MATHEMATICAL MODEL ANTENNAS, BASED ON MODULATED PLAZMON-POLARITON STRUCTURES

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Presents a review of development of the theory of plasmon-polariton structures and nanoantennas based on them. It shows the method for solving the problem of excitation of impedance structures, modulated by multiple periodic sequences of impulse functions by an external source of electromagnetic field. The algorithm and the recurrence formula are proposed for the construction of mathematical models of a wide class of elements of infocommunication systems based on structures with N-fold periodicity. A comparative analysis is provided of the influence of the form of impulse sequences on the formation of spatial distribution of radiation field. Parameters for the design of nanoantennas, spatial filters, interferometers, commutators, superlenses and supercollimators have been calculated.

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1. INTRODUCTION

1.1. PREREQUISITES FOR DEVELOPMEN SUBSECTIONT OF ANTENNAS BASED OF PLASMONIC STRUCTURES

An increased interest one can observe today in the development of the elements of infocommunication systems based on plasmon-polariton structures [1]. This can be explained by several reasons.

- The potential of microelectronics for solving new problems of increasing productivity of computing systems is running out. The inertia of electronic processes of transfer, processing and storage of information appeared to be an insolvable problem. Photonic processes are not inertial. Therefore, they are coming to replace electronic processes under the new banners: "Photonics" and "Plasmonics".
- The theory and technological solutions for construction of the elemental base of modern infocommunication systems based on electromagnetic, including photonic, processes were elaborated in the last century. However, the existing technologies and the level of manufacture in those days were not prepared for the largescale manufacture of such systems.
- In the end of the last century and in the beginning of this millennium key discoveries in many fields of science and technology have been made owing to which it became possible to study, develop and manufacture products based on nanotechnologies on a mass scale [2].

1.2. PLASMON-POLARITONS ASA VARIETY OF SURFACE WAVES

Surface electromagnetic waves (SEW) as a process that occurs at the two media interface and is guided by that interface was first described by A. Sommerfeld in 1899 [3] and mathematically substantiated by J. Zenneck in 1907 [4]. The first experiments with SEW in optics were made by R. Wood in 1902 [5], and Wood anomalies were explained by U. Fano [6] on the basis of the concept of SEW (1941). These works gave an impetus to the first wave of large-scale investigations of the peculiarities of excitation and transformation of SEW guided by two media interface [7 - 9]. The diversity of theoretical problems being solved and the results of their practical application in microwave and optical wave ranges are demonstrated in a number of reviews of the second half of the last century (Harvey [10], Miller and Talanov [11]). The works of A.F. Chaplin, O.N. Tereshin and their followers [12 - 23] played an important role for the development of theory and technology of SEW antennas.

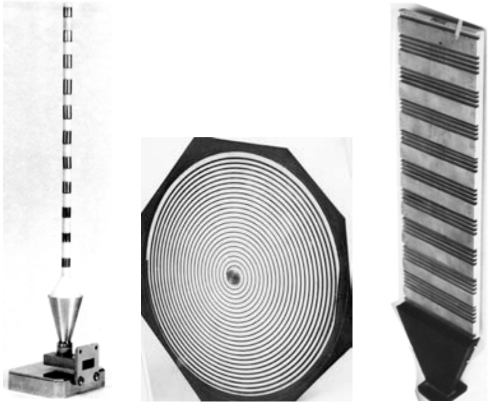
The second wave of investigations of SEW (plasmons) guided by the interface of nanodimensional structures emerged in the beginning of the second millennium. The intensity of these investigations can be judged from reviews of the works on SEW [24 - 26]. These reviews cover 376 works.

Objective of this report is to present the results of development of the theory and technology of nanoantennas, based on plasmon-polariton structures with N-fold periodicity, and the perspectives of their development.

2. PLASMON-POLARITON STRUCTURES WITH N-FOLD PERIODICITY

The new line of works, enveloping the works aiming at the development of electrodynamic methods of solving the problems of excitation of structures with complex impedance boundary conditions, of mathematical and computer models for the problems of analysis and synthesis of the construction of impedance structures, elements of infocommunication systems based on SEW structures and, particularly, PPS structures, was formed in early 60's-80's of the last century [23] at Moscow Power Engineering Institute and further developed at Lviv Polytechnical Institute (now National University «Lviv Polytechnic»). Over a dozen of projects have been accomplished at the National University "Lviv Polytechnic" to develop commercial prototypes of SEW modulated antennas for centimetric and millimetric waves (Fig. 1) [17 - 23]. These samples are prototypes (macromodels) of nanoantennas based on plasmon-polariton structures. In these structures, modulation of construction of dielectric structures (see Fig. 1,a,b) and structures with "artificial dielectric" rectangle structures with "artificial dielectric" (see Fig. 1,c) with an interval of quasi-periodic sequence (see Fig. 1,c) of rectangle functions is used.

For the first time in such structures there has been discovered experimentally [17] and explained theoretically [20] the effect of emission along the normal to the direction of propagation of SEW fundamental spatial harmonic. Earlier in their works Yevstropov and Talanov [8, 9] pointed at the absence of emission by such structures along the normal to the direction of propagation of SEW fundamental harmonic ('normal effect').



a *b* *c*

Fig. 1. Surface-wave antennas

These works are particularly topical at present time for the tasks of mastering the optical (nanometric) range. However, to design superlenses, supercollimators, nanoantennas, high resolution microscopes, spatial filters, commutators, transformers of spatial harmonics in nanometric range it is necessary to extend a great number of laws of modulation of PPS construction parameters, investigate properties of very large arrays with tens or hundreds of thousands of emitters [21]. To investigate electrodynamic and optical properties of such structures in nanometric range the most affordable in terms of economic costs are methods of mathematical modeling. Construction of PPS mathematical models with complex laws of modulation of their design parameters is based on the works of the author of the report, accomplished during the last 30 years and continuing development of A.F. Chaplin's works.

2.1. THEORY

Theory of nanoantennas based on modulated PPS includes cylindrical (see Fig. 1,a), disk (see Fig. 1,b), planar (Fig. 1,c) and other metal-dielectric structures, whose dielectric permeability varies in compliance with the law, resulting from overlapping of multiple periodic sequences of impulse functions. The form of impulse functions (IF) can be rectangular, triangular, trapezoidal and may also have the form of Gaussian functions, half-period cosine curve, delta function. PPS can be excited by arbitrary distribution of external sources of electric or magnetic currents: by filament, strip of electric current, lattice of strips of electric or magnetic currents, plane wave incident at an arbitrary angle in two-dimensional problem for the plane interface between two media. The theory of nanoantennas, rectennas, collimators, superlenses, interferometers, spatial harmonics filters, super-resolution microscopes, transmission lines, attenuators, splitters, commutators, routers, digital filters and logic elements as well as other elements of infocommunication systems is based on the effective mathematical models (MM). These models have been developed on the basis of analytical solutions with a rigorous statement of a number of problems of electromagnetic excitation of PPS, one of the design parameters of which (dielectric or magnetic permeability, thickness of dielectric layer) have an N-fold periodicity.

Examples of twofold periodicity are shown in Fig. 2.

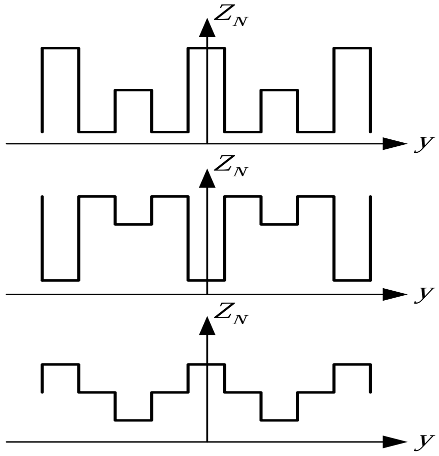
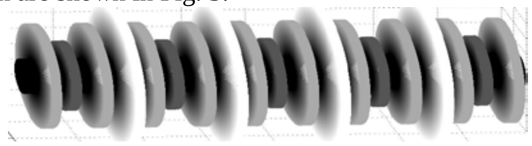


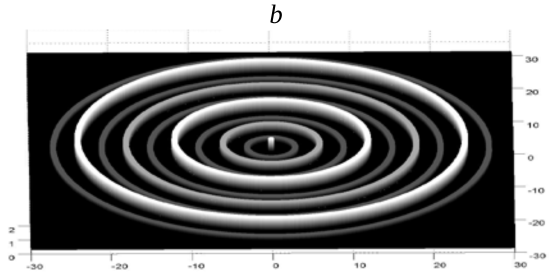
Fig. 2. Examples of twofold periodicity of PPS

Examples of threefold periodicity of PPS modulation are shown in Fig. 3.



a

b



c

Fig. 3. Threefold periodicity of modulation of cylindrical (a) and disk (b) PPS

2.2. DEVELOPMENT OF A.F. CHAPLIN METHOD OF ANALYSIS OF PERIODIC STRUCTURES

In his work [14] (1981) A.F. Chaplin published the method of rigorous solution of the problem of excitation of periodically heterogeneous impedance structures in case of modulation of impedance plane by one- and twofold periodic sequences of delta functions. A modified method of A.F. Chaplin for solving the problems of excitation of periodically heterogeneous impedance structures in case of modulation of impedance plane by N-fold periodic sequences of rectangle IF similar to delta function was presented in the work [20] (1984). There was obtained an asymptotic solution of the problem, whose accuracy grew with the transition of rectangular impulse into delta function. Solution of the problem for the case of modulation of impedance plane by

N-fold periodic sequences of triangular IF, similar to delta functions was found in this paper.

2.2.1. PROBLEM STATEMENT. DERIVATION OF FUNDAMENTAL RELATIONS

Let us assume that the infinite plane (Fig. 4) is the interface between two media (1) and (2) and allows a description by impedance boundary condition (1) [11, 16]:

$$Z_E(y) = E_y(y)/H_x(y)_{z=0}\,, \qquad (1)$$

where E_y and H_x – components of intensity of electric and magnetic fields. Suppose, distribution of the external sources of the field in volume V′c with cross-section value $S(y',z')$ does not depend on the coordinate x, values of surface impedance (SI) $Z_E(y)$ do not depend on coordinate x. This allows to use presentation presentation of the field in the upper half-space (z ≥ 0) as a superposition of two-dimensional electric and magnetic waves [14].

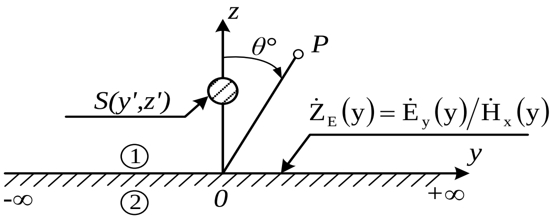


Fig. 4. Modulated impedance plane

Let us examine the field of electric waves (E-waves). We shall write impedance boundary condition (2) as the initial relation for the total field of E-waves, consisting of the field of external sources and the field reflected from the plane (Fig. 1), as follows [15]:

$$Z_E(y) = \frac{i}{\omega\varepsilon'_a}\,\frac{\int\limits_{-\infty}^{\infty}\left[f_1(\chi) - F^e(\chi)\right]\dfrac{e^{-i\chi y}}{\chi}\,d\chi}{\int\limits_{-\infty}^{\infty}\left[f_1(\chi) + F^e(\chi)\right]\dfrac{e^{-i\chi y}}{\chi\sqrt{\chi^2-k^2}}\,d\chi}\,, \qquad (2)$$

where $F^e(\chi)$ – spectral density of the function of distribution of external sources for E-waves; $f_1(\chi)$ – spectral density of reflected field; k – wave number for free space ($k = 2\pi/\lambda$); λ – wavelength; $k^2 = \omega^2\varepsilon'_a\mu'_a$ – generalized spatial number (dimensionality rad/m); ω – circular frequency (rad/s); $\omega = 2\pi/T$; T – period of source electromagnetic oscillations; ε'_a and μ'_a – medium dielectric and magnetic permeability; $F^e(\chi)$ – determined by external field sources filling volume V′ [15]:

$$F^e(\chi) = \frac{1}{4\pi}\int\limits_S\left[\frac{\chi^2}{i\omega\varepsilon'_a}\,j_z^э - i\chi\left(\frac{\pm\sqrt{\chi^2-k^2}}{i\omega\varepsilon'_a}\,j_y^э + j_x^м\right)\right]\to \qquad (3)$$
$$\to * e^{i\chi y' \mp\sqrt{\chi^2-k^2}z'}dy'dz',$$

$j_z^э, j_y^э, j_x^м$ – the given distributions of external electric and magnetic currents.

Analysis problem statement. Let distribution of SI $Z_E(y)$ be described by the following mathematical model:

$$Z_E(y) = Z_0 + Z_{M_1}\sum_{n=-\infty}^{\infty} tripuls\left(\frac{y-nd_1}{\Delta}\right), \qquad (4)$$

where Δ – IF width; d_1 – sequence period of IF; n – infinite sequence of whole numbers; Z_{M_1} – amplitude of triangular IF; Z_0 – SI constant component; tripuls – plays the role of operator setting the form of triangular IF. The law of SI distribution is presented in Fig. 5 in the form of the graph.

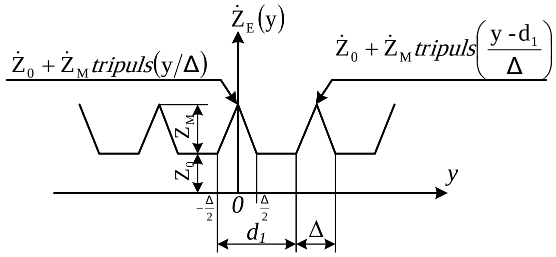


Fig. 5. The law of impedance distribution

It is necessary to find E-waves field in upper half-space of plane (z y 0), that satisfies impedance boundary condition (2).

2.2.2. SOLUTION OF FORMULATED PROBLEM

To solve the problem we shall present equation (2) in identical form:

$$i\omega\varepsilon'_a Z_E(y)\int\limits_{-\infty}^{\infty}\xi_1(\chi)e^{-i\chi y}d\chi \Rightarrow$$
$$\to -\int\limits_{-\infty}^{\infty}[\sqrt{\chi^2-k^2}\,\xi_1(\chi) - 2\Phi^e(\chi)]e^{-i\chi y}d\chi\,, \qquad (5)$$

where $\xi_1(\chi) = \left[f_1(\chi) + F^e(\chi)\right]\big/\chi\sqrt{\chi^2-k^2}$,

$$\Phi(\chi) = F^e(\chi)/\chi\,. \qquad (6)$$

Here $\xi_1(\chi)$ – unknown spectral density of electric surface currents (6), distributed along axis y. Substitute expression (4) in (5) and apply to the derived equation Fourier transform (FT) of the form

$$\tilde{f}(\chi) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} f(y)e^{i\chi y}dy. \qquad (7)$$

Then, applying convolution theorem to FT, we shall proceed to the following equation:

$$i\omega\varepsilon'_a Z_0\xi_1(\chi) + i\omega\varepsilon'_a\xi_{T_1,\Delta}(\chi) \Rightarrow$$
$$-\sqrt{\chi^2-k^2}\,\xi_1(\chi) + 2\Phi^e(\chi), \qquad (8)$$

where

$$\xi_{T_1,\Delta}(\chi) = Z_{M_1}\frac{\Delta}{2d_1}\sum_{n_1=-\infty}^{\infty}\sin c(n_1\pi\Delta/2d_1)\xi_1(\chi - n_1T_1) -$$

function that describes influence of modulation of SI on the field propagating over impedance surface;

$$sinc(n_1\pi\Delta/2d_1) = \frac{sin(n_1\pi\Delta/2d_1)}{n_1\pi\Delta/2d_1}; \qquad T_1 = 2\pi/d_1.$$

From equation (8) we shall find $\xi_1(\chi)$:

$$\xi_1(\chi) = \xi_0(\chi) - G(\chi)\xi_{T_1,\Delta}(\chi)\,, \qquad (9)$$

with denoted:

$\xi_0(\chi) = 2\Phi(\chi)/\beta_0(\chi)$ – solution of formulated problem for the case of absence of SI modulation:

$$(Z_{M_1} = 0)\,;\; G(\chi) = i\omega\varepsilon'_a/\beta_0(\chi)\,;$$
$$\beta_0(\chi) = \sqrt{\chi^2-k^2} + i\omega\varepsilon'_a Z_0\,. \qquad (10)$$

To determine the unknown function $\xi_{T_1,\Delta}(\chi)$ we shall multiply the left and the right sides of equation (9) by expression:

$$Z_{M_1}\frac{\Delta}{2}sinc\left[(\chi-\eta)\Delta/4\right]$$

and then apply to the left and the right sides of derived equation FT of the form:

$$f(m_1 d_1)=\int_{-\infty}^{\infty}\tilde{f}(\chi)e^{-im_1 d_1\chi}d\chi . \quad (11)$$

The reform we shall obtain the following relation:

$$Z_{M_1}\frac{\Delta}{2}\int_{-\infty}^{\infty}\xi_1(\chi)\sin c[(\chi-\eta)\Delta/4]\,e^{-im_1 d_1\chi}d\chi=\rightarrow$$

$$\rightarrow Z_{M_1}\frac{\Delta}{2}\int_{-\infty}^{\infty}\left\{\xi_0(\chi)-G(\chi)\xi_{T_1,\Delta}(\chi)\right\}\times\rightarrow$$

$$\rightarrow\sin c[(\chi-\eta)\Delta/4]e^{-im_1 d_1\chi}d\chi . \quad (12)$$

Multiplying then both sides of the equation (12) by multiplier $\exp(im_1 d_1\eta)$ and, summing up both of these sides by m_1 from $-\infty$ to ∞, we proceed to equation:

$$Z_{M_1}\frac{\Delta}{2d_1}\sum_{n_1=-\infty}^{\infty}sinc(n_1\pi\Delta/2d_1)\xi_1(\eta-n_1T_1)=\xi_{T_1,\Delta}(\eta)=\rightarrow$$

$$=Z_{M_1}\frac{\Delta}{2d_1}\sum_{n_1=-\infty}^{\infty}\left\{\xi_0(\eta-n_1T_1)-G(\eta-n_1T_1)\xi_{T_1,\Delta}(\eta-n_1T_1)\right\}\rightarrow$$

$$\rightarrow\times\sin c(n_1\pi\Delta/2d_1). \quad (13)$$

In transition from equation (9) to equation (13) relation [15] was used:

$$\sum_{m_1=-\infty}^{\infty}e^{im_1 d_1(\eta-\chi)}=T_1\sum_{n_1=-\infty}^{\infty}\delta(\eta-\chi-n_1T_1)\,;\,T_1=2\pi/d_1 .$$

Then, bearing in mind existence of the following expression

$$\lim_{\Delta\to 0}\xi_{T_1,\Delta}(\chi-n_1T_1)=\lim_{\Delta\to 0}\xi_{T_1,\Delta}(\chi)=\xi_{T_1}(\chi),$$

we find from equation (13) expression for the function $\xi_{T_1,\Delta}(\chi)$:

$$\xi_{T_1}(\chi)=\frac{\lim\limits_{\Delta\to 0}Z_{M_1}\frac{\Delta}{2d_1}\sum\limits_{n_1=-\infty}^{\infty}\xi_0(\chi-n_1T_1)\sin c(n_1\pi\Delta/2d_1)}{\lim\limits_{\Delta\to 0}D_{1,\Delta}(\chi)},$$

where

$$D_{1,\Delta}(\chi)=1+Z_{M_1}\frac{\Delta}{2d_1}\sum_{n_1=-\infty}^{n_1=\infty}G(\chi-n_1T_1)\sin c(n_1\pi\Delta/2d_1). \quad (14)$$

Relation (9) with formulas (14) taken into account gives solution of formulated problem in analytical form.

$$\xi_1(\chi)\cong\xi_0(\chi)-G(\chi)\rightarrow$$

$$\rightarrow*\frac{Z_{M_1}\frac{\Delta}{2d_1}\sum\limits_{n_1=-\infty}^{\infty}\xi_0(\chi-n_1T_1)\sin c(n_1\pi\Delta/2d_1)}{D_{1,\Delta}(\chi)}. \quad (15)$$

The first term of formula (15) describes spectral density of the field reflected from the plane with constant impedance Z_0. The analysis of this field can be found in monograph [15]. The second term of formula (15) describes spectral density of the field, appearing through the effect of periodic modulation on the fundamental surface wave.

Modified A. Chaplin's method was generalized in the works [20, 21, 29, 30] for the case of electromagnetic waves and more complex laws of modulation of surface impedance, that represent the sum of constant component Z_0 and superimposed one on the other N-fold periodic sequences of IF of rectangular, triangular, trapezoidal and impulses other forms without limitation on their amplitude Z_{M_i} (i= 1, 2, 3,…N) provided, however, that the width $\Delta<<\lambda$.

2.2.3. GENERALIZED SOLUTION OF THE PROBLEM, RECURRENCE FORMULA

We shall set N-fold periodicity of triangular form impulses, to the following MM:

$$Z_E(y)=Z_0+\sum_{i=1}^{N}Z_{M_i}\sum_{n=-\infty}^{\infty}tripuls(\frac{y-nd_i}{\Delta})$$

$$d_N=d_1\prod_{i=1}^{N}p_i , \quad (16)$$

where d_1 – value of the shortest period (a random nonnegative number), p_i – sequence of integers. Recurrence formula to construct solutions of the problems of SEW structures excitation by a random source of the field has the following form [20]:

$$\xi_N(\chi)\cong\xi_{N-1}(\chi)-G(\chi)\rightarrow$$

$$\rightarrow*\frac{Z_{M_N}\frac{\Delta}{2d_N}\sum\limits_{n_1=-\infty}^{\infty}\xi_{N-1}(\chi-n_NT_N)\sin c(n_N\pi\Delta/2d_N)}{D_{1,\Delta}(\chi)\prod\limits_{m=1}^{N}D_{m,\Delta}(\chi)}, \quad (17)$$

$$D_{N,\Delta}(\chi)=1+Z_{M_N}\frac{\Delta}{2d_N}\rightarrow$$

$$\rightarrow*\sum_{n_N=-\infty}^{\infty}\frac{G(\chi-n_NT_N)}{\prod\limits_{m=1}^{N}D_{m-1,\Delta}(\chi-n_NT_N)}\sin c(n_N\pi\Delta/2d_N)\cdot$$

Formula (17) allows construction in a closed symbolic form solution of the problem for N, provided that the solution is known of the problem for N-1 number. It is shown in the work [21, 22], that the obtained solutions of electrodynamics problems in a given formulation present a new class of branched continued fractions with N-branches of branching with complex-valued components. They are of special interest for the mathematical theory of branching continued fractions, founded by Professor V.Ya. Skorobogatko [21, 27, 28].

2.2.4. CALCULATION OF THE FIELD OF NANOANTENNA BASED ON PLASMON-POLARITON STRUCTURE

Let PPS be described by conditions (1) and excited by current source (see Fig. 4) with coordinates

$$z=0;y=0;x=0;$$

$$j_x^M(x,y)=I_{x_0}^M rect(\frac{x}{b})\delta(y-0). \quad (18)$$

Substitution of (18) in (3), and then in (5) will produce expression of density of the function of distribution of the field external sources:

$$\Phi(\chi)=\Phi(\chi-n_1T_1)=-i(I_{x_0}^M/4\pi)=\Phi_0 ;$$

$$\Phi(\chi_2)=\Phi_0\sin c(b*\chi_2). \quad (19)$$

Substitution of (19) in (15) taking into account (14) for $D_{1,\Delta}(\chi)$ leads to MM, describing generalized directivity diagram (DD) of the structure:

$$\xi_1(\chi) \cong \Phi_0 \varphi_0(\chi) \varphi_{1,\Delta}(\chi), \qquad (20)$$

where $\phi_0(\chi) = 2/\beta_0(\chi)$, $\phi_{1,\Delta}(\chi) = 1/D_{1,\Delta}(\chi)$, $\Delta << \lambda$.

Substitution of (20) $\chi = k\sin\theta^0$ gives the following expression for calculation of DD in MATLAB.

$$\hat{E}(\theta^0) = \frac{\cos(\theta^0)}{[\cos(\theta^0) - \hat{Z}_0](1 - \hat{Z}_1 \dfrac{\Delta}{2d_1} * \to} \qquad (21)$$

$$\to \sum_{n=-N}^{N} \frac{\sin c(n\pi\Delta / 2d_1)}{\sqrt{(\sin\theta^0 - n\lambda / d_1)^2 - 1} - \hat{Z}_0}).$$

$$\hat{E}(\phi^0) = \Phi_0 \left| \sin c(\sin(\phi^0) * b / \lambda) \right|.$$

Results of calculation of field distribution of PPS modulated by periodic rectangular (Figs. 6, 8) and triangular form (Figs. 7, 9) impulse sequences are shown below for comparative analysis.

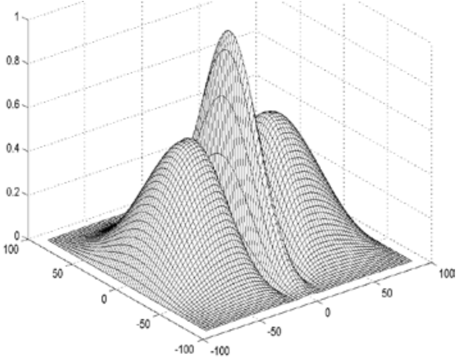


Fig. 6. Rectangular; $d = 0.63\lambda$

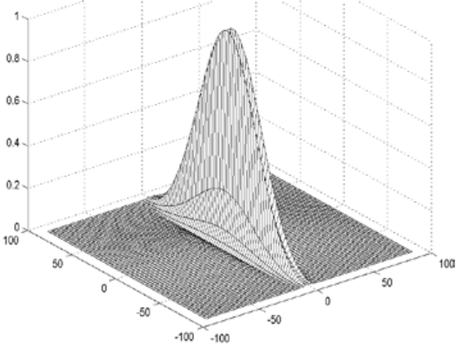


Fig. 7. Triangular; $d = 0.63\lambda$

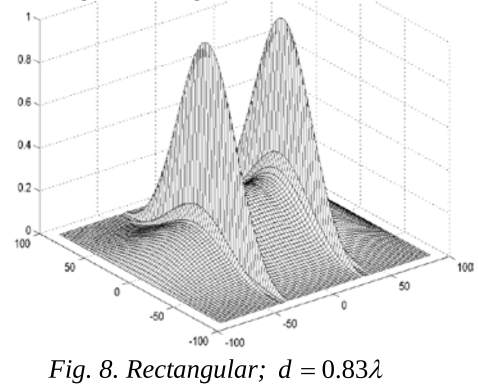


Fig. 8. Rectangular; $d = 0.83\lambda$

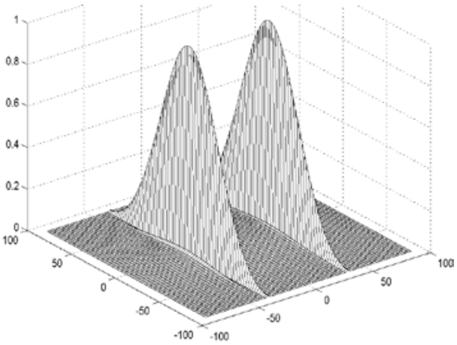


Fig. 9. Triangular; $d = 0.83\lambda$

CONCLUSIONS

Analysis of the results of computer modeling of periodically heterogeneous plasmon-polariton structures allows making the conclusion of a tremendous potential of the modified method of A.F. Chaplin for development of mathematical models of a wide class of surface wave antennas, including nanoantennas, superarrays of optical range, and numerous elements of infocommunication systems based on modulated nanodimensional structures.

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МАТЕМАТИЧЕСКИЕ МОДЕЛИ АНТЕНН НА ОСНОВЕ ПЛАЗМОН-ПОЛЯРИТОННЫХ СТРУКТУР

В.В. Гоблик

Предложена теория наноантенн на основе модулированных плазмон-поляритонных структур. Приведён метод решения задач возбуждения сторонним источником электромагнитного поля импедансных структур, модулированных наложенными друг на друга периодическими последовательностями импульсных функций. Предложен алгоритм и рекуррентная формула для построения математических моделей широкого класса элементов инфокоммуникационных систем на основе структур с N-кратной периодичностью. Исследовано влияние формы импульсных функций на формирование поля такими структурами.

МАТЕМАТИЧНІ МОДЕЛІ АНТЕН НА ОСНОВІ ПЛАЗМОН-ПОЛЯРИТОННИХ СТРУКТУР

В.В. Гоблик

Запропонована теорія наноантен на основі плазмон-поляритонних структур. Наведено метод розв'язування задач збудження стороннім джерелом електромагнітного поля імпедансних структур, модульованих накладеними одна на одну періодичними послідовностями імпульсних функцій. Запропоновано алгоритм і рекурентну формулу для побудови математичних моделей широкого класу елементів інфокомунікаційних систем на основі структур з N-кратною періодичністю. Досліджено вплив форми імпульсних функцій на формування поля такими структурами.