LOW-FREQUENCY GENERATION BY IONIZING FEMTOSECOND LASER PULSE SUPPLIED BY ITS SECOND OR HALF-HARMONIC

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We calculate the free-electron residual current density excited in a gas by ionizing two-color femtosecond laser pulses containing an intense component centered at 800-nm and a weaker 400-nm (second-harmonic) or 1600-nm (half-harmonic) component with the use of quantum-mechanical (with the three-dimensional time-dependent Schrödinger equation solved numerically) and semiclassical approaches. The efficiency of residual current excitation by two-color pulses with additional second- and half-harmonic components is compared.

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INTRODUCTION

Laser-plasma terahertz sources using intense ionizing femtosecond laser pulses attract nowadays great attention in context of applications in remote sensing, time-domain spectroscopy, imaging, etc. [1]. The two-color schemes employing two-color femtosecond pulses ionizing a gas are some of the most popular laser-plasma schemes and can provide high output terahertz field amplitudes (up to 8 MV/cm) with bandwidth exceeding 100 THz [2 - 6]. Taking into consideration the accessibility of the working medium (which can simply be the ambient air [1 - 3; 5 - 9]) one can find such schemes to be of great interest for investigation.

The ways to generate the additional field for a two-color pulse in second- and of half-harmonic cases are different: the second harmonic is usually obtained from a nonlinear crystal [1 - 5, 7, 8], while the half-harmonic can be generated in an optical parametric amplifier [6, 9]. In this work, we compare the terahertz generation efficiency in these two schemes via *ab initio* numerical calculations.

The main source of low-frequency terahertz radiation in the two-color schemes is the plasma oscillation in the long wakefield of the laser pulse. The amplitude of this oscillation is proportional to the free-electron residual current density (RCD) [9 - 13] remaining in plasma behind the passed laser pulse. Since the additional field in ionizing two-color pulse is usually generated with the main field used as a pump, it is reasonable to compare the half- and the second-harmonic schemes by analyzing the dependencies of RCD on the additional component intensity I_1 with main component intensity I_0 fixed.

1. MODEL EQUATIONS

The electric field of the two-color laser pulse is assumed to be linearly polarized along the x axis, and the field x component is given as follows,

$$E(t)=[E_0\cos\omega_0 t+E_1\cos(\omega_1 t+\varphi)]\exp\left(-2\ln 2\,\frac{t^2}{\tau_p^2}\right). \quad (1)$$

Here E_0 and E_1 are maximum amplitudes of the main and additional components, respectively, $E_0 >> E_1$; ω_0 is the main field carrier frequency; ω_1 is the additional field carrier frequency, $\omega_1 = 2\omega_0$ in case of additional second harmonic and $\omega_1 = \omega_0/2$ in case of additional half harmonic; φ is the phase shift between the main and the additional field carriers; τ_p is the laser pulse duration (intensity full width at half-maximum).

1.1. THE QUANTUM-MECHANICAL APPROACH

The quantum-mechanical approach to calculating RCD employs the three-dimensional time-dependent Schrödinger equation for an electron wavefunction $\psi(\mathbf{r}, t)$ in hydrogen atom under the action of electric field specified above and the electric field of the nucleus,

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2-\frac{e^2}{r}-eE(t)x\right)\psi(\mathbf{r},t). \quad (2)$$

Here $\hbar$ is the reduced Planck constant; e and m are the electron charge and mass, respectively. This equation is solved numerically, and the RCD is calculated from the resulted wavefunction. The details on the approach and simulations methods are described in [12]. This approach (unlike the semiclassical approach) accounts for such features of electron dynamics as bound-bound transitions and free-bound ones with non-zero velocities of free state, recombination, free-electron motion under joint action of laser and nucleus fields.

1.2. THE SEMICLASSICAL APPROACH

The semiclassical approach to calculating RCD is used in this work as an approximation to the comprehensive quantum-mechanical solution in cases of high computational complexity. Within this approach, the RCD is found from equations for the free-electron density $N(t)$ and the residual current density j_{RCD} in laser-produced plasma [9-13]:

$$N(t)=N_g\left(1-\exp\left[-\int_{-\infty}^{t}w(|E(t')|)dt'\right]\right), \quad (3)$$

$$j_{RCD}=\frac{e^2}{m}\int_{-\infty}^{+\infty}N(t)E(t)dt. \quad (4)$$

Here N_g is the initial density of neutral particles, and $w(|E|)$ is the probability of atom ionization per unit time as a function of electric field at some time instant. We use the analytical formula for the probability of the tunneling ionization from [14],

$$w(|E|)=4\omega_a\frac{E_a}{|E|}\exp\left(-\frac{2E_a}{3|E|}-12\frac{|E|}{E_a}\right), \quad (5)$$

where $\omega_a = 4.13 \times 10^{16}$ s^{-1} and $E_a = 5.14 \times 10^9$ V/cm are the atomic units of frequency and field strength, respectively.

2. RESULTS OF NUMERICAL CALCULATIONS

In our calculations, we used the pulses with duration $\tau_p = 30$ fs and wavelength $\lambda_0 = 2\pi c/\omega_0 = 800$ nm. In all the calculations performed, we find the optimum phase shift value φ_{opt} corresponding to the maximal RCD produced, and choose that value for the result presentation. Thus, we focus on the dependences $j_{RCD}^{(1/2)}(I_0, I_1)$ and $j_{RCD}^{(2)}(I_0, I_1)$ of the RCD on intensities of the main and additional laser components in two compared cases (with $\omega_1 = \omega_0/2$ and $\omega_1 = 2\omega_0$).

We start with comparing the results for RCD from quantum-mechanical and semiclassical calculations for pulses with $I_1/I_0 = 0.04$ for both half-harmonic and second-harmonic cases to determine the validity region of the semiclassical approach. The result of comparison is illustrated in Fig. 1 (for additional second-harmonic field) and Fig. 2 (for additional half-harmonic field), where the RCD versus the intensity I_0 is plotted. It can be seen that there is a good (both quantitative and qualitative) agreement between semiclassical and quantum-mechanical calculations for intensities $I_0 \geq 10^{14}$ W/cm^2 which correspond to the tunnel ionization.

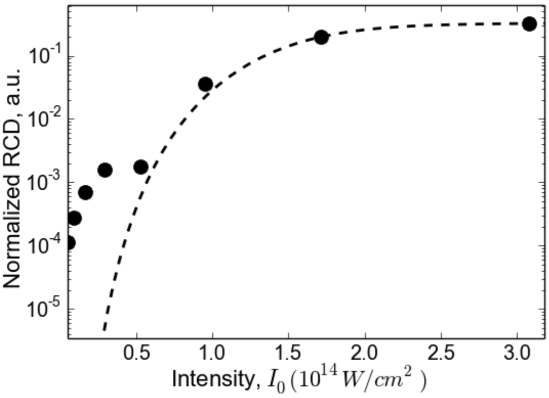

Fig. 1. The residual current density (normalized to N_g) from quantum-mechanical (dots) and semiclassical (dashed line) calculations as a function of the intensity I_0 of main laser component in ionizing two-color laser pulse with the intensity $I_1 = 0.04I_0$ of the additional second harmonic at the optimum phase shift; the laser pulse duration is $\tau_p = 30$ fs, the laser wavelength is $\lambda_0 = 800$ nm

The results of quantum-mechanical and semiclassical computations of RCD dependence on the intensities of the additional field are presented in Fig. 3 for pulses with the main field intensity $I_0 = 3 \times 10^{14}$ W/cm^2. These dependences on the additional field intensities have power scalings at not so large I_1, $j_{RCD}^{(1/2)} \sim I_1$ and $j_{RCD}^{(2)} \sim I_1^{1/2}$. The scalings can also be obtained analytically from semiclassical approach (see Ref. [10, 13]).

Thus, one can find the value of intensity $I^*(I_0, \alpha)$ such that $j_{RCD}^{(1/2)}(I_0, I^*) = j_{RCD}^{(2)}(I_0, \alpha I^*)$ where α is an arbitrary positive constant.

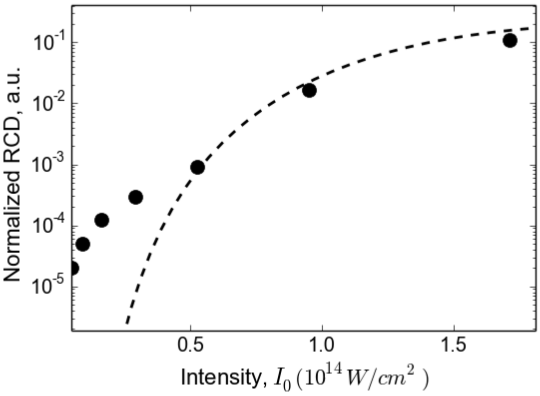

Fig. 2. The residual current density (normalized to N_g) from quantum-mechanical (dots) and semiclassical (dashed line) calculations as a function of the intensity I_0 of main laser component in ionizing two-color laser pulse with the intensity $I_1 = 0.04I_0$ of the additional half-harmonic at the optimum phase shift; the laser pulse duration is $\tau_p = 30$ fs, the laser wavelength is $\lambda_0 = 800$ nm

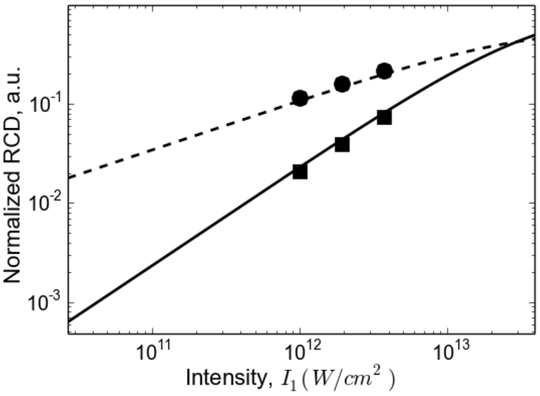

Fig. 3. The dependence of normalized residual current density (at the optimum phase shift) on the intensity I_1 of additional field from quantum-mechanical calculations (circles and squares) and semiclassical calculations (solid and dashed lines) for second-harmonic additional field (dashed line, circles) and half-harmonic additional field (solid line, squares); $I_0 = 3 \times 10^{14}$ W/cm^2

This intensity value corresponds to the equal terahertz yields in the cases of half- and second-harmonic additional fields with the main field intensity and the ratio α of the additional field generation efficiencies in these two cases fixed. In other words, with I_0 and α given, the use of the half-harmonic additional field is preferable (rather than the second-harmonic one) if half-harmonic intensity $I_1 > I^*(I_0, \alpha)$. We use the semiclassical approach to find the function $I^*(I_0, \alpha)$, which is presented in Fig. 4.

As it is seen, the ratio I^*/I_0 has complex behavior as function of α and I_0 within the range 10^{14} W/cm$^2 < I_0 < 1.5 \times 10^{14}$ W/cm^2, but it saturates and is almost independent of I_0 for $I_0 > 1.5 \times 10^{14}$ W/cm^2: while I^* is a strong function of α at low intensities, it depends on α much weaker at higher intensities. This saturation may be explained by the depletion of neutral particles by ionization on the laser pulse front with main component being intense enough. For $\alpha = 1$, I^*/I_0 saturates to 0.085. Thus, for equal second- and half-harmonic additional fields, using half-harmonic is preferable when the intensity additional field is greater than 8...9%. Overall, in a wide range of laser pulse total intensities, the half harmonic becomes preferable for effective terahertz generation when half-harmonic generation efficiency is greater then several percents (5...10%).

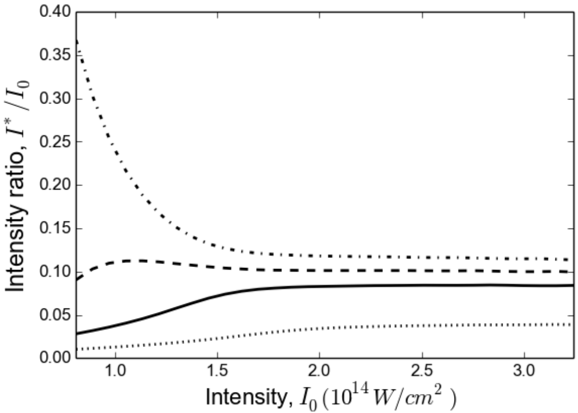

Fig. 4. The minimum half-harmonic intensity I^* *(normalized to the main field intensity) required for the addition of half-harmonic to be at least as effective as the addition of second harmonic versus the main field intensity* I_0 *at various values of efficiency ratios of additional field generation in half-harmonic and second-harmonic cases:* $\alpha = 0.5$ *(dotted line),* $\alpha = 1$ *(solid line),* $\alpha = 1.3$ *(dashed line),* $\alpha = 2$ *(dash dotted line)*

CONCLUSIONS

The excitation of residual current density is studied numerically for two-color laser-plasma scheme of terahertz generation with the use of the quantum-mechanical and semiclassical approaches. This scheme employs ionizing femtosecond two-color laser pulses which consist of a strong main field and a weaker additional field at doubled or halved frequency of the main field. We compare the results of semiclassical and quantum-mechanical approaches and prove that the semiclassical approach can be used for calculation of the residual current density for high enough intensities of the main field in the cases of both second- and half-harmonic additional fields. By comparing the residual current density in these two cases, we show that the addition of the half-harmonic field provides more efficient terahertz generation than the addition of the second harmonic for half-harmonic intensity greater than several percents of the total laser pulse intensity in a wide range of laser pulse parameters.

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НИЗКОЧАСТОТНАЯ ГЕНЕРАЦИЯ ПРИ ИСПОЛЬЗОВАНИИ ИОНИЗИРУЮЩИХ ФЕМТОСЕКУНДНЫХ ЛАЗЕРНЫХ ИМПУЛЬСОВ С ДОБАВОЧНЫМИ ВТОРОЙ ИЛИ ПОЛОВИННОЙ ГАРМОНИКАМИ

И.Д. Ларюшин, Л.С. Кузнецов, В.А. Костин, А.А. Силаев, Н.В. Введенский

С использованием квантово-механического (основанного на численном решении трёхмерного нестационарного уравнения Шрёдингера) и полуклассического подходов рассчитана остаточная плотность тока свободных электронов, возбуждаемая в газе двухцветными лазерными импульсами, содержащими сильное основное поле с длиной волны 800 нм и более слабую добавку второй (400 нм) либо половинной (1600 нм) гармоник. Проведено сравнение эффективности возбуждения остаточного тока двухцветными импульсами с добавкой второй и половинной гармоник.

НИЗЬКОЧАСТОТНА ГЕНЕРАЦІЯ ПРИ ВИКОРИСТАННІ ІОНІЗУЮЧИХ ФЕМТОСЕКУНДНИХ ЛАЗЕРНИХ ІМПУЛЬСІВ З ДОДАТКОВОЮ ДРУГОЮ АБО ПОЛОВИННОЮ ГАРМОНІКАМИ

І.Д. Ларюшин, Л.С. Кузнецов, В.А. Костін, О.А. Силаєв, М.В. Введенський

З застосуванням квантово-механічного (заснованого на числовому розв'язанні тривимірного нестаціонарного рівняння Шрьодингера) та полукласичного підходів розрахована залишкова густина струму вільних електронів, що збуджена в газі двокольоровими лазерними імпульсами, які складаються з сильного основного поля з довжиною хвилі 800 нм та більш слабкого додатку другої (400 нм) або половинної (1600 нм) гармонік. Проведено порівняння ефективності збудження залишкового струму двокольоровими імпульсами з додатком другої та половинної гармонік.