A.A. VLASOV AND COLLISIONLESS LANDAU DAMPING

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The main results obtained by A.A. Vlasov in 1944 and published in 1945 are presented. The solution of the initial problem for small perturbations of the longitudinal field in collisionless plasma is givn. For the example of the model function of electron velocity distribution is shown that small perturbations of the longitudinal electric field varies with the plasma frequency and decay with time due to their absorption by the plasma electrons. Thus, at least a year before the publication of the famous work L.D. Landau, by A.A. Vlasov was obtained the decrement for collisionless damping of plasma oscillations, known today as Landau damping.

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1. The first, who drew attention to the fact that the Boltzmann transport equation, strictly speaking, is not applicable to describe the gas with Coulomb interaction of the particles was L.D. Landau [3]. The fact that the condition of applicability of the Boltzmann equation to describe the gas of neutral particles (with short-range interaction) – conditions of the gas can be written as:

$$an^{1/3} << 1, \qquad (1)$$

where a is a characteristic dimension of the interaction of particles, and n is their density. Condition (1) is not applicable in the case of gas charged particles with Coulomb interaction as $a \to \infty$, or more exact, complete scattering cross section of such particles diverges. For this reason, diverges also the collision integral in the Boltzmann equation, the logarithmic divergence and takes place both over large distances and at also low distances as well.

L.D. Landau pointed out that the electron – ion plasma due to the polarization of the plasma potential is screened Coulomb interaction between the particles, the characteristic size of the screening is the Debye radius:

$$r_d = \sqrt{\frac{T}{4\pi e^2 n}}\,. \qquad (2)$$

Here T is the average energy of the random motion of particles equal to their temperature. Given the interaction of particles in the screening of plasma, Landau cuts the collision integral in the Boltzmann equation over long distances. At short distances, it also cuts the integral, requiring little interaction energy on average compared to their average kinetic energy (temperature):

$$\frac{e^2}{r_{cp}} = e^2 n^{1/3} << T. \qquad (3)$$

As a result, L.D. Landau received the Boltzmann equation with collision integral [3]:

$$\frac{\partial f_\alpha}{\partial t} + \vec{u}\frac{\partial f_\alpha}{\partial \vec{r}} + e_\alpha\left\{\vec{E} + \frac{1}{c}\left[\vec{v}\times\vec{B}\right]\right\}\frac{\partial f_\alpha}{\partial \vec{p}_\alpha} =$$

$$= \frac{\partial}{\partial p_{\alpha i}}\left[D_{ij}\frac{df_\alpha}{dp_{\alpha j}} - A_i f_\alpha\right], \qquad (4)$$

$$D_{ij} = \sum_\beta \int d\vec{p}_\beta I_{ij}^{\alpha\beta}\left(\vec{p}_\alpha, \vec{p}_\beta\right) f_\beta\left(\vec{p}_\beta\right),$$

$$A_i = \sum_\beta \int d\vec{p}_\beta I_{ij}^{\alpha\beta}\left(\vec{v}_\alpha, \vec{v}_\beta\right)\frac{\partial f_\beta}{\partial p_{\beta j}}.$$

Here $I_{ij}^{\alpha\beta} = 2\pi e_\alpha^2 e_\beta^2 L(u^2\delta_{ij} - u_i u_j)/u^3$, where $\vec{u} = \vec{v}_\alpha - \vec{v}_\beta$; $\vec{v}_\alpha = d\vec{r}_\alpha/dt$; $\vec{v}_\alpha = d\vec{r}_\alpha/dt$, and $f_\alpha(\vec{p}_\alpha, \vec{r}_\alpha, t)$ – is the single-particle distribution function of particles of species, at last L is, so-called, Coulomb logarithm equal

$$L = \ln\left(\frac{T}{e^2 n^{1/3}}\right) >> 1. \qquad (5)$$

Inequality (5), is equivalent to (3), but stronger. Electric and magnetic fields in the equation (4) are deemed to be given. Inequality (5) is essential in the derivation of the Landau collision integral converging in the equation (4). It is a little average distance between particles compared to the characteristic size of their interaction – Debye radius. In other words, in the Debye sphere interaction particles should be large number of particles, and therefore each particle simultaneously in plasma interacts with many particles through interaction with the fields produced by them. It is a basic requirement applicability gas approximation for describing plasma. Apparently, the recording condition (5), L.D. Landau this circumstance did not pay attention. Solving the problem of relaxation of small perturbations of the distribution function of the electrons in the plasma, it is completely neglected fields in the equation (4) [3]. As a result, as expected, it has been shown that the relaxation of the electron distribution function and all the other values calculated using this function iar completely determined by electron collisions:

$$\delta f_e(t) \sim \delta f_e(0)\exp(-\nu_{ei} t),$$

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m}}\frac{e^2 e_i^2 L}{T_e^{3/2}}. \qquad (6)$$

L.D. Landau did not doubt the correctness of the result, and until 1946 he defended.

2. However, in 1938 A.A. Vlasov published a paper [4], which shows that under the condition (5) the interaction of particles with fields created by particles themselves far superior to their direct interaction by pair collisions. Therefore, in equation (3) the right side is small,

and in the first approximation, can be neglected. In this approximation, the equation (3) is called the Vlasov equation. The very same equation (3) should be supplemented by equations for electromagnetic fields:

$$\mathrm{rot}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \qquad \mathrm{div}\,\vec{E} = 4\pi \sum_{\alpha} e_{\alpha} \int f_{\alpha} d\vec{p},$$
$$\mathrm{rot}\,\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\sum_{\alpha} e_{\alpha}\int \vec{v} f_{\alpha} d\vec{p}, \quad \mathrm{div}\,\vec{B} = 0. \qquad (7)$$

Equation (3), supplemented by equations (7), called the system of Vlasov-Maxwell equations with self-consistent field.

In [4], based on equations (3) without the right-hand side (collisionless limit) and (7) solved the problem on their own are not damped oscillations in the electron plasma. For the type of oscillation $\sim \exp(-i\omega t + i\vec{k}\vec{r})$ when the real frequency and wave vector was obtained dispersion equation[1]:

$$1 - \frac{4\pi e^2}{k^2} P \int \frac{\vec{k}\partial f_0 / \partial \vec{p}}{\omega - \vec{k}\vec{v}} = 0. \qquad (8)$$

Here P means that the improper integral is to be understood in the sense of principal value, the equilibrium electron distribution function is normalized by the density $\int d\vec{p} f_0 = n.$ In the long-wavelength limit of the equation (8) A.A. Vlasov received oscillation spectrum:

$$\omega = \omega_p + \frac{3k^2 v_{Te}^2}{2\omega_p}, \qquad (9)$$

where ω_p – is the plasma frequency and v_{Te} – the velocity of the thermal motion of the electrons. Formula (9) explains well the cycle of experiments [5].

3. No one work A.A. Vlasov has not been so sharp and well-deserved criticism as the work [4]. First of all, the criticism was followed by L.D. Landau, who in 1946 published his famous work [2]. In this work we were criticized dispersion equation (8) and improper integral contained therein. L.D. Landau decided to start the task and noted that the resulting improper integral should be understood as

$$\frac{1}{\omega - \vec{k}\vec{v}} = \frac{P}{\omega - \vec{k}\vec{v}} - i\pi\delta(\omega - \vec{k}\vec{v}), \qquad (10)$$

and that limited to the principal value of no grounds. As a result, the frequency spectrum of (9) was obtained by the imaginary.

Amendment corresponding to the damping of the oscillations with time [2]:

$$Jm\omega = -\frac{\pi\omega_p^3 k v_T}{2k^3 n}\frac{\partial f_0}{\partial \varepsilon}\left(v = \frac{\omega_p}{k}\right), \qquad (11)$$

where $\varepsilon = mv^2/2 \approx m.\omega_p^2/2k^2$. In the scientific literature this attenuation of plasma waves has been called "Landau damping", and its physical nature is the Cerenkov absorption of the waves with the spectrum (9) plasma electrons.

L.D. Landau criticized the validity of applicability of the self-consistent field and the existence of the dispersion equation (9). The article [2] L.D. Landau writes: "These equations are applied to the study of plasma oscillations A.A. Vlasov [3]. However, most of his findings are wrong. A.A. Vlasov was looking for solutions of the type $\exp(-i\omega t + i\vec{k}\vec{r})$ and determined the dependence of the frequency ω on the wave vector $\vec{k}$, "... and the" really does not exist any specific dependence of ω on $\vec{k}$; and the dependence $\omega(\vec{k})$ may be given arbitrary. "Even more specifically expressed in the four academic article [9]: "We leave aside mathematical errors A.A. Vlasov admitted them in solving equations and led him to the conclusion that "the dispersion equation" ... and further: "study author again fluctuations were based on non-existent" dispersion equation. "As must say "no comment"! All of the development of plasma physics and solid state physics has shown that it is self-consistent field method and the dispersion equation defined the modern development of these areas of science[2].

4. All of the above criticism of A.A. Vlasov was developed in the scientific press in 1946. But, apparently, it sounded at workshops earlier. After L.D. Landau was a Professor of Theoretical Physics, Faculty of Physics, Moscow State University, whose staff while was and A.A. Vlasov, and V.A. Fok, head of the department in 1943-1944 (at a time when the department of superintendence was removed Vlasov). Otherwise, you cannot explain the appearance of add-in [1], which a statement of his doctoral thesis (without supplements), protected in 1942 (in Ashgabat, where the Physics Department was evacuated for the first time during the Great Patriotic War). In this appendix in the third paragraph discusses the dispersion equation and its form. It is shown that the equation in the form (9) without limiting the integration of the principal value of the improper integral exists and determines the temporal evolution of the initial perturbations. And in the case of CW integration should be the principal value. What undamped wave exists in the

[1] Private Message of L.S. Kuzmenkov, graduate student A.A. Vlasov, is referring to A.A. Vlasov that the seminar of the department of theoretical physics of L.D. Landau denied existence as the dispersion equation and collisionless damping of small oscillations. That's what A.A. Vlasov said, "After I got the dispersion relations for longitudinal and transverse waves, I found solutions to their various distribution functions of different frequencies and wavelengths. In particular, it was shown that there is a mechanism of collisionless damping of longitudinal waves is analyzed in an electron plasma waves with not a Maxwell velocity distribution, in particular, the distribution of Fermi. I decided to report results at a scientific seminar of the Department of Theoretical Physics. After the report was made L.D. Landau said that the next workshop he will show that my result is wrong. He said and published in JETP the same results under his own name. Since then, the collisionless damping is called Landau damping.

[2] Select L.D. Landau contour of integration to calculate the improper integral with a singularity on the real axis for the Maxwell distribution is not strictly justified [6]. Result (11), however, corresponds to the result obtained using the formula Sakhotskogo-Plemel [7, 8], which is applicable for the function (12), as did A.A. Vlasov.

plasma can be seen from the formula (12), for example, in the limit, when the wave-absorbing and electron in the plasma is not. The very initial problem was solved in the second paragraph supplement entitled "Cauchy problem". Using Laplace transform. The author accurately calculates the improper integral model for the electron distribution function (as we use the notation and normalization $\int d\vec{v} f_0 = n$:

$$f_0(v) = \frac{v_{Te}}{\pi} \frac{n}{v^2 + v_{Te}^2} \cdot . \qquad (12)$$

Improper integral in solving the equations for the results of a longitudinal field for the time of initial perturbations calculated exactly. The result of the calculation field potential for small long-wave perturbations author writes as follows[3]:

$$\varphi(t) = \varphi(0)\exp(-kv_{Te}t)\cos\omega_p t . \qquad (13)$$

Thus, A.A. Vlasov in 1944-1945 received not only the frequency of longitudinal oscillations of the collisionless plasma, but also their damping decrement, which is called "Landau damping". It remains to add that the results of the A.A. Vlasov in 1944 were awarded the newly established Award of Lomonosov Moscow State University. Published, these results were in [1].

In conclusion, we emphasize once again that, as described, A.A. Vlasov is not only the author of the Vlasov equation, and the collisionless Landau damping.

[3] The exact calculation, taking into account the next order of powers of $\vec{k}$ vector gives:

$$\mathrm{Re}\,\omega = \omega_p + \frac{3k^2 v_{Te}^2}{16\omega_p}, \quad \mathrm{Im}\,\omega = kv_T \frac{\omega_p^2}{\omega^2}\left(1 - 2\frac{k^2 v_T^2}{\omega_p^2}\right).$$

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А.А. ВЛАСОВ И БЕССТОЛКНОВИТЕЛЬНОЕ ЗАТУХАНИЕ ЛАНДАУ

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Представлены основные результаты, полученные А.А. Власовым в 1944 г. и опубликованные в 1945 г. Даны решения начальной задачи для малых возмущений продольного поля в бесстолкновительной плазме. Для примера была взята модельная функция распределения электронов по скоростям. Показано, что малые возмущения продольного электрического поля изменяются с частотой плазмы и затухают со временем в результате их поглощения электронами плазмы. Таким образом, по крайней мере за год до публикации знаменитой работы Л.Д. Ландау, А.А. Власовым было получено бесстолкновительное затухание плазменных колебаний, известное сегодня как затухание Ландау.

А.О. ВЛАСОВ ТА БЕЗЗІТКНЕНЕВЕ ЗАГАСАННЯ ЛАНДАУ

А.А. Рухадзе

Представлено основні результати, що отримані А.А. Власовим у 1944 р. і опубліковані в 1945 р. Надано рішення початкової задачі для малих збурень поздовжнього поля в беззіткненневій плазмі. Для прикладу було взято модельну функцію розподілу електронів за швидкостями. Показано, що малі збурення поздовжнього електричного поля змінюються з частотою плазми і затухають з часом через їх поглинання електронами плазми. Таким чином, принаймні за рік до публікації знаменитої праці Л.Д. Ландау, А.А. Власовим було отримано беззіткненеве загасання плазмових коливань, відоме сьогодні як загасання Ландау.