SUPPRESSION OF VORTICAL TURBULENCE IN PLASMA IN CROSSED ELECTRICAL AND MAGNETIC FIELDS DUE TO FINITE LIFETIME OF ELECTRONS AND IONS AND DUE TO FINITE SYSTEM LENGTH

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The dispersion equation, describing the instability development of vortex turbulence excitation in cylindrical plasma in crossed radial electric and axial magnetic fields with taking into account the longitudinal inhomogeneity and finite time of leaving of plasma electrons and ions from the system, has been derived. It is shown that the finite length of system time and finite time of system leaving of plasma electrons and ions leads to the appearance of the instability threshold and to decrease of growth rate of its development.

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INTRODUCTION

In plasma [1-3] a turbulence has been excited in crossed radial electrical and longitudinal magnetic fields by gradient of external magnetic field. This turbulence is a distributed vorticity. In this paper the excitation and damping of similar vortical turbulence, excited in cylindrical plasma in crossed radial electrical E_{0r} and longitudinal magnetic H_0 fields [4], is investigated theoretically. From the general nonlinear equation, presented in article [3], for vorticity the dispersion relation, which describes the instability development of vortex turbulence excitation, has been derived. It is shown that the finite length of system time and finite time of system leaving of plasma electrons and ions leads to the appearance of the instability threshold and to decrease of growth rate of its development.

EXCITATION OF VORTICES

Let us derive the dispersion relation. We take into account that the ions pass with velocity V_{bi} through system of length L during time, approximately equal $\tau_i = \frac{L}{V_{bi}}$. We also take into account that the electrons pass through system and are renovated in system also during finite time, τ_e. Damping of perturbations of densities and velocities of electrons and ions at recovery of their unperturbed values we describe, using $\nu_i \equiv \frac{1}{\tau_i}$, $\nu_e \equiv \frac{1}{\tau_e}$.

We use the electron hydrodynamic equations

$$\frac{\partial \vec{V}}{\partial t} + \nu_e\left(\vec{V} - \vec{V}_{\theta o}\right) + \left(\vec{V}\vec{\nabla}\right)\vec{V} = = \left(\frac{e}{m_e}\right)\vec{\nabla}\phi + \left[\vec{\omega}_{He}, \vec{V}\right] - \left(\frac{V_{th}^2}{n_e}\right)\vec{\nabla}n_e,$$

$$\frac{\partial n_e}{\partial t} + \frac{(n_e - n_{oe})}{\tau_e} + \vec{\nabla}\left(n_e\vec{V}\right) = 0 \ . \qquad (1)$$

Also we use the ion hydrodynamic equations

$$\frac{\partial \vec{V}_i}{\partial t} + \nu_i\left(\vec{V} - \vec{V}_{bi}\right) + \left(\vec{V}_i\vec{\nabla}\right)\vec{V}_i = -\left(\frac{q_i}{m_i}\right)\vec{\nabla}\phi \ ,$$

$$\frac{\partial n_i}{\partial t} + \frac{(n_i - n_{oi})}{\tau_i} + \nabla\left(n_i\vec{V}_i\right) = 0 \qquad (2)$$

and Poisson equation for the electrical potential, ϕ,

$$\Delta\phi = 4\pi\left(en_e - q_i n_i\right) \ . \qquad (3)$$

Here $\vec{V}$, n_e are a velocity and density of electrons; V_{th} is the electron thermal velocity; $V_{\theta o}$ is the electron azimuth drift velocity in crossed fields; $\vec{V}_i$, n_i, q_i, m_i are the velocity, density , charge and mass of ions.

As it will be visible from the further, the dimensions of the vortical perturbations are much larger than the electron Debye radius, $r_{de} \equiv \frac{V_{th}}{\omega_{pe}}$, then the last term in (1) can be neglected. Here $\omega_{pe} \equiv \left(\frac{4\pi n_{oe}e^2}{m_e}\right)^{1/2}$, n_{oe} is the unperturbed electron density.

From equations (1) one can derive non-linear equations

$$d_t\left[\frac{(\alpha - \omega_{He})}{n_e}\right] = \left[\frac{(\alpha - \omega_{He})}{n_e}\right]\partial_z V_z - \frac{\alpha\nu_e}{n_e} \ , \qquad (4)$$

$$d_t V_z + \nu_e V_z = \left(\frac{e}{m_e}\right)\partial_z\phi$$

describing both transversal and longitudinal electron dynamics. Here

$$d_t \equiv \partial_t + \left(\vec{V}_\perp\vec{\nabla}_\perp\right) \ , \ \partial_z \equiv \frac{\partial}{\partial z} \ , \ \partial_t \equiv \frac{\partial}{\partial t} \ , \qquad (5)$$

$\vec{V}_\perp$, V_z are the transversal and longitudinal electron velocities, α is the vorticity, the characteristic of electron vortical motion, $\alpha \equiv \vec{e}_z \mathrm{rot}\vec{V}$.

Taking into account higher linear terms, from (1) one can obtain

$$\vec{V}_\perp \approx \left(\frac{e}{m_e\omega_{He}}\right)\left[\vec{e}_z, \vec{\nabla}_\perp\phi\right] + \left(\frac{e}{m_e\omega_{He}^2}\right)\left(\partial_t + \nu_e\right)\vec{\nabla}_\perp\phi \quad (6)$$

From (6) we derive

$$\alpha \equiv \mu\omega_{He} \approx -\frac{2eE_{or}}{rm_e\omega_{He}} - \frac{eE_{or}}{m_e}\partial_r\left(\frac{1}{\omega_{He}}\right) + \frac{e}{m_e\omega_{He}}\Delta_\perp\varphi +$$

$$+\frac{e}{m_e}(\partial_r\varphi)\partial_r\left(\frac{1}{\omega_{He}}\right) + \frac{e}{m_e}(\partial_t + \nu_e)\vec{e}_z\left[\vec{\nabla}_\perp, \left(\frac{1}{\omega_{He}^2}\right)\vec{\nabla}_\perp\varphi\right],$$

$$\vec{\nabla}\phi \equiv \vec{\nabla}\varphi - \vec{E}_{or}. \quad (7)$$

Here E_{or} is the radial focusing electric field, φ is the electric potential of the vortical perturbation; $-\frac{2eE_{or}}{rm_e\omega_{He}} = \left(\frac{\omega_{pe}^2}{\omega_{He}}\right)\left(\frac{\Delta n}{n_{oe}}\right) \equiv \eta\omega_{He}$, $\Delta n \equiv n_{oe} - n_{oi}\frac{q_i}{e}$.

From (7) $\alpha \approx \left(\frac{\omega_{pe}^2}{\omega_{He}}\right)\left(\frac{\delta n_e}{n_{oe}}\right)$ approximately follows. Thus the vortical motion begins, as soon as the electron density perturbation, δn_e, appears.

We use that, as it will be shown below, the characteristic frequencies of perturbations approximately equal to ion plasma frequency, ω_{pi}.

As beam ions have large mass and propagate through system with velocity V_{bi}, we describe their dynamics in linear approximation. We derive ion density perturbation from eq.s (2)

$$\delta n_i = -n_{io}\left(\frac{q_i}{m_i}\right)\frac{\Delta\phi}{\left(\omega - k_z V_{ib} + i\nu_i\right)^2}. \quad (8)$$

Here k, ω are wave number and frequency of perturbation, V_{bi} is the unperturbed longitudinal velocity of the ions. Substituting (8) in Poisson equation (3), one can obtain

$$\frac{\beta\Delta\phi}{4\pi e} = \delta n_e \ , \ \beta = 1 - \frac{\omega_{pi}^2}{\left(\omega - k_z V_{ib} + i\nu_i\right)^2},$$

$$n_e = n_{oe} + \delta n_e. \quad (9)$$

Let us consider instability development in linear approximation. Then we search the dependence of the perturbation on z , θ in the form $\delta n_e \propto \exp\left(ik_z z + i\ell_\theta\theta\right)$. Then from (4) we derive

$$d_t\left(\frac{\omega_{He}}{n_e}\right) = \alpha\frac{\nu_e}{n_e} - \left(\frac{e\omega_{He}}{m_e n_{oe}}\right)\frac{ik_z^2\phi}{\left(\omega - \ell_\theta\omega_{\theta o} + i\nu_e\right)},$$

$$\omega_{\theta o} \equiv \frac{V_{\theta o}}{r}. \quad (10)$$

From (5), (6), (9), (10) we obtain, using the radial gradient of the short coil magnetic field, the following linear dispersion relation, describing the instability development

$$1 - \frac{\omega_{pi}^2}{\left(\omega - k_z V_{bi} + i/\tau_i\right)^2} - \frac{\omega_{pe}^2\left(\ell_\theta/r\right)\partial_r\left(1/\omega_{He}\right)}{\left(\omega - \ell_\theta\omega_{\theta o} + i/\tau_e\right)k^2} -$$

$$-\frac{\omega_{pe}^2}{\left(\omega - \ell_\theta\omega_{\theta o} + i/\tau_e\right)^2}\frac{k_z^2}{k^2} = 0. \quad (11)$$

From (11) for quick $V_{ph} \approx V_{\theta o}$ vortical perturbations we obtain

$$k_z = 0 \ , \ \omega = \omega^{(o)} + \delta\omega \ , \ |\delta\omega| << \omega^{(o)},$$

$$\omega^{(o)} = \omega_{pi} = \ell_\theta\omega_{\theta o} \ , \ \omega_{\theta o} = \left(\frac{\omega_{pe}^2}{2\omega_{He}}\right)\left(\frac{\Delta n}{n_{oe}}\right),$$

$$\Delta n \equiv n_{oe} - \frac{q_i n_{oi}}{e} \ , \ \delta\omega = i\gamma_q \ ,$$

$$\gamma_q \approx \left(\frac{\omega_{pe}}{k}\right)\sqrt{\left(\frac{\omega_{pi}}{2}\right)\left(\frac{\ell_\theta}{r}\right)\left|\partial_r\left(\frac{1}{\omega_{He}}\right)\right|} - \frac{1}{2}\left(\frac{1}{\tau_e} + \frac{1}{\tau_i}\right). \quad (12)$$

From (11) for slow $V_{ph} << V_{\theta o}$ vortical perturbations we obtain

$$\gamma_s = \gamma_{s0} - \frac{1}{3}\left(\frac{1}{\tau_e} + \frac{2}{\tau_i}\right), \ \gamma_{s0} \approx \left(\frac{\sqrt{3}}{2^{4/3}}\right)\left[\omega_{pi}^2\ell_\theta\omega_{\theta o}\right]^{1/3}$$

$$k^2 = -\left(\frac{1}{V_{\theta o}}\right)\omega_{pe}^2\partial_r\left(\frac{1}{\omega_{He}}\right), \ \mathrm{Re}\,\omega_s = \frac{\gamma_{s0}}{\sqrt{3}}. \quad (13)$$

One can see that τ_e and τ_i decrease growth rates and lead to appearance of thresholds of instability development.

Let us consider now, how finite $k_z \neq 0$ influences on growth rate of the instability development. From (11) we obtain the growth rate of the excitation of slow homogeneous turbulence with taking into account k_z.

$$\gamma_s \approx \left(\frac{\sqrt{3}}{2^{4/3}}\right)\omega_{pi}^{2/3}\left(\ell_\theta\omega_{\theta o} - k_z V_{bi}\right)^{1/3} \times$$

$$\times\left\{1 - \frac{k_z^2}{\left[2k_z^2 + \left(\frac{\ell_\theta}{r}\right)\left(\ell_\theta\omega_{\theta o} - k_z V_{bi}\right)\left|\partial_r\left(\frac{1}{\omega_{He}}\right)\right|\right]}\right\}^{1/3}. \quad (14)$$

From (14) one can see that both the taking into account the longitudinal dynamics of ions and electrons results in reduction of the growth rate. The perturbations with least $k_z\left(\approx\frac{\pi}{L}\right)$ have maximum growth rate, that is with the largest longitudinal dimensions, close to system length.

CONCLUSIONS

The dispersion equation, describing the instability development of vortex turbulence excitation in cylindrical plasma in crossed radial electric and axial magnetic fields with taking into account the longitudinal inhomogeneity and finite time of leaving of plasma electrons and ions from the system, has been derived. It is shown that the finite length of system time and finite time of system leaving of plasma electrons and ions leads to the appearance of the instability threshold and to decrease of growth rate of its development.

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ПОДАВЛЕНИЕ ВИХРЕВОЙ ТУРБУЛЕНТНОСТИ В ПЛАЗМЕ В СКРЕЩЕННЫХ ЭЛЕКТРИЧЕСКОМ И МАГНИТНОМ ПОЛЯХ ЗА СЧЕТ КОНЕЧНОГО ВРЕМЕНИ УХОДА ЭЛЕКТРОНОВ И ИОНОВ И КОНЕЧНОЙ ДЛИНЫ СИСТЕМЫ

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Получено дисперсионное уравнение, описывающее развитие неустойчивости возбуждения вихревой турбулентности в цилиндрической плазме в скрещенных радиальном электрическом и продольном магнитном полях с учетом продольной неоднородности и конечного времени ухода электронов и ионов плазмы из системы. Показано, что конечная длина системы и конечное время покидания системы электронами и ионами плазмы приводят к появлению порога неустойчивости и уменьшению инкремента ее развития.

ПОДАВЛЕННЯ ВИХРОВОЇ ТУРБУЛЕНТНОСТІ В ПЛАЗМІ В СХРЕЩЕНИХ ЕЛЕКТРИЧНОМУ І МАГНІТНОМУ ПОЛЯХ ЗАВДЯКИ КІНЦЕВОМУ ЧАСУ ВИХОДУ ЕЛЕКТРОНІВ ТА ІОНІВ І КІНЦЕВІЙ ДОВЖИНІ СИСТЕМИ

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Отримано дисперсійне рівняння, що описує розвиток нестійкості збудження вихрової турбулентності в циліндричній плазмі в схрещених радіальному електричному та поздовжньому магнітному полях з урахуванням поздовжньої неоднорідності і кінцевого часу залишання системи електронами та іонами плазми. Показано, що кінцева довжина системи і кінцевий час залишання системи електронами і іонами плазми призводять до появи порога нестійкості та зменшення інкремента її розвитку.