FRACTAL ANALYSIS OF THE FRACTAL ULTRA-WIDEBAND SIGNALS

L.F. Chernogor[1], O.V. Lazorenko[2], A.A. Onishchenko[3]
[1]V. N. Karazin Kharkiv National University, Kharkov, Ukraine;
[2]Kharkiv National University of Radio Electronics, Kharkov, Ukraine;
[3]V.V. Dokuchaev Kharkiv National Agricultural University, Kharkov, Ukraine
E-mail: Leonid.F.Chernogor@univer.kharkov.ua;
E-mail: Oleg-Lazorenko@yandex.ua;
E-mail: andrywa87@mail.ru

The results of fractal analysis of the fractal ultra-wideband (FUWB) signals were proposed. With usage of the continuous wavelet transform the time-frequency structure of that signals was investigated. Calculating the box and the regularization dimensions for each model signal with various its parameters values, three different estimators were applied. The optimal estimations of the fractal dimension value for each FUWB signal model were defined.

PACS: 05.45.Df, 07.50.Qx

INTRODUCTION

Resent time there is a significant interest to the non-traditional signal type applications, in particular, to usage of the ultra-wideband (UWB) signals, which should be considered as one of such signal types [1 - 4]. One of modern UWB signal classes proposed by authors in 2004 is the fractal ultra-wideband (FUWB) signals [5, 6].

But the new signal type usage needs to apply the modern signal analysis methods. The wavelet analysis was appeared to be such good and forward-looking one [7 - 9]. Nevertheless, the fractal signals, in particular the FUWB signals, require to apply a specific investigation method, developed specially for such signals. The fractal analysis can be considered as the such one. This idea seems to be modern, useful and actual.

1. FUWB SIGNAL MODELS

1.1. FUWB SIGNAL DEFINITION

By the B. Mandelbrot's definition [10], fractal is a set, whose topological dimension is greater then Hausdorf's one. The fractal function is supposed to be a function with fractal properties [11, 12].

The fractal UWB signal is defined as a UWB signal with self-affine property and fractal dimension [5 - 9]. In this paper only self-similar FUWB signals are discussed.

1.2. FUWB SIGNAL MODELS USED

Many simple numerical and analytical FUWB signal models in time-domain were proposed [7, 8].

All investigations described in this paper were performed for a lot of these models, as analytical, as numerical. But being strongly limited by the paper volume requirements, we show only the results obtained for some analytical FUWB signal models.

Analytical FUWB signals are based on the well-known fractal functions (see, for example, [10]), which have been some modified. They are given by:

$$FUWB_1(t) = \left(1 - \sum_{n=1}^{\infty} \alpha^n \cos(\pi\beta^n(2t-1))\right)\operatorname{sign}(2t-1)\Theta(t),$$

$$\operatorname{sign}(t) = \begin{cases} 1, & t > 0; \\ 0, & t = 0; \\ -1, & t < 0; \end{cases}$$

$$\Theta(t) = \eta(t) - \eta(t-1),$$

$$\eta(t) = \begin{cases} 1, & t \ge 0; \\ 0, & t < 0; \end{cases} \quad 0 < \alpha < 1,\ \alpha\beta \ge 1;$$

$$FUWB_2(t) = \sum_{n=1}^{\infty} n^{-2} \sin(2\pi n^2 t)\Theta(t);$$

$$FUWB_3(t) = \frac{2}{\pi^{\gamma}} \sum_{n=1}^{\infty} n^{-2\gamma} \sin(2\pi n^2 t)\Theta(t);$$

$$FUWB_4(t) = \left[1 - b^{2D-4}\right] \times \frac{\sum_{n=0}^{M} b^{(D-2)n} \cos\left(2\pi s b^n t + \psi_n\right)}{1 - b^{(2D-4)(M+1)}};$$

where $\gamma > 0{,}5$, t is the time variable; α, β and γ are the numerical parameters of the signal, b is the time scale parameter, s is the frequency scale parameter, D is the fractal dimension of the signal, $1 < D < 2$, ψ_n is the phase distributed randomly at the interval $\left[0, 2\pi\right]$, M is the harmonics number (if $M \to \infty$, we obtain a mathematical fractal). These analytical models are based on the Weierstrass, Riman and Riman-Weierstrass functions.

2. FRACTAL DIMENSIONS

It is well known that a fractal theory was built for the *mathematical* fractals, but in practice we investigate the *physical* fractals [10 - 12]. Therefore, the fractal numerical characteristics describing the fractal properties of the investigated signals should be adapted for such case.

One of the basic characteristics of a fractal (more precisely, of coarse, of a mono-fractal) is the Hausdorf's dimension. But for physical fractals represented in digital form, the Minkovsky's dimension D_M can be considered as a good approximation for the Hausdorf's dimension.

It should be pointed, at the present time, there many different fractal dimensions, which are supposed to be an estimation of the Hausdorf's dimension [13, 14].

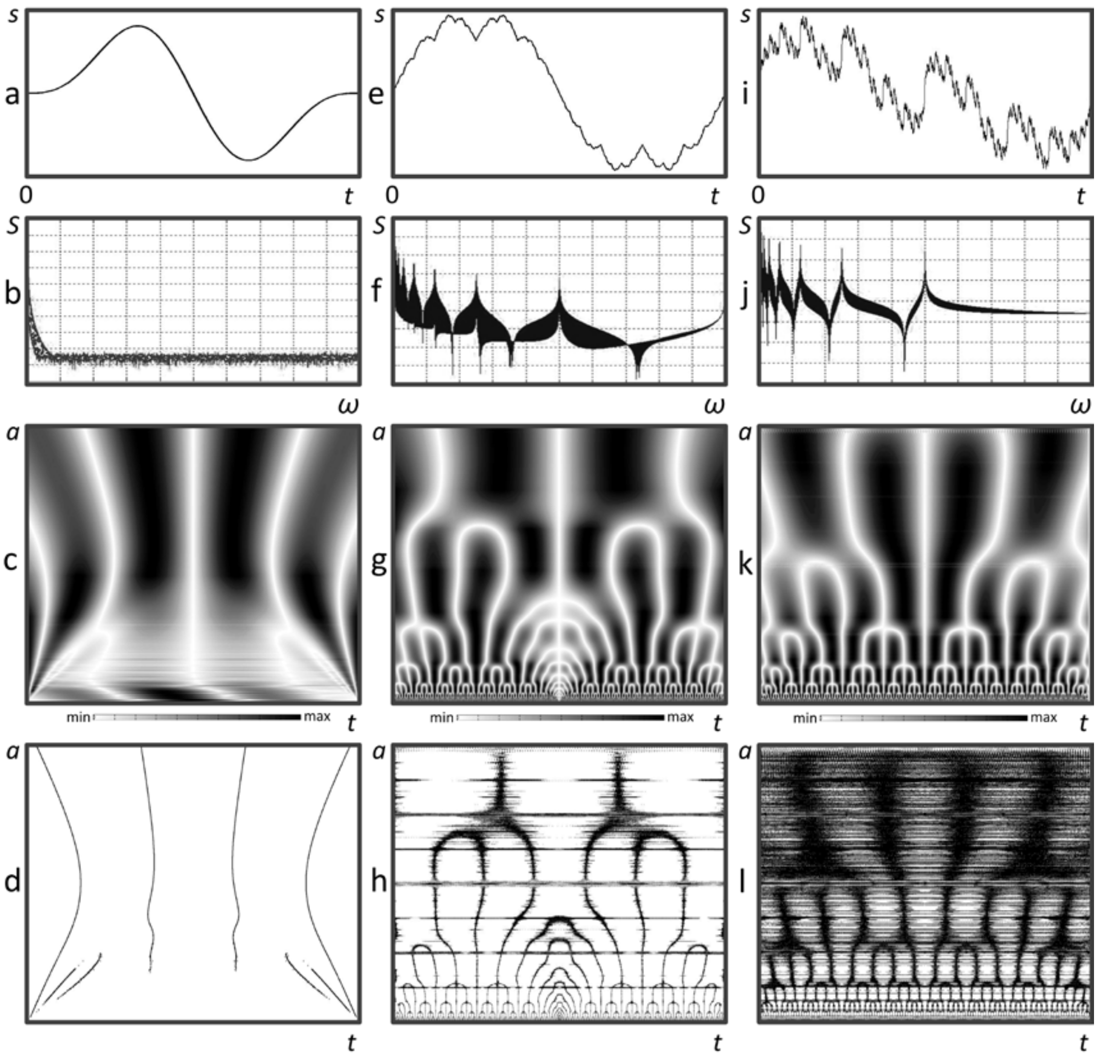

Fig. 1. The frequency and the time-frequency characteristics of the investigated model signals:
a – non-fractal model $UWB(t)$ with $N = 2$ in time domain; b – its ODFT SDF module; c – its CWT SDF;
d – its CWT SDF skeleton; e – model $FUWB_1(t)$ with $a = 0,5$ and $b = 2$ in time domain; f – its ODFT SDF module; g – its CWT SDF; h – its CWT SDF skeleton; i – model $FUWB_4(t)$ with $D = 0,5$ in time domain;
j – its ODFT SDF module; k – its CWT SDF; l – its CWT SDF skeleton.
CWT SDF calculations were performed with usage of the Morlet wavelet

Moreover, a practical calculation of such dimensions is appeared to be not so simple, as it seems [13].

Just because two different fractal dimensions, namely the box dimension and the regularization dimension, are considered as the good approximations of the Minkovsky's dimension D_M, and they were chosen for the investigations, described below.

Three different estimators based on the box counting method, the variation method and the regularization method [13] allow to obtain two estimations of the box dimension (D_B and D_V correspondently) and one estimation of the regularization dimension D_R for the investigated signal. The usage of three estimators is needed, since each of them has the best accuracy on the limited interval of a fractal dimension, as it had been shown in the paper [13].

3. CALCULATION RESULTS

3.1. FREQUENCY AND TIME-FREQUENCY STRUCTURE

As the time structure of a FUWB signal is fractal, its frequency and time-frequency structures are waited to be fractal too. A signal frequency structure is represented by the spectral density function (SDF) module of the one-dimensional Fourier transform (ODFT), as it is usually performed. A signal time-frequency structure is described by the continuous wavelet transform (CWT) SDF and its skeleton. As it is known, *a skeleton* is a picture describing the ridges of a two-dimensional real function. *A ridge* is a line at plane, which connects the points of the function minima or maxima.

The calculation results performed for the non-fractal UWB signal model $UWB(t)$ and for the FUWB models $FUWB_1(t)$ and $FUWB_4(t)$ are shown at the Fig. 1. The fractal frequency and time-frequency structures of

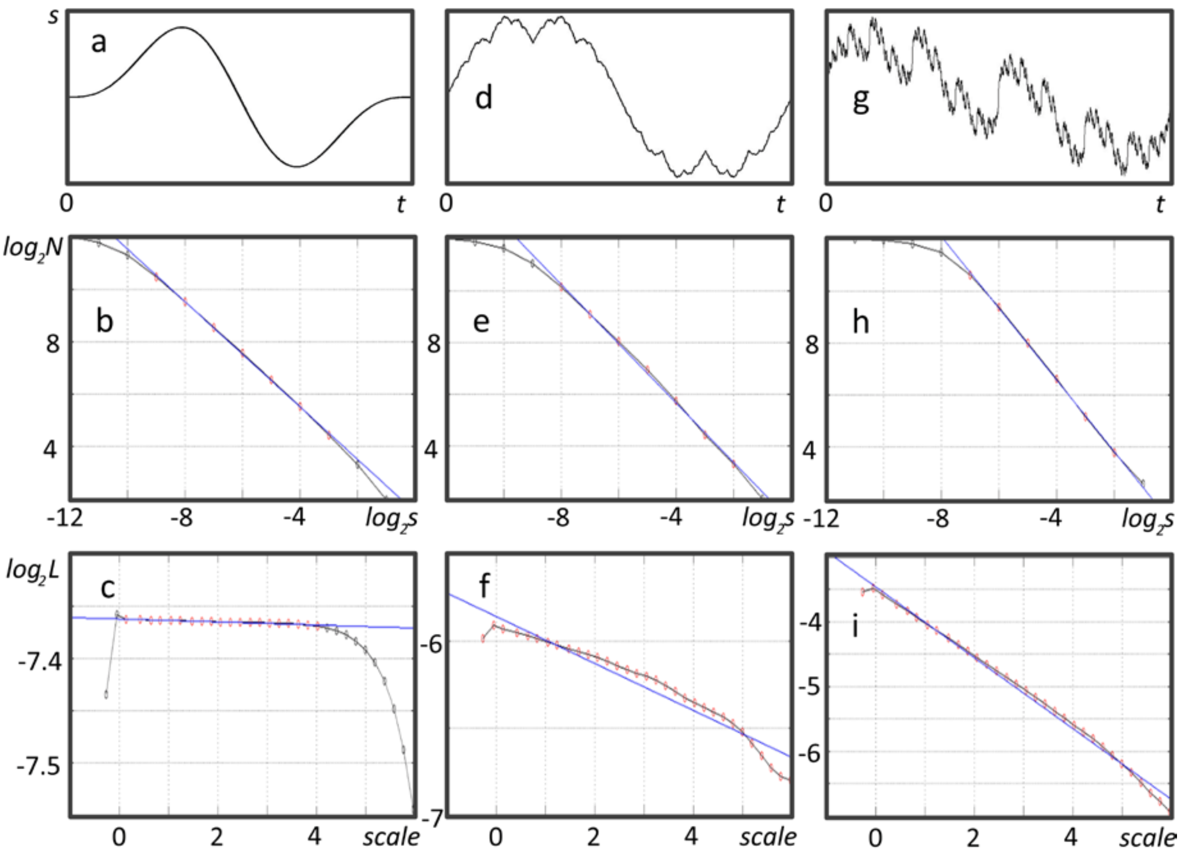

Fig. 2. The estimating of fractal dimensions of the investigated model signals: a – non-fractal model $UWB(t)$ with $N = 2$ in time domain; b – a sample of plot for its box dimension calculation with box method usage; c – a sample of plot for its regularization dimension calculation; d – model $FUWB_1(t)$ with $a = 0,5$ and $b = 2$ in time domain; e – a sample of plot for its box dimension calculation with box method usage; f – a sample of plot for its regularization dimension calculation; g – model $FUWB_4(t)$ with $D = 0,5$ in time domain; h – a sample of plot for its box dimension calculation with box method usage; i – a sample of plot for its regularization dimension calculation

the FUWB signals are clearly visible (Figs. 1, f-h; j-l), comparing with the corresponding characteristics of the non-fractal UWB signal (Figs. 1, b-d). All the CWT characteristics were obtained with usage of the Morlet wavelet, which has a fractal nature too. It was found, that if instead of a fractal wavelet any non-fractal one is used, the more smoothed picture will be obtained.

3.2. FRACTAL DIMENSION ESTIMATIONS

It is very interesting to estimate a fractal dimension for each FUWB signal model and to investigate a dependence of this dimension from the signal parameters. Suddenly, but in the most cases there are not any analytical forms of such dependence.

Three approximations of the Minkovsky's dimension D_M described above were calculated for all FUWB signal models. An example of these results are presented in the Table.

Calculating these dimensions for the model $FUWB_4(t)$, when the true value of the fractal dimension is known previously, it was found, that the D_B estimator is appeared to be the best for $1 < D \leq 1.3$, the D_V estimator – for $1.3 < D \leq 1.6$, and the D_R estimator – for $1.6 < D \leq 2$. These results are well agreed with the results described in the paper [13].

Fractal dimensions estimations of the signal model $FUWB_1(t)$ with α and β

α	β	D_R	D_B	D_V
0.2	6	1.29	1.31	1.21
0.3		1.41	1.37	1.34
0.4		1.52	1.44	1.45
0.5		1.60	1.48	1.54
0.6		1.67	1.54	1.61
0.7		1.73	1.62	1.66
0.8		1.77	1.58	1.69
0.9		1.82	1.55	1.71

The fractal dimensions estimations were performed with usage of the FracLab2.1 toolbox for MATLAB.

CONCLUSIONS

- The frequency and the time-frequency structures of the model FUWB signals were shown to be fractal.
- The fractal dimensions of the model FUWB signals were estimated, and their dependences from signal parameters were found.
- The bounds of the best efficiency of the each fractal dimension estimator were defined.

REFERENCES

1. D.L. Moffatt, R.J. Puskar. Subsurface Electromagnetic Pulse Radars // *Geophysics (41).* 1976, № 3, p. 506-518.
2. E.M. Kennaugh. The K-Pulse Concept // *IEEE Transactions on Antennas and Propagation.* 1981, № 2, p. 327-331.
3. L.Yu. Astanin, A.A. Kostylev. *Principles of Ultra-Wideband Radar Measurements*. Moscow: “Radio i Svyaz”, 1989 (in Russian).
4. H.F. Harmuth. *Nonsinusoidal Waves for Radar and Radio Communication*. New York: “Academic Press”, 1981.
5. L.F. Chernogor, O.V. Lazorenko. Fractal Ultrawideband Signals *// Proc. Ultrawideband and Ultrashort Impulse Signals.* Sevastopol. 2004, p. 118-120.
6. O.V. Lazorenko, L.F. Chernogor. Fractal ultrawideband signals // *Radio Physics and Radio Astronomy (10).* 2005, №1, p. 62-84.
7. O.V. Lazorenko, L.F. Chernogor. *Ultrawideband Signals and Processes: Monograph.* Kharkov: V.N. Karazin Kharkiv National University, 2009.
8. O.V. Lazorenko, A.A Potapov, L.F. Chernogor. *Fractal Ultrawideband Signals,* in book: *Informational security: the encryption methods*/ Ed.-in-ch. E.M. Suharev. Moscow: “Radiotechnika”, 2011 (in Russian).
9. L.F. Chernogor, O.V. Lazorenko, A.A. Potapov. Wavelet Analysis of Model Fractal Ultra-Wideband Signals // *Proc. Ultrawideband and Ultrashort Impulse Signals.* Sevastopol, 2012, p. 291-293.
10. B.B. Mandelbrot. *The Fractal Geometry of Nature.* CA-Freeman, San Francisco, 1982.
11. A.A. Potapov. *Fractals in Radio Physics and Radar: The Sample Topology*. Moscow: “University book”, 2005.
12. A.A. Potapov. *About Fractal Radio Systems, Fractional Operators, Scaling and not only...// Fractals and Fractional Operators* (the collective monograph) / Ed.-in-chief. A.H. Gilmutdinov, Kazan’: “Fan Publishing House”, 2011.
13. P. Goncalves. Fractal Dimension estimation: Empirical Mode Decomposition Versus wavelets *// Proc. ICASSP.* 2007, p. 1153-1156.
14. J. Theiler. Estimating Fractal Dimensions // *J. Opt. Soc. Am. (6).* 2011, №6, p. 1055-1073.

Article received 01.06.2015

ФРАКТАЛЬНЫЙ АНАЛИЗ ФРАКТАЛЬНЫХ СВЕРХШИРОКОПОЛОСНЫХ СИГНАЛОВ

Л.Ф. Черногор, О.В. Лазоренко, А.А. Онищенко

Предложены результаты фрактального анализа фрактальных сверхширокополосных (ФСШП) сигналов. С использованием непрерывного вейвлет-преобразования исследована время-частотная структура таких сигналов. Для вычисления клеточной и регуляризационной размерностей каждого модельного сигнала при различных значениях его параметров использовано три различных метода оценивания. Получены оптимальные оценки величины фрактальной размерности для каждого модельного ФСШП-сигнала.

ФРАКТАЛЬНИЙ АНАЛІЗ ФРАКТАЛЬНИХ НАДШИРОКОСМУГОВИХ СИГНАЛІВ

Л.Ф. Чорногор, О.В. Лазоренко, А.А. Онищенко

Запропоновано результати фрактального аналізу фрактальних надширокосмугових (ФНШС) сигналів. З використанням безперервного вейвлет-перетворення досліджено часо-частотну структуру таких сигналів. Для обчислення кліткової та регуляризаційної розмірностей кожного модельного сигналу за різних значень його параметрів використано три різні методи оцінювання. Отримано оптимальні оцінки величини фрактальної розмірності для кожного модельного ФНШС-сигналу.