AREAS OF APPLICABILITY OF KINEMATIC AND DYNAMIC THEORIES OF PARAMETRIC X-RAY RADIATION

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The areas of applicability of kinematic and dynamic theories of parametric X-ray radiation (PXR) of relativistic particles in a crystal are considered. It is shown that dynamic diffraction of the PXR is possible in crystallographic planes with low Miller indexes in the central part of the PXR reflection only, where the PXR yield is weak and masked by other kinds of diffracted radiation. Therefore the properties of the PXR reflection are well described by the kinematic PXR theory. Results of the analysis are compared to experimental data.

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INTRODUCTION

Radiation of fast particles in a medium with periodically varying permittivity was first considered in [1]. Later, the kinematic theory of such radiation produced by relativistic particles moving in a crystal was proposed in [2]. In modern literature this radiation is names as parametric X-ray radiation (PXR). Therefore we will use this term in the present paper. The PXR reflection is emitted by relativistic charged particles moving inside a crystal, the energy and the direction of the PXR propagation in the PXR reflection are close to conditions of the X-rays diffraction. Therefore shortly after [2], the dynamic PXR theory that takes into account the diffraction of the PXR in a crystal was proposed in refs. [3, 4]. According to the dynamic PXR theory, the PXR undergoes to diffraction at the same family of crystallographic planes were it is was produced and can be emitted in the vicinity of the velocity vector of incident particle. In the experimental researches (see reviews [5, 6] and also [7 - 12]), the main properties of observed PXR in the PXR reflection were well described by the kinematic PXR theory, meanwhile the dynamic PXR in forward direction was not observed. However, discussions about existence and possibilities of observation of the dynamic PXR were continued, see e.g. [13 - 16] and brief review of the discussions in [17]. At last, weak spectral peaks of diffracted PXR were observed in narrow angular range in the vicinity of the incident particles velocity vector $\vec{V}$. Below we will discuss conditions, when the dynamic PXR diffraction is sufficient and when the kinematic approach is enough for description of properties of radiation in the PXR reflection.

1. THE PXR FREQUENCY AND THE BRAGG FREQUENCY

The frequency of the PXR spectral peak ω_{PXR} [2, 7]

$$\omega_{PXR} = g \cdot V \cdot \sin\phi \cdot \left(1 - \frac{V \cdot \sqrt{\varepsilon}}{c}\cos\theta\right)^{-1} \tag{1}$$

is some different from the Bragg frequency ω_B in the crystal in the same direction

$$\omega_B = \frac{cg^2}{2\sqrt{\varepsilon}\left|\vec{\Omega} \cdot \vec{g}\right|}, \tag{2}$$

where g is the module of the reciprocal lattice vector; ε is the permittivity in the crystal; ϕ is the angle between the particle velocity vector $\vec{V}$ and the crystallographic planes; θ is the angle between the observation direction and the particle velocity vector $\vec{V}$; $\vec{\Omega}$ is the unit vector in the direction of the PXR emission. Explicit expression for normalized difference of Bragg (2) and PXR (1) frequencies in approximation of small angles $\delta_\perp, \delta_\| << 1$ in the vicinity of the PXR reflection center has been derived in [19]

$$\frac{\omega_B - \omega_{PXR}}{\omega_{PXR}} \approx \frac{\gamma^{-2} + |\chi_0| + \eta^2}{4\sin^2\phi} = \frac{\gamma_{eff}^{-2} + \eta^2}{4\sin^2\phi}, \tag{3}$$

where $\eta = \sqrt{\delta_\perp^2 + \delta_\|^2}$ is the angle of declining from the center of the PXR reflection in arbitrary direction. One can see from (3) that the PXR frequency is always below of the Bragg frequency and the difference of frequencies is minimal in the area of the center of the PXR reflection at $\eta = 0$, where the PXR yield is close to zero. In the area of the maximal PXR yield at $\eta = \gamma_{eff}^{-1}$, the difference of frequencies (3) is $\frac{\gamma_{eff}^{-2}}{2\sin^2\phi}$. The difference of frequencies (3) quickly increases at further increasing of the angle η of declining from the center of the PXR reflection.

2. CONDITIONS FOR DYNAMIC DIFFRACTION OF THE X-RAY RADIATION

It is known that the effective dynamic X-ray diffraction is possible in the case when the monochromatic X-ray beam declines from the exact Bragg direction within the angular limits

$$\Delta\vartheta = \pm\frac{C|\chi_g|}{\sin 2\phi}, \tag{4}$$

where C is the coefficient equal to unity for perpendicular (σ) polarization and $|\cos 2\phi|$ for parallel (π) polarization of radiation, χ_g is the Fourier component of dielectric susceptibility, and quickly reduces outside the limits (4), see [20]. In our case of fixed direction of radiation corresponding limits of the deviation of X-ray radiation frequency can be found by the differentiation of the Bragg law. They are

$$\frac{\Delta\omega_B}{\omega_B} = \pm\frac{C\left|\chi_g\right|}{2\sin^2\phi}. \qquad (5)$$

3. CONDITIONS FOR DIFFRACTION OF THE PXR

Comparing (3) and (5), let us estimate conditions when the effective manifestations of the PXR diffraction are possible. They are possible in the case when the deviation of the PXR frequency (3) are within limits (5), at

$$\frac{\dfrac{\omega_B - \omega_{PXR}}{\omega_{PXR}}}{\left|\dfrac{\Delta\omega_B}{\omega_B}\right|} \le 1. \qquad (6)$$

Inserting (3) and (5) into (6), we obtain estimation of the angular size of the central part of the PXR reflection σ_D, where effective manifestations of the PXR dynamic diffraction are possible

$$\eta \le \sigma_D = \gamma_{eff}^{-1}\sqrt{\frac{2C\left|\chi_g\right|}{\gamma_{eff}^{-2}} - 1}. \qquad (7)$$

The condition for existence of such angular area is the positive sign of the radicand in eq. (7)

$$2C\left|\chi_g\right| > \gamma_{eff}^{-2}. \qquad (8)$$

It is interesting that both the condition for existence of the PXR diffraction (8) and the angular size of the central part in the PXR reflection where the diffraction is possible (7) are independent of the direction of the PXR reflection emission. Below we will consider the most favorable for existence of the dynamic diffraction case of high enough incident particles energy, when one can neglect the value γ^{-2} in comparison to $\left|\chi_0\right|$, in other words at brightly expressed longitudinal Ter-Mikaelian density effect [2, 8, 13]

$$\gamma^{-2} << \left|\chi_0\right|. \qquad (9)$$

At condition (9), the angular size of the PXR reflection $\gamma_{eff}^{-1} = \sqrt{\left|\chi_0\right|}$ does not depend on the incident particle energy, the estimation of the angular size of the central area σ (7) depends on the crystal properties and polarization only

$$\eta \le \sigma = \sqrt{\left|\chi_0\right|}\sqrt{\frac{2C\left|\chi_g\right|}{\left|\chi_0\right|} - 1}, \qquad (10)$$

and condition (8) takes the shape

$$2C\left|\chi_g\right| > \left|\chi_0\right|. \qquad (11)$$

Note that always $\sigma > \sigma_D$. The angular size of the area (10) can be expressed (normalized) in units of the angular size of the PXR reflection $\sqrt{\left|\chi_0\right|}$.

$$\sigma_N = \frac{\sigma}{\sqrt{\left|\chi_0\right|}} = \sqrt{\frac{2C\left|\chi_g\right|}{\left|\chi_0\right|} - 1}. \qquad (12)$$

One can consider the inequalities (8) and (11) as an estimations criterion for choose of the crystallographic plane where the PXR dynamical diffraction is possible. One can use the expressions (7), (10), (12) for estimation of the angular size of the central part in the PXR reflection, where the dynamical diffraction is possible. Note, that less expressed effects of dynamical diffraction and increased radiation attenuation are possible in the vicinity of the merges of the angular area (7), (10), (12) as well as the manifestations of the secondary maxima of pendulous solution in the vicinity of the area in thin crystals.

4. RESULTS AND DISCUSSION

Below we will consider the case of perpendicular polarization at $C = 1$. In this case the condition for existence of the PXR diffraction (11) and angular size of the central part (12) depend on crystal properties only,

$$2\left|\chi_g\right| > \left|\chi_0\right|, \qquad (13)$$

$$\sigma_N = \frac{\sigma}{\sqrt{\left|\chi_0\right|}} = \sqrt{\frac{2\left|\chi_g\right|}{\left|\chi_0\right|} - 1}. \qquad (14)$$

The most popular crystal in experimental studies of the PXR is the Si singlecrystal. Therefore let us perform estimations for this crystal. Calculations show that at PXR frequencies exceeding atomic frequencies and out of resonance frequencies the condition (13) is fulfilled for three crystallographic planes with non-zero structure factors – (111), (220) and (400) only, and the angular size of the area σ_N (14) is 0.275, 0.45 and 0.077 of the angular part of the PXR reflection angular size respectively. This means that small central angular part of the PXR reflection only, where the PXR yield is minimal, can be diffracted. Similar situation is realized for Ge singlecrystal, where the angular size of the field σ_N (14) is 0.47, 0.67, 0.45 and 0.18 for crystallographic planes with non-zero structure factors satisfying to condition (13) – they are crystallographic planes (111), (220), (400) and (422) respectively. One can expect the most prominent manifestations of the PXR diffraction are possible from crystallographic planes (220) in both crystals. The situation with other crystals demands of separate analysis.

Experiments for observation of two-dimensional angular distribution of the yield in the PXR reflection from Si crystal (220) crystallographic plane were performed in refs. [10, 12]. In both research main properties of the PXR reflection are well described by the kinematic PXR theory and any manifestations of the dynamic phenomena were not found. However, this does not exclude the possibility of diffraction in forward direction of some part of radiation from the central area of the PXR reflection.

The diffracted PXR from the Si (111) crystallographic plane has been observed in ref. [17]. The diffracted PXR has been observed in forward direction in the angular range about $\pm 0.36\sqrt{\left|\chi_0\right|}$ relative to the incident particles velocity vector (see Fig. 3 in ref. [17]). These angles are close to above obtained estimation of the angular range for Si (111) plane $\pm 0.275\sqrt{\left|\chi_0\right|}$.

It is evident that the kinematic theory can not describe properties of the PXR that is diffracted in forward direction and has been observed in refs. [17, 18]. The manifestations of the dynamical effects in the central area of the PXR reflection are unnoticeable because of the low PXR intensity in this area and masking influence of other kinds of diffracted radiation, like as transition radiation at entrance surface of the crystal and bremsstrahlung radiation in the crystal. Therefore the effects of diffracted PXR should be weakly expressed in the experiments on observation of the PXR reflection and properties of the PXR reflection are well described by the kinematic PXR theory. Note, that the PXR frequency and the frequency of other kinds of diffracted radiations are close one to another but different (see eq. (3)) that gives the possibility for experimental resolving of their yields by frequency [19].

CONCLUSIONS

The estimations presented in the paper show, that conditions for effective manifestations of the dynamical PXR diffraction in the considered crystals are executed for low-indexed crystallographic planes in the central area of the PXR reflection only, where the PXR yield is close to zero and masked by other types of radiation of incident particles. Therefore the influence of this phenomenon on the yield in the PXR reflection is insignificant and the main properties of the PXR in the PXR reflection are very well described by the kinematic PXR theory. This circumstance does to exclude the existence of the diffraction of some part of the central area in the PXR reflection in forward direction that has been observed in papers [17, 18].

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ОБЛАСТИ ПРИМЕНИМОСТИ КИНЕМАТИЧЕСКОЙ И ДИНАМИЧЕСКОЙ ТЕОРИЙ ПАРАМЕТРИЧЕСКОГО РЕНТГЕНОВСКОГО ИЗЛУЧЕНИЯ

А.В. Щагин

Анализируются области применимости кинематической и динамической теорий параметрического рентгеновского излучения (ПРИ) релятивистских частиц в кристалле. Показано, что динамическая дифракция ПРИ возможна только на кристаллографических плоскостях с малыми индексами Миллера и только в центральной области рефлекса ПРИ, где выход ПРИ мал и маскируется другими видами подвергшихся дифракции излучений. Поэтому свойства излучения в рефлексе ПРИ хорошо описываются кинематической теорией ПРИ. Результаты анализа сравниваются с экспериментальными данными.

ОБЛАСТІ ЗАСТОСОВНОСТІ КІНЕМАТИЧНОЇ ТА ДИНАМІЧНОЇ ТЕОРІЙ ПАРАМЕТРИЧНОГО РЕНТГЕНІВСЬКОГО ВИПРОМІНЮВАННЯ

А.В. Щагин

Аналізуються області застосовності кінематичної та динамічної теорій параметричного рентгенівського випромінювання (ПРВ) релятивістських частинок у кристалі. Показано, що динамічна дифракція ПРВ можлива тільки на кристалографічних площинах з малими індексами Міллера і тільки в центральній області рефлексу ПРВ, де вихід ПРВ малий і маскується іншими видами, що піддалися дифракції, випромінювань. Тому властивості випромінювання в рефлексі ПРВ добре описуються кінематичною теорією ПРВ. Результати аналізу порівнюються з експериментальними даними.