FORMATION OF DISCRETE SPATIAL TEMPORAL STRUCTURES IN THE NONSTATIONARY INHOMOGENEOUS PLASMA

V.A. Buts, I.K. Kovalchuk
National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine
E-mail: vbuts@kipt.kharkov.ua

The traditional approach in the linear electrodynamics is using of spectral method. However, in many cases, the physical process is defined by large number of spectral components. In this case the spectral approach is not effective. It is necessary to use methods that are more general. In the present work the Klein-Gordon equation to define dynamics of fields in the nonstationary plasma was obtained. It was shown that in decaying plasma electromagnetic fields vanish faster than plasma decay.

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INTRODUCTION

In the linear electrodynamics to describe processes spectral approach is used usually. It is effective for stationary processes, slowly varying and slightly inhomogeneous matter. Such approach is not effective when parameters are changing rapidly because it is necessary to take into account large number of spectral components. In these cases sometimes it is possible do not use spectral approach and to solve problem as the whole (not expanding into spectral components). Such formulation of problem leads to the need to solve more complex equations, that may be nonlinear and in partial derivatives in general case. The analyses of these equations is possible by means numerical methods only (see, for example [1]). So latter to analyze such equations we are restrict ourselves by linear approximation. We will see that they will be Klein-Gordon equation for nonstationary plasma.

As it is known for this equation it may be formulated problem on eigenfunctions and eigenvalues with discrete specter. Moreover such discrete value may be plasma density. In other words there is class of solutions which may be exist at some certain values of plasma density. This work is devoted to investigation of this problem. Comparative analysis was carried out for temporal dynamics of plasma density and electromagnetic fields. It was shown that fields in plasma disappear more quickly than density of decaying plasma is changed.

BASIC EQUATION

To obtain equations describing electromagnetic field in the isotropic plasma except the Maxwell equations we use hydrodynamics ones for electrons. Laplace transformation on time was used for all of these equations. The expressions for Laplace transforms of electron density and velocity perturbations were obtained. Substituting them into the equations for electromagnetic field Laplace transforms and performing inverse Laplace transformation we obtain the following set of nonlinear integral-differential equations with partial derivatives for finding electromagnetic fields:

$$\begin{aligned}
&\Delta\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}-\frac{\omega_p^2}{c^2}\vec{E}-grad\, div\vec{E}=\\
&=\frac{\omega_p^2}{c^2}\frac{e}{m}\left(\int_0^t\vec{E}(t')dt\nabla\right)\left(\int_0^t\vec{E}(t')dt'\right)-\\
&-\frac{e}{mc^2}\vec{E}\,div\vec{E}-\frac{e}{mc^2}div\frac{\partial\vec{E}}{\partial t}\int_0^t\vec{E}(t')dt'-\\
&\frac{e^2}{m^2c^2}div\vec{E}\left(\int_0^t\vec{E}(t')dt\nabla\right)\left(\int_0^t\vec{E}(t')dt'\right)-\\
&-\frac{e^2}{m^2c^2}div\frac{\partial\vec{E}}{\partial t}\int_0^t\left(\int_0^{t'}\vec{E}(t'')dt''\nabla\right)\left(\int_0^{t'}\vec{E}(t'')dt''\right)dt',
\end{aligned}\quad(1)$$

where $\vec{E}$ – vector of electric field; m – electron mass; c – light velocity. The analysis of set (1) is possible by numerical methods. So latter we restrict ourselves by linear case only. To obtain some analytical results we will do latter simplifications. Namely we will restrict ourselves by one dimensional case and one component of electric field. In this case we obtain one scalar equation that has next form:

$$\frac{\partial^2 E_x}{\partial z^2}-\frac{\partial^2 E_x}{\partial\tau^2}=\frac{\omega_p^2}{c^2}E_x,\quad(2)$$

where $\tau=ct$. Equation (2) is Klein-Gordon equation (see [2, 3]). Similar equation is used in quantum mechanics to define discrete mass spectra of elementary particles (see [4]). Equation (2) allows to formulate task for eigenfunctions E_x and eigenvalues $\frac{\omega_p^2}{c^2}$. We will consider case of nonstationary plasma when it density riches maximal value in the certain moment of times. After this the plasma density is decreased. Such dynamics is possible in the case when plasma density initially increase under influence of outer processes and after plasma begin to decay as result of termination of external action. In this case expression for plasma density (square of plasma frequency) may be presented as follows:

$$\frac{\omega_p^2}{c^2}=\frac{\bar{\omega}_p^2}{c^2}+\Omega(\tau),$$

where $\bar{\omega}_p$ – average value of plasma frequency; $\Omega(\tau)$ – addition due to by external action on plasma. In the case of interest expression for $\Omega(\tau)$ may be expanded in

Taylor series in the vicinity of its maximal value and presented as follow:

$$\Omega(\tau) = \frac{\partial^2 \Omega(0)}{\partial \tau^2}\tau^2 .$$

In the case when plasma density reaches maximal value the next condition must be satisfied

$$-\Omega_{0\tau} = \frac{\partial^2 \Omega(0)}{\partial \tau^2} < 0 .$$

After taking into account the above assumptions, equation will have form:

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{\partial^2 E_x}{\partial \tau^2} + \Omega_{0\tau}\tau^2 E_x = \frac{\bar{\omega}_p^2}{c^2} E_x . \quad (3)$$

In the similar form Klein-Gordon equation in the theory of elementary particles were considered in work [5]. In the quantum mechanics problems the term that is similar to third member in the left part of equation (3) take into account influence of external potential on elementary particle motion. In the general case such potential may depend as on coordinate as on time.

We will solve equation (3) by method of separation of variables. For this electric field will be presented as follows:

$$E_x(z,\tau) = Z(z)T(\tau) . \quad (4)$$

After substitution of expression (4) into equation (3) after simple transformations we will obtain two ordinary differential equations for $Z(z)$ and $T(\tau)$ which have form:

$$\begin{aligned} &\frac{d^2 Z(z)}{dz^2} = \lambda_z Z, \\ &\frac{d^2 T(\tau)}{dz^2} - \Omega_{0\tau}\tau^2 T = \lambda_\tau' T, \end{aligned} \quad (5)$$

and relationship between λ_z, λ_τ' and $\bar{\omega}_p^2 / c^2$ has next form:

$$\lambda_z - \lambda_\tau' = \frac{\bar{\omega}_p^2}{c^2} . \quad (6)$$

Here λ_z and λ_τ' are values, that to be determined. They are eigenvalues for spatial and temporal parts. Latter we will consider plasma that is placed between ideally conducting planes that are perpendicular to z axis and located on some distance from each other. In this case the solution of first equation of set (5) is standing wave and eigenvalues must satisfy to condition $\lambda_z < 0$. The set of such eigenvalues as it is known will be discrete.

Latter we will consider second equation of set (5) describing temporal dynamics of interesting us process. Previously we will make replacement of independent variable:

$$\varsigma = \sqrt[4]{\Omega_{0\tau}}\,\tau .$$

Now equation for $T(\varsigma)$ function will look like:

$$\frac{d^2 T(\varsigma)}{d\varsigma^2} - \varsigma^2 T(\varsigma) = \lambda_\tau T(\varsigma) , \quad (7)$$

where $\lambda_\tau = \lambda_\tau' / \sqrt{\Omega_{0\tau}}$. In the quantum mechanics (see [6]) equations of this type describe wave function of linear oscillator. To define solutions bounded on entire temporal axis we will present $T(\varsigma)$ in the following form:

$$T(\varsigma) = \exp\left(-\frac{\varsigma^2}{2}\right)\chi(\varsigma) . \quad (8)$$

Equation for $\chi(\varsigma)$ has form:

$$\frac{d^2\chi(\varsigma)}{d\varsigma^2} - 2\varsigma\frac{d\chi(\varsigma)}{d\varsigma} - (1+\lambda_\tau)\chi(\varsigma) = 0 . \quad (9)$$

When next condition is satisfied

$$-(1+\lambda_\tau) = 2j , \quad (10)$$

solutions of equation (9) are Hermit polynomials ([6, 7]).

From above presented materials, we obtain following expression for eigenfunction for problem formulated in equations (2) and (3).

$$E_{xj}(z,\tau) = C\sin\left(\sqrt{|\lambda_z|}z\right)\exp\left(-\frac{\sqrt{\Omega_{0\tau}}\tau^2}{2}\right)H_j\left(\sqrt[4]{\Omega_{0\tau}}\tau\right), \quad (11)$$

where C is an arbitrary constant, $H_j(y)$ is Hermit polynomials of j order. The condition for average plasma density defining by expression (6) will have next form:

$$\frac{\bar{\omega}_p^2}{c^2} = -|\lambda_z| + \sqrt{\Omega_{0\tau}}(2j-1) . \quad (12)$$

Taking into account that λ_z belongs to discrete set of values (we consider plasma between two ideally conducting planes) from expression (12) it follows that solutions (11) for equation (3) are eigenfunctions and corresponding to them eigenvalues are discrete and defined by expression (12). As it is seen time of electric field existence defining by expression (11) in order of magnitude is

$$t_{field} \sim \frac{\sqrt{2}}{\sqrt[4]{\Omega_{0\tau}}\,c} . \quad (13)$$

It is possible to consider more general case, when plasma is not only nonstationary, but spatially inhomogeneous. Above we investigated plasma that had temporal maximum. It is possible to consider simple for analytical investigation case of inhomogeneous plasma that besides temporal maximum has spatial minimum. The varying spatial part of plasma density may be expanded into Taylor series on spatial coordinate. In this case spatial equation will be similar to temporal (7). It solution will be similar to expression (11).

Let us make some estimations for $\Omega_{0\tau}$. We use correlation

$$\frac{\omega_p^2}{c^2} = \frac{\bar{\omega}_p^2}{c^2}\left(1 - \frac{\Omega_{0\tau}c^2}{\bar{\omega}_p^2}\tau^2\right) .$$

The estimation for character time of plasma existence following from this expression is

$$\tau_{pl} \sim \frac{\bar{\omega}_p}{c\sqrt{\Omega_{0\tau}}} . \quad (14)$$

On other hand time of plasma existence may be obtained from character time of recombination that is defined by relationship

$$\tau_r \sim \frac{c}{\beta n_0} , \quad (15)$$

where $\beta \sim 10^{-7}$ sm^3/s – coefficient of recombination, n_0 – initial density of plasma. Equating these two times ((14) and (15)) we will obtain connection between $\sqrt{\Omega_{0\tau}}$ and initial plasma density:

$$\sqrt{\Omega_{0\tau}} \sim \frac{\omega_p \beta n_0}{c^2}. \qquad (16)$$

Substituting numerical values of constant it is possible to obtain another expression for $\sqrt{\Omega_{0\tau}}$:

$$\sqrt{\Omega_{0\tau}} = 0.62 \bullet 10^{-23} n_0^{3/2}. \qquad (17)$$

Substituting expression for $\sqrt{\Omega_{0\tau}}$ into (12) we will obtain cubic equation for definition of discrete values ω_p when eigenfunction (11) exists. This equation has form:

$$\omega_p^3 - \frac{4\pi e^2}{\beta m_e (2j-1)} \omega_p^2 - \frac{4\pi e^2 c^2}{\beta m_e (2j-1)} |\lambda_z| = 0. \qquad (18)$$

The following is estimations of character times for plasma density $n_0 \sim 10^{10}$ sm^{-3}. For time of existence of eigenfields defining by expression (11) from (13) we obtain $t_{field} \sim 10^{-7}$ s. Time of plasma existence in accordance (15) is $\tau_{pl} \sim 10^{-3}$ s. As it is seen time of existence of eigenfields is more less than time of plasma existence.

Thus, it was shown that in the spatial inhomogeneous and nonstationary plasma it is possible existence of spatially-temporal structures.

CONCLUSIONS

Thus in our work we have abandoned from spectral approach in the analysis of spatially-temporal dynamics of fields. In this case in the linear approximation it was possible to obtain Klein-Gordon equation for these fields. It was found it eigenfunctions and eigenvalues. Analysis of these functions shown that time of field existence is more less than time of plasma density change of decaying plasma. To estimate of this time plasma recombination was used. Currently, authors have no physical explanation of fast field disappearance in decaying plasma. Such fields disappear in almost homogeneous plasma.

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ОБРАЗОВАНИЕ ДИСКРЕТНЫХ ПРОСТРАНСТВЕННО-ВРЕМЕННЫХ СТРУКТУР В НЕСТАЦИОНАРНОЙ НЕОДНОРОДНОЙ ПЛАЗМЕ

В.А. Буц, И.К. Ковальчук

Традиционный подход в линейной электродинамике – использование спектрального метода. Однако во многих случаях физический процесс определяется большим числом спектральных компонент. В этом случае спектральный подход не эффективен. Необходимо использовать более общие методы. В настоящей работе для определения динамики полей в нестационарной плазме получено уравнение Клейна-Гордона. Показано, что в распадающейся плазме электромагнитные поля исчезают быстрее, чем происходит распад плазмы.

СТВОРЮВАННЯ ДИСКРЕТНИХ ПРОСТОРОВО-ЧАСОВИХ СТРУКТУР У НЕСТАЦІОНАРНІЙ НЕОДНОРІДНІЙ ПЛАЗМІ

В.О. Буц, І.К. Ковальчук

Традиційний підхід у лінійній електродинаміці – використання спектрального методу. Але в багатьох випадках процес визначається великою кількістю спектральних складових. У цьому випадку спектральний підхід не є ефективним. Необхідно використовувати більш загальні методи. У цій роботі для визначення динаміки полів у нестаціонарній плазмі одержано рівняння Клейна-Гордона. Показано, що в плазмі, яка розпадається, електромагнітні поля зникають швидше, ніж триває розпад плазми.