilon = 0$). Such a problem can be solved using the system of equations (2), i.e., analyzing distributed scheme. The result of this analysis is the same as in the analysis of similar discrete scheme. Now suppose that the position of one of the walls of the potential vibrates slightly. The amplitude of this vibration is low, and the frequency is sufficiently large. Thus, it is assumed that on the system acts small high-frequency disturbances. The question is: "What effects will this perturbation in the discrete and distributed schemes?".

ISSN 1562-6016. ВАНТ. 2015. №6(100)

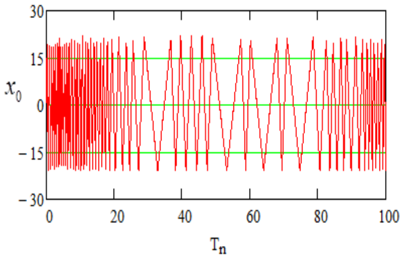

Fig. 4. The dependence of the position of a particle in a distributed scheme Fermi acceleration

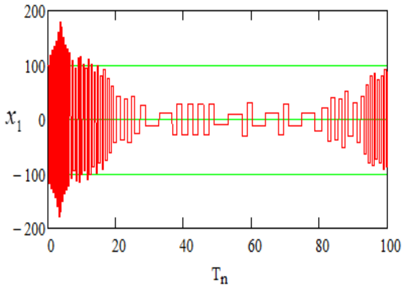

Fig. 5. The dependence of the velocity of a particle in a distributed scheme Fermi acceleration

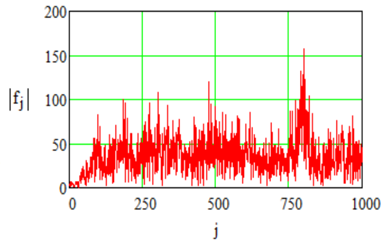

Fig. 6. The spectrum of motion of a particle

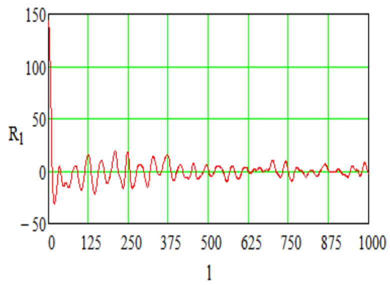

Fig. 7. The autocorrelation function of the particle motion

As for the discrete model, we can expect that the dynamics of particles will remain regular and little changed under the influence of a small perturbation, *if the parameter* K *is small*. The distributed scheme, as will be seen below, is very sensitive to such perturbations. As an example, consider the following parameters acceleration scheme: $b = 10$; $x_0(0) = 1$; $x_1(0) = 200$; $\varepsilon = 0.005$; $\omega = 1000$; $K = 0.1$; $N = 32.770$; length of "realization": 100; TOL=CTOL=10^{-10}. For these values of the parameters of particle the dynamics in a discrete pattern is a regular. In the distributed scheme such perturbation qualitative changes the dynamics of particles. This dynamics becomes chaotic. As an example, in Figs. 8 and 9 shows the dependence of position and velocity of the particles in the distributed system. We see a noticeable influence of a small perturbation. Moreover, in the Figs. 10, 11 are presented statistical processing of these dynamics. It can be seen that the spectrum is broadened, and the correlation function decreases sufficiently rapidly.

This result indicates that, simulating the dynamics of particles in the potentials the discrete models must be used with caution. In such models, there is no region of phase space in which the uniqueness theorem is violated. Therefore, they are little sensitive to small perturbations. The real physical systems can be abnormally sensitive to small perturbations.

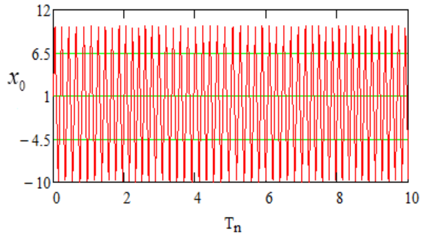

Fig. 8. The dependence of the position of a particle in a distributed scheme Fermi acceleration

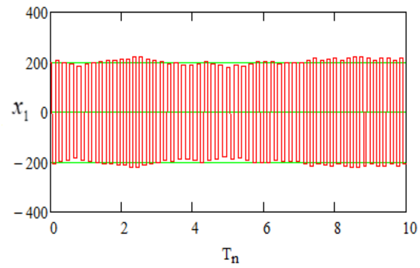

Fig. 9. The dependence of the velocity of a particle in a distributed scheme Fermi acceleration

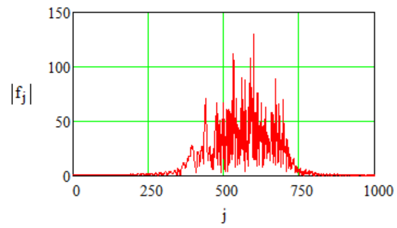

Fig. 10. The spectrum of motion of a particle

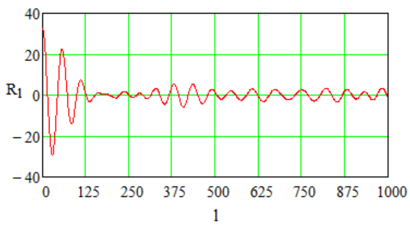

Fig. 11. The autocorrelation function of the particle motion

CONCLUSIONS

Let us formulate the most important results:

1. One of the most important results is the proof of the fact that singular solutions must be considered when analyzing the dynamics of various physical systems. Accounting of these solutions can significantly extend the range of parameters studied systems in which can be realized chaotic regimes. Such regimes may appear in systems with one degree of freedom, and even in systems completely integrable. This fact violates one of the basic paradigms of dynamic chaos that chaotic regimes can occur only in systems with 1.5 or more degrees of freedom. In addition, the presence of singular solutions provides an additional mechanism for the appearance of unpredictability and irreversibil-

ity of the dynamics of physical systems. Let us remind that there are two common mechanisms of unpredictability and irreversibility. The first is the mechanism of local instability and uncertainty (even though very small) in the initial conditions. The second mechanism is associated with the presence of noise.

2. The physical nature of chaotic dynamics, which was appear as the result taking into account singular solutions is different from the nature of the appearance of the conventional dynamic chaos. The unpredictability of the phase trajectories in this mechanism occurs only when these trajectories passing points of singular solutions. Such mechanism of occurrence unpredictability is more like to the mechanism unpredictability which appear when one throw the dice with an unlimited number of faces.

3. Note also that even in those cases when the phase trajectories do not fall strictly in the points of singular solutions, but are close enough to them, singular solutions must also be taken into account. In this case, the structure of the phase space in the vicinity of singular solutions is such that even small perturbations can significantly change the dynamics of the physical system. This feature of the particle dynamics was visible in the distributed scheme Fermi acceleration. These features one must to have in the mind at modeling physical processes. Especially during the transition from continuous mathematical models to discrete models.

4. In [3, 4] it was shown that the number of singular solutions is growing rapidly with growing of the number of the freedom in physical systems, which studied. Therefore, the results formulated above are particularly important for more complex systems, for systems with many degrees of freedom.

The above, as well as in [1 - 4], the main attention was paid to the fact that the inclusion of singular solutions significantly expands the range of parameters of physical systems with chaotic behavior. However, it can be expected that the taking into account of such solutions can be useful for opening some new features of the dynamics of physical systems which one studied.

REFERENCES

1. V.A. Buts. Anomalous influence of small perturbations // *Physical bases of instrumentation.* November. 2011, p. 76-87.
2. V.A. Buts. Chaotic motion of dynamic systems with “one” degree of freedom // *Problems of Atomic Science and Technology. Series “Nuclear Physics Investigations”*. 2012, № 1, p. 328-332.
3. V.A. Buts. Singular solutions and dynamic chaos // *Problems of Atomic Science and Technology.* 2015, № 4, p. 232-236.
4. VA Buts. The role of singular solutions in analyzing the dynamics of physical systems // *Physical bases of instrumentation.* December, 2015.
5. E. Fermi. On the Origin of the Cosmic Radiation // *Phys. Rev.* 1949, v. 75, p. 1169.
6. S. Ulam // *Proc. 4th Berkeley Symp. on Math, and Probability.* 1961, v. 3, p. 315.
7. A.J. Lichtenberg, M.A. Lieberman. *Regular and Stochastic Motion.* Springer-Verlag, 1983, p. 499.
8. G.M. Zaslavsky, B.V. Chirikov About mechanism Fermi acceleration in one dimension case // *Dokl. USSR Academy of Sciences.* 1964, v. 159, p. 306.
9. G.M. Zaslavsky. *Stochasticity of dynamic systems.* Moscow: “Science”, 1984.

Article received 20.10.2015

ОСОБЕННОСТИ ДИНАМИКИ ЧАСТИЦ В СХЕМЕ УСКОРЕНИЯ ФЕРМИ

В.А. Буц

На примерах дискретной и распределенной математических моделей механизма ускорения Ферми показана полезность, а в некоторых случаях, и необходимость учета особых решений. Также показана роль влияния областей фазового пространства, в которых нарушаются условия выполнения теоремы единственности на динамику изучаемых физических систем. Показано, что динамика частиц в распределенной и дискретной схемах ускорения Ферми может существенно отличаться. Это различие связано с учетом в распределенной модели влияния областей фазового пространства, в которых нарушена теорема единственности. Обсуждается роль особых решений.

ОСОБЛИВОСТІ ДИНАМІКИ ЧАСТИНОК У СХЕМІ ПРИСКОРЕННЯ ФЕРМІ

В.А. Буц

На прикладах дискретної та розподіленої математичних моделей механізму прискорення Фермі показана корисність, а в деяких випадках, і необхідність врахування особливих рішень. Також показана роль впливу областей фазового простору, в яких порушуються умови виконання теореми єдності на динаміку досліджуваних фізичних систем. Показано, що динаміка частинок у розподіленій та дискретній схемах прискорення Фермі може істотно відрізнятися. Ця відмінність пов'язана з урахуванням у розподіленій моделі впливу областей фазового простору, у яких порушена теорема єдності. Обговорюється роль особливих рішень.