STRUCTURAL-PHASE TRANSITIONS AND STATE FUNCTION IN UNSTABLE CONVECTIVE MEDIUM

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Convection in a thin layer of liquid (gas) with temperature independent viscosity between poorly heat conducting boundaries is studied within framework of the Proctor-Sivashinsky model. We have shown by numerical simulation of the Proctor-Sivashinsky model that the state with certain topology can be described by the state function, which is the sum of squared mode amplitudes of spatial temperature spectrum. The transitions between these states are characterized by splashes in time-derivative of this function and different meta-stable structures, corresponding to different values of the state function have different visually distinguishable topologies.

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Nonlinear systems with many degrees of freedom can undergo non equilibrium phase transitions characterized by a large variety of spatial or spatiotemporal patterns. Transitions and competition among these patterns of different symmetries are fundamental problems, which have attracted considerable interest in recent decades. Convection in a horizontal fluid layer subject to a vertical temperature gradient is very convenient for their study [1 - 5] due to its relative simplicity and great variety of observed patterns.

The Proctor-Sivashinsky model [6, 7] is found to be very attractive for studying the processes of pattern formation in systems, which possess a preferred characteristic spatial scale of interaction between the elements of future structure. This model was developed for description of convection in a thin layer of liquid between poorly conducting horizontal boundaries. Authors of [8] have found the stationary solutions with a small number of the spatial modes, one of which (convective cells) was steady and the second one turned out to be unstable (convective rolls). A particular future of the model is that it forces a preferred spatial scale of interaction, leaving the system a chance of selecting the symmetry during evolution. It was found, that the type of symmetry and hence the characteristics of the structure are determined by the minima of the potential of interaction between modes lying on a circle in k-space. Even within the Proctor-Sivashinsky model not all processes and the phenomena were studied. The detailed analysis of instability leading to the formation of a metastable structure (convective rolls) will be presented below. Earlier, it was found that at first stage of the instability evolution the metastable long-lived state (the curved quasi-one-dimensional convective rolls) arises. And later, after a lapse of time (which is considerably greater than the reverse linear increment of the process), the system transforms to the steady state (square convective cells) [9, 10]. The detailed treatment of the Proctor-Sivashinsky model presented below shows that this structural transition demonstrates all the characteristics of the second order phase transition (the continuity of the sum of squared mode amplitudes over the spectrum that the same, the continuity of density of this value and discontinuity of its time derivative. The existence of preferred scale (the distance between the regular spatial perturbations) and the possibility to select the type of symmetry (the regular spatial configuration) motivate the interest to this physical model, particularly for description of processes in solid state physics, where the characteristic distance between elements of spatial structures (atoms, molecules) in their condensed state is almost invariable. The objective of this work is investigation of the mechanisms of pattern formation and mode competition in convective medium. The nature and evolution of structural phase transitions between patterns of different topology are considered.

THE PROCTOR-SIVASHINSKY MODEL

When the Rayleigh number Ra exceeds the critical value corresponding to the onset of convective flow, the three-dimensional convection begins in a thin layer of liquid between poorly conducting horizontal plates heated from below [2], which can be described by the Proctor-Sivashinsky equation [6, 7]. This equation determines the dynamics of temperature field in the horizontal plane (*x*,*y*):

$$\dot{\Phi} = \varepsilon^2\Phi + \gamma\cdot\nabla(\Phi\nabla\Phi) - (1-\nabla^2)^2\Phi + \\ + \frac{1}{3}\nabla\left(\nabla\Phi\left|\Phi\right|^2\right) + \varepsilon^2 f, \tag{1}$$

where f is the random function describing the external noise, and the quantity ε determines the convection threshold overriding, which is assumed to be sufficiently small ($0<\varepsilon<1$). The term $\gamma\nabla(\Phi\nabla\Phi)$ describes the temperature dependence of viscosity. Further, we assume $\gamma = 0$ for simplicity. In this case we shall find the solution in the form

$$\Phi = \varepsilon\sum_j a_j \exp(i\vec{k}_j\vec{r}) \tag{2}$$

with $|\vec{k}_j|=1$. Renormalizing the time units $\propto\varepsilon^2$, we obtain the evolution equation for slow amplitudes a_j:

$$\dot{a}_j = a_j - \sum_{m=1}^{N} V_{mj}\,|\,a_m\,|^2\,a_j\;, \tag{3}$$

where interaction coefficients are determined as follows

$$V_{jj}=1, \tag{4}$$

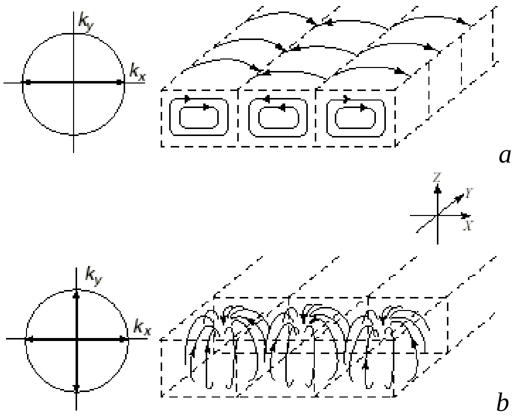
$$V_{ij}=(2/3)\left(1-2\left(\vec{k}_i\vec{k}_j\right)^2\right)=(2/3)\left(1+2\cos^2\vartheta\right). \tag{5}$$

Here ϑ is the angle between vectors $\vec{k}_i$ and $\vec{k}_j$.

The instability interval in *k*-space represents a ring with average radius equal to unit and the width is order of relative above-threshold parameter ε, i.e. much less than unity. During the development of the instability, the effective growth rate of modes that are localized outside of the very small neighborhood near the unit circle will decrease due to the growth of the nonlinear terms and can change sign which will lead to a narrowing of the spectrum to the unit circle in the *k*-space. Since the purpose of further research will be the study of stability of spatial structures with characteristic size of order $2\pi / k \propto 2\pi$ and the important characteristic for visualization of simulation results will be evidence of these structures, so we restrict ourselves by considering some idealized model of the phenomenon, assuming that the oscillation spectrum is already located on the unit circle in the *k*-space.

SIMULATION RESULTS

It was shown in [9, 11] that in the absence of temperature dependence of viscosity and when the number of modes is sufficiently large, the system delayed the development while remaining in a dynamic equilibrium. Development of perturbations in the system, as shown by the numerical analysis will be as follows [9]. Starting from initial fluctuations, the modes over a wide range of ϑ begin grow. The value of the quadratic form of the spectrum $I = \sum_j a_j^2$ can be estimated to obtain as result a value close to 0.75. It was shown that in the absence of temperature dependence of viscosity and when the number of modes is sufficiently large, the system delayed the development while remaining in a dynamic equilibrium. For further development – "crystallization", one of the modes must get a portion of the energy, which excesses some threshold value. That is, in these case, it is necessary a certain level of noise (fluctuations).



a

b

Fig. 1. Convective structures: rolls (a); square cells (b)

If one of the modes gets the proper amount of energy, then the process of formation of a simplest convective structure – rolls begins (Fig. 1,a). Note that in the nature, the thin clouds also can form the roll structure. The value of *I* in this case tends to unity ($I \to 1$). However, this state is not stable and then we can see the next structural transition: convective rolls are modulated along the axis of fluid rotation, and the typical size of this modulation phases down. In this transition state, the system stays for a sufficiently long time (which slightly increases within some limits with increase in the number of modes), and the value $I \approx 1.07$ remains constant during this time. After a rather long time, ten times more than the inverse linear growth rate of the initial instability only the one mode "survives" from newly formed "side" spectrum, which amplitude is comparable with the amplitude of the primary leading mode. In the end, the stable convective structure – square cells is generated (Fig. 1,b), and the quadratic form I reaches the value of $I = 1.2$.

Further researches of this process have found the following dynamics of quadratic form $I = \sum_j a_j^2$ with time (Fig. 2). Exactly after the first peak of the derivative, the metastable structure – a system of convective rolls is formed, and up to the moment when the second burst have appeared with value of $I \approx 1$ it remains unchanged. The next burst indicates the onset of a secondary metastable structure with a new value of $I \approx 1.07$.

After the second burst of the quadratic form derivative, a stable structure of squared convective cells is started to build up. Such behavior proves the existence of structural-phase transitions in the system.

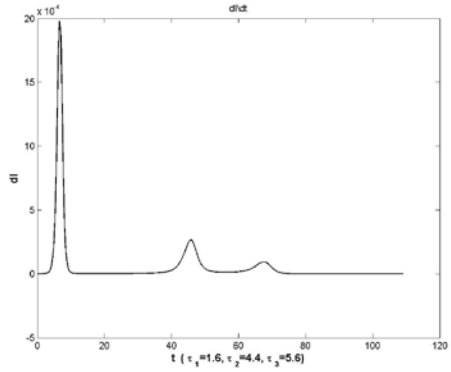


Fig. 2. The evolution of the derivative dI/dt (in relative units) of the integral quadratic form $I = \sum_j a_j^2$

Generally speaking, the characteristic times of relaxation processes during evolution of the system to more equilibrium state are determined as usual by the difference of the state function values before the transition and after it. The greater this difference, the faster the transition from one state to another. It is important to keep in mind that the sequence of state transitions is determined by the characteristic times of instabilities (which play the role of relaxation processes) that provide a cascade evolution of the system to the most equilibrium state. Initially, the fastest relaxation processes take place that associated with large difference of the state function values corresponding to different equilibrium states.

Let us verify that in this case all the phenomena occur in the same order and within the framework of the foregoing scenario. The numerical analysis of the model allows confirming these considerations.

It can be seen that the times of state formation τ_n are inversely proportional to $I = \sum_i A_i^2$ the difference between the values $I_n^{(+)} = (\sum_i A_i^2)_n^{(+)}$ after *n*-th structural phase transition $I_n^{(+)} = (\sum_i A_i^2)_n^{(+)}$ and before it

$$I_n^{(-)} = (\sum_i A_i^2)_n^{(-)},$$

$$\tau_n \sim \{(\sum_i A_i^2)_n^{(+)} - \sum_i A_i^2)_n^{(-)}\}^{-1} = \Delta I_n^{-1}. \qquad (6)$$

It follows from this that

$$\tau_3 / \tau_2 \approx \Delta I_2 / \Delta I_3. \qquad (7)$$

Thus, we have shown by numerical simulation of the Proctor-Sivashinsky model that the state with certain topology can be described by the state function, which is the sum of squared mode amplitudes. The transitions between these states are characterized by splashes in time-derivative of this function and different meta-stable structures, corresponding to different values of the state function have different visually distinguishable topologies.

The fact that the metastable states are characterized by specific values of the state function was highlighted in our earlier works [11 - 13]. The numerical study, presented in this paper, confirmed two observations:

1) the difference between the values of the state function before and after the structural phase transition is inversely proportional to the characteristic time of the corresponding structural-phase transition;

2) the evolution of the planar convective structure under consideration demonstrates all the features of a relaxation process, i.e. the fast structural-phase transition is succeeded by more slow ones. Thus, a fuller picture of the process becomes clear.

CONCLUSIONS

The special feature of the Proctor-Sivashinsky model with temperature independent viscosity is the existence of three possible metastable states, which correspond to patterns of different symmetries. The times of structural transitions between these metastable states are much less than the times of their existence. Each state has a definite topology and can be characterized by definite steady value of the state function. The metastable states are destroyed with time for the instabilities, the growth rate of which can be evaluated from the amplitude of splashes of time-derivative of the state function. It is shown, that the characteristic times of the instabilities, which destroy the previous state and form a new one are inversely proportional to the difference between the values of the state function before and after the structural phase transition. In addition, we show that the faster relaxation processes, i.e. structural phase transitions take priority over more slow ones.

REFERENCES

1. F.H. Busse, N. Riahi. Nonlinear convection in a layer with nearly insulating boundaries // *J. Fluid Mech.* 1980, v. 96, p. 243.
2. S. Chandrasekhar. *Hydrodynamic and hydromagnetic stability*. Dover Publication Inc.: New York, 1970, 704 p.
3. A.V. Getling. Structures in heat convection // *Usp. Fiz. Nauk*. 1991, v. 11, p. 1-80.
4. M.C. Cross, P.C. Hohenberg. Pattern formation outside of equilibrium // *Reviews of modern physics*. 1993, v. 65, № 3, p. 851.
5. E. Bodenschatz et al. Transitions between patterns in thermal convection // *Physical review letters*. 1991, v. 67, № 22, p. 3078.
6. J. Chapman, M.R.E. Proctor. Nonlinear Rayleigh-Benard convection between poorly conducting boundaries // *J. Fluid Mech*. 1980, № 101, p. 759-765.
7. V. Gertsberg, G.E. Sivashinsky. Large cells in nonlinear Rayleigh-Benard convection // *Prog. Theor. Phys.* 1981, № 66, p. 1219-1229.
8. B.A. Malomed, A.A. Nepomniachtchi, M.P. Tribel'skii. Two-dimensional quasi-periodic structures innonequilibrium systems // *Zh. Eksp. Teor. Fiz.* 1989, v. 96, p. 684-700.
9. A.V. Kirichok, V.M. Kuklin. Allocated Imperfections of Developed Convective Structures // *Physics and Chemistry of the Earth Part A.* 1999, № 6, p. 533-538.
10. I.V. Gushchin, A.V. Kirichok, V.M. Kuklin. Pattern formation in convective media (review) // *Journal of Kharkiv National University. Series «Nuclei, Particles, Fields»*. 2013, № 1040, Issue 1/57, p. 4-27.
11. E.V. Belkin, I.V. Gushchin, A.V. Kirichok, V.M. Kuklin. Structural transitions in the model of Proctor-Sivashinsky // *Problems of Atomic Science and Technology. Series «Plasma Electronics and New Methods of Acceleration»*. 2010, № 4, p. 296-298.
12. I.V. Gushchin, A.V. Kirichok, V.M. Kuklin. Pattern formation in unstable viscous convective medium // *Problems of Atomic Science and Technology. Series «Plasma Electronics and New Methods of Acceleration» (8)*. 2013, № 4, p. 251-256.
13. I.V. Gushchin, E.V. Belkin. Modeling of noise influence on the formation of spatial structures in the Proctor-Sivashinsky model // *Contemporary problems of mathematics, mechanics and computing sciences.* V.N. Karazin Kharkiv National University, 2011, p. 226-231.

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СТРУКТУРНО-ФАЗОВЫЕ ПЕРЕХОДЫ И ФУНКЦИЯ СОСТОЯНИЯ В НЕСТАБИЛЬНОЙ КОНВЕКТИВНОЙ СРЕДЕ

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Конвекция в тонком слое жидкости (газа) между плохо проводящими тепло поверхностями рассмотрена в условиях применимости модели Проктора-Сивашинского при отсутствие зависимости вязкости от температуры. С помощью численного анализа показано, что каждое состояние может быть описано с помощью функции состояния, которая равна сумме квадратов мод спектра пространственного распределения температуры на поверхности. Переход между состояниями характеризуется изменением производной по времени от этой функции. Различие между метастабильными состояниями, которые отличаются топологией, определяется разными значениями функции состояния.

СТРУКТУРНО-ФАЗОВІ ПЕРЕХОДИ ТА ФУНКЦІЇ СТАНУ В НЕСТАБІЛЬНОМУ КОНВЕКТИВНОМУ СЕРЕДОВИЩІ

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Конвекцію в тонкому шарі рідини (газу) між поверхнями, що недостатньо добре проводять тепло, розглянуто в умовах придатності моделі Проктора-Сівашинського за відсутністю залежності в’язкості від температури. З використанням числового моделювання показано, що кожен стан може бути представлений за допомогою функції стану, що дорівнює сумі квадратів мод просторового спектра температури на поверхні. Перехід між станами характеризується значенням похідної за часом від функції станів. Різниця між метастабільними станами, які відрізняються топологією, визначається різними значеннями функції стану.