STRONG-FIELD APPROXIMATION FOR ANALYTICAL CALCULATION OF THE RESIDUAL CURRENT DENSITY EXCITED BY GAS IONIZATION WITH AN INTENSE TWO-COLOR LASER PULSE

A.A. Romanov[1,2], A.A. Silaev[1,2], N.V. Vvedenskii[1,2]
[1]Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod, Russia;
[2]Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, Russia
E-mail: vved@appl.sci-nnov.ru

On the basis of analytical solution of the time-dependent Schrödinger equation the excitation of residual current density (RCD) in a gas ionized by two-color laser pulse is studied. We find general analytical expression for the RCD for arbitrary values of the Keldysh parameter, which coincides with the semiclassical calculations in the case of tunneling regime of ionization.

PACS: 52.50.-b, 32.80.Rm

INTRODUCTION

This work is focused on analytical investigation of the excitation of quasi-dc residual current density (RCD) due to gas ionization by ultrashort laser pulses. At present, this phenomenon is of great interest due to the possibility of using it to convert efficiently laser pulses into low-frequency radiation, in particular, into the radiation of terahertz frequency band [1 - 10]. Various ionization-driven mechanisms for generating residual currents in the plasma are being considered at present. When multicycle laser pulses are used, the RCD can be generated due to gas ionization by two-color laser pulses [3, 4, 6 - 10] or due to the asymmetry of the ionized medium [11]. In the case of using few-cycle laser pulses, free electrons can be accelerated by the electric field of the ionizing laser pulse itself [1, 2, 4, 12 - 15].

Previous analytical studies of this phenomenon were based on the so-called semiclassical approach, which includes the hydrodynamic equation for the plasma current density, and the adopted model expression for the tunneling ionization probability per time unit [4, 10, 15]. However, the range of applicability of the semiclassical approach is limited by the parameters of laser pulses corresponding to the tunneling regime of ionization, when the Keldysh parameter [16] (defined by the ratio of atomic ionization energy and the average kinetic energy of an electron in a laser field) is much less than unity [12].

In this work we calculate RCD analytically by solution of the time-dependent Schrödinger equation using the strong-field approximation used in the pioneer work of Keldysh [16]. We assume that the quasi-dc RCD is generated due to gas ionization by two-color laser pulse, which contains a strong field at the fundamental frequency and a low-intensity field at the doubled frequency. We find the general analytical expression for the time derivative of the low-frequency current density and simplify it in the case of the low Keldysh parameter. In this case we obtain closed-form formula for the RCD and show that it coincides with the corresponding formula obtained on the basis of semiclassical approach.

1. STATEMENT OF THE PROBLEM

We assume that the electric field $\mathbf{E}(t)$ of the laser pulse is polarized linearly along the z axis. In order to ensure equality to zero of the integral of $\mathbf{E}(t)$, the electric field is given via the vector potential $\mathbf{A}(t)$:

$$\begin{cases} \mathrm{E}(t) = -\dfrac{1}{c}\dfrac{d\mathrm{A}}{dt},\ \mathrm{A}(t) = -\hat{\mathrm{z}}\dfrac{cE_0}{\omega_0}a(t) \\ a(t) = f(t)\left[\sin(\omega_0 t) + \dfrac{\alpha}{2}\sin(2\omega_0 t + \phi)\right]. \end{cases} \quad (1)$$

Here, $\hat{\mathrm{z}}$ is the unit vector along the z axis, E_0 is the peak amplitude of the main field, $\alpha << 1$ is the ratio of the amplitudes of the additional and main fields, ω_0 is the fundamental (carrier) frequency, φ is the phase shift between the carriers of additional and main fields, $f(t)$ is the pulse envelope, and c is the speed of light in vacuum. For the sake of certainty, we will assume that the envelope has the Gaussian form

$$f(t) = \exp\left(-2\ln 2\, t^2/\tau_p^2\right). \quad (2)$$

Here, τ_p is the intensity full-width at half maximum (FWHM). We neglect the interaction of atoms with each other assuming that the gas density is sufficiently low. In addition, we do not take into account the polarization response of plasma assuming that the maximum density of plasma is much less than the critical density and plasma frequency is $\omega_p << \tau_p^{-1}$.

The quantum-mechanical approach for calculation of the RCD is based on the solution of time-dependent Schrödinger equation for the electron wave function ψ:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) - e\mathbf{rE}\right)\psi . \quad (3)$$

Here, $\hbar$ is the Planck constant, $U(\mathbf{r})$ is the potential of the parent ion. For the sake of simplicity, we assume that the gas consists of hydrogen atoms and $U(\mathbf{r})$ is the Coulomb potential, $U(\mathbf{r}) = -e^2/r$. The RCD of free electrons is written as

$$\mathrm{j}_{\mathrm{RCD}} = \frac{eN_g}{m}\langle\psi_f \,|\, \hat{\mathrm{p}} \,|\, \psi_f\rangle\Big|_{t\to\infty}, \quad (4)$$

where N_g is the undisturbed gas density, $\hat{\mathrm{p}} = -i\hbar\nabla$ is the momentum operator, and ψ_f is the projection of the wave function on the continuum states.

2. ANALYTICAL RESULTS

Since the duration of the laser pulse is sufficiently large, the density of free electrons increases during many periods of the electric field. It allows one to approximate the RCD by the following integral

$$\mathbf{j}_{\text{RCD}} = \int_{-\infty}^{\infty} \frac{\overline{\partial \mathbf{j}}}{\partial t} dt \qquad (5)$$

where

$$\frac{\overline{\partial \mathbf{j}}}{\partial t} = \frac{eN_g}{m} \int \mathbf{p} \overline{W}(\mathbf{p},t) d^3 p \qquad (6)$$

is the average (low-frequency) growth rate of RCD, which is equal to the time derivative of the low-frequency current density $\bar{\mathbf{j}}$. Here, $\overline{W}(\mathbf{p},t)$ is the averaged over the field period momentum distribution of the ionization probability per unit time.

In order to calculate $\overline{W}(\mathbf{p},t)$ we assume that the envelope of the pulse is constant, i. e., the vector potential is

$$\mathrm{A}_f(t) = -\hat{z} \frac{cE_f}{\omega_0} [\sin(\omega_0 t) + (\alpha/2)\sin(2\omega_0 t + \phi)], \quad (7)$$

where the field amplitude $E_f = E_0 f(t)$ is the function of the "slow" time. We use the strong-field approximation used in the Keldysh work [16]. In this approximation the interaction of free electrons with the parent ion is neglected. At the same time, it is assumed that the laser pulse intensity is small enough to neglect the depletion of the atomic ground state. In this case the momentum distribution of the ionization probability per unit time is expressed as the sum of the probabilities of n-photon processes:

$$\overline{W}(\mathbf{p},t) = \frac{2\pi}{\hbar^2 \omega_0} |L(\mathbf{p})|^2 \sum_{n=n*}^{\infty} \delta\left(\frac{\Delta E}{\hbar\omega_0} - n\right). \qquad (8)$$

Here, $n* = \langle \tilde{I}_p/(\hbar\omega_0) + 1 \rangle$ is the minimum possible number of absorbed photons (the expression $\langle x \rangle$ denotes the integer part of the number x), $\tilde{I}_p = I_p + U_p$, I_p is the atom ionization energy, $U_p = U_{p0}(1+\alpha^2/4)$ is ponderomotive energy in the two-color field, $U_{p0} = e^2 E_f^2/4m\omega_0^2$ is ponderomotive energy in the field of fundamental field, $\Delta E = p^2/2m + \tilde{I}_p$ is the energy for detachment and acceleration of the electron.

The function $L(\mathbf{p})$ in the formula (7) describes the envelope of the momentum distribution of the ionization probability. Taking into account that the photon energy is much smaller than the ionization energy, i. e., $n_0 = I_p/\hbar\omega_0 >> 1$, the function $L(\mathbf{p})$ is written as

$$L(\mathbf{p}) = 2i \frac{\omega_0}{e} \left(\frac{\pi\hbar^3 I_p^{3/2}}{1 + p_\perp^2/2mI_p} \right)^{1/2} \sum_{t_s} \frac{e^{iS(\mathbf{p},t_s)/\hbar}}{E(t_s)}. \quad (9)$$

Here,

$$S(\mathbf{p},t) = \int_0^t \left(\frac{(\mathbf{p} + e\mathbf{A}_f/c)^2}{2m} + I_p \right) dt'. \qquad (10)$$

is the part of the action of a free electron that is independent of the coordinates, $p_\perp$ is the module of the transverse momentum, t_s is the stationary points of $S(\mathbf{p},t)$, and s is the index numbering these points. The values of t_s satisfy the equation

$$\left. \frac{\partial S(\mathbf{p},t)}{\partial t} \right|_{t_s} = 0, \qquad (11)$$

and have a positive imaginary part and a real part lying in the interval $[0, 2\pi/\omega_0$.

The action of the second harmonic field is taken into account in the phases in Eq. (9) by considering terms linear with α. Under the condition $\alpha \ll \gamma, \gamma^{-1}$ the stationary points of action are found in the absence of additional field:

$$\begin{cases} t_1 = \omega_0^{-1} \arcsin\left(\gamma q_z + i\gamma\sqrt{1+q_\perp^2} \right), \\ t_2 = \pi/\omega_0 - (t_1)^*. \end{cases} \qquad (12)$$

Here, where q_z is the projection of dimensionless momentum $\mathbf{q} = \mathbf{p}/\sqrt{2mI_p}$ on the z axis, $q_\perp$ is the module of the transverse dimensionless momentum, $\gamma = \sqrt{2I_p/U_{p0}}$ is the adiabacity parameter of Keldysh.

Following the work [16] we assume that the main contribution in $\overline{W}(\mathbf{p},t)$ is given by the small values of final electron momentum, $q^2 \ll 1$. It makes possible to neglect the momentum dependence of the pulse in the pre-exponential factors and use a Taylor series expansion of q up to the quadratic terms in the exponential factor:

$$\overline{W}(\mathbf{p}) \approx \sqrt{\frac{2I_p}{m^3}} \frac{4\pi^2\hbar}{\omega_0} \frac{\gamma^2}{\gamma^2+1} \exp(2n_0 b(\mathbf{q}))$$
$$\times [\cosh(2n_0 a(\mathbf{q})) + \varepsilon(\mathbf{q},\gamma)] \sum_{n=n^*}^{\infty} \delta\left(\frac{\Delta E}{\hbar\omega_0} - n\right). \qquad (13)$$

Here,

$$a(\mathbf{q}) = \alpha\gamma\cos\varphi\left(\frac{2}{3} + q_\perp^2\right),$$
$$b(\mathbf{q}) = \frac{\gamma q_z}{\sqrt{\gamma^2+1}}(q_z - \alpha\gamma\sin\varphi) + \frac{\sqrt{\gamma^2+1}}{2\gamma} \qquad (14)$$
$$-\left(q^2 + 1 + \frac{1}{2\gamma^2}\right) \operatorname{arcsh}\gamma.$$

The term $\varepsilon(\mathbf{q},\gamma)$ in Eq. (13) is associated with the intercycle interference of two electron trajectories originating from electron ionization from neighboring half-cycles. It is a rapidly oscillating function of the momentum for arbitrary values of γ. Therefore, when calculating the integral characteristics such as average ionization probability and current density the term ε can be neglected. The rest of the function $\overline{W}(\mathbf{p})$ is a product of the smooth envelope and the sum of delta functions corresponding the spheres defined by the energy conservation law. The maximum of the smooth envelope is located at the some optimal momentum:

$$\mathbf{p}_{opt} = \mathbf{z}\frac{\alpha\gamma\sqrt{2mI_p}\sin\varphi}{2\left(1-\sqrt{1+\gamma^{-2}}\,\mathrm{arcsh}\,\gamma\right)}. \qquad (15)$$

It can be seen that for $\alpha = 0$ the optimal momentum is $\mathbf{p}_{opt} = 0$ and the function $\overline{W}(\mathbf{p})$ is symmetric in the longitudinal momentum. Therefore, in the absence of the second harmonic the average photoelectron momentum is zero. The addition of a small field at the doubled frequency breaks the symmetry of the momentum distribution of the ionization probability. It leads to the excitation of nonzero residual current density.

Substituting the expression (13) in Eq. (6) we obtain an expression for the derivative of the low-frequency current density

$$\frac{\overline{\partial\mathbf{j}}}{\partial t} = \mathbf{z}eN_g\sqrt{\frac{8I_p^3}{m\hbar^2}}\frac{\gamma^2}{1+\gamma^2} \times\exp\left(-\frac{2\tilde{I}_p}{\hbar\omega_0}\left(\mathrm{arcsh}\gamma-\gamma\frac{\sqrt{\gamma^2+1}}{2\gamma^2+1}\right)\right) \times\sum_{n=n^*}^{+\infty}\exp\left(-2n_0q_n^2\,\mathrm{arcsh}\gamma\right)P_n. \qquad (16)$$

Here, $q_n = (n/n_0-(1+1/2\gamma^2))^{1/2}$ is the dimensionless momentum of the electron that absorbed n photons and

$$P_n = \int_{-q_n}^{q_n}\cosh\left(2n_0\alpha\gamma\cos\varphi(2/3+q_n^2-q_z^2)\right) \times\exp\left[\frac{2n_0\gamma}{\sqrt{\gamma^2+1}}\left(q_z^2-\alpha\gamma\sin\varphi\,q_z\right)\right]q_z dq_z. \qquad (17)$$

The expression obtained is rather complicated. However, it can be significantly simplified in the limits of high and small Keldysh parameter γ. In this work we simplify the obtained expression for $\gamma << 1$, which corresponds to the tunneling regime of ionization. In this case, the derivative of the low-frequency current density is represented as a product of the average ionization probability per unit time $\overline{w} = \int\overline{W}(\mathbf{p},t)d^3\mathbf{p}$, and the most most probable electron velocity $\mathbf{v}_{opt} = \mathbf{p}_{opt}/m$:

$$\frac{\overline{\partial\mathbf{j}}}{\partial t} = eN_g\mathbf{v}_{opt}\overline{w}, \qquad (18)$$

where

$$\mathbf{v}_{opt} = \mathbf{z}\frac{3}{2}\frac{e\alpha E_f}{m\omega_0}\sin\varphi. \qquad (19)$$

Average ionization probability per unit time is calculated by the method similar to that is used to calculate the $\overline{\partial\mathbf{j}}/\partial t$,

$$\overline{w} = Q_0\sqrt{\frac{3\pi}{2\sqrt{2}}}\frac{I_p}{\hbar}\left(\frac{E_f}{E_a}\right)^{1/2}\exp\left(-\frac{2E_a}{3E_f}\right) \times\cosh\left(\frac{2E_a\alpha}{3E_f}\cos\varphi\right), \qquad (20)$$

where $Q_0 = 1$. Note, that the strong-field approximation neglects the interaction of the free electron with its parent ion. The account of this interaction leads to the addition of the correction factor $Q_0 = (16/2^{1/4}\pi)(E_a/E_f)$ [16, 17].

In the case of $\alpha << E_f/E_a$ the expression (18) exactly coincides with the analogous formula obtained by semiclassical approach [18]. Using Eqs. (18) and (20) it is easy to find an analytical expression for the RCD. To do this, we use the method used previously in [10, 15, 18]. In this method, the time dependence of the average ionization probability is approximated by a Gaussian function with a characteristic scale $\tau_i = 2[\ddot{f}(0)E_0\overline{w}'(E_0)/\overline{w}(E_0)]^{-1/2}$. As a result, the RCD is

$$\mathbf{j}_{\mathrm{RCD}} \approx \frac{3}{2}\alpha\sigma j_{osc}\sin\varphi, \qquad (21)$$

where $\sigma = \sqrt{(\pi/2)}\overline{w}(E_0)\tau_i$ is the final degree of ionization and $j_{osc} = e^2N_gE_0/m\omega_0$ is the maximum oscillatory current density in the field of fundamental harmonic.

CONCLUSIONS

The excitation of residual current density due to gas ionization by ultrashort laser pulses was studied on the basis of the strong-field approximation for the solution of the time-dependent Schrödinger equation. It is assumed that the laser pulse contains the main field at the fundamental frequency and the additional field at the doubled frequency. We have found the general analytical expression for the time derivative of the low-frequency current density, which is significantly simplified in the case of Keldysh parameter $\gamma << 1$ corresponding to tunneling regime of ionization. In this case the photocurrent is determined by the product of the average ionization probability per unit time and the most probable velocity of the electron (corresponding to the maximum of the velocity distribution function), in good agreement with the results given by the semiclassical approach. When the condition $\gamma << 1$ is not satisfied, such factorization is impossible and the dependence of the residual current density on the laser pulse parameters may differ significantly from the results obtained by the semiclassical approach.

ACKNOWLEDGEMENTS

The derivation of general analytical expressions for ionization probability and time derivative of low-frequency current density was supported by the Government of the Russian Federation (Agreement No. 14.B25.31.0008) and the Russian Foundation for Basic Research (Grants Nos. 14-02-00847 and 14-02-31722). The derivation of closed-form analytical formula for RCD in the case of small Keldysh parameter was supported by the Russian Science Foundation (Grant No. 15-12-10033).

REFERENCES

1. V.B. Gildenburg, N.V. Vvedenskii. Optical-to-THz wave conversion via excitation of plasma oscillations in the tunneling-ionization process // *Physical Review Letters*. 2007, v. 98, p. 245002-1-4.
2. H.-C. Wu, J. Meyer-ter Vehn, Z.-M. Sheng. Phase-sensitive terahertz emission from gas targets irradiated by few-cycle laser pulses // *New Journal of Physics*. 2008, v. 10, p. 043001-1-10.
3. K.Y. Kim, A.J. Taylor, J.H. Glownia, G. Rodriguez. Coherent control of terahertz supercontinuum generation in ultrafast laser-gas interactions // *Nature Photonics*. 2008, v. 2, p. 605-609.
4. K.-Y. Kim. Generation of coherent terahertz radiation in ultrafast laser-gas interactions // *Physics of Plasmas*. 2009, v. 16, p. 056706-1-8.
5. V.A. Kostin, N.V. Vvedenskii. Ionization-induced conversion of ultrashort Bessel beam to terahertz pulse // *Optics Letters*. 2010, v. 35, p. 247-249.
6. M.D. Thomson, V. Blank, H.G. Roskos. Terahertz white-light pulses from an air plasma photo-induced by incommensurate two-color optical fields // *Optics Express*. 2010, v. 18, p. 23173-23182.
7. T.I. Oh, Y.S. You, N. Jhajj, E.W. Rosenthal, H.M. Milchberg, K.Y. Kim. Intense terahertz generation in two-color laser filamentation: energy scaling with terawatt laser systems // *New Journal of Physics.* 2013, v. 15, p. 075002-1-17.
8. B. Clough, J. Dai, X.-C. Zhang. Laser air photonics: beyond the terahertz gap // *Materials Today*. 2012, v. 15, p. 50-58.
9. M. Clerici, M. Peccianti, B.E. Schmidt, L. Caspani, M. Shalaby, M. Giguère, A. Lotti, A. Couairon, F. Légaré, T. Ozaki, D. Faccio, R. Morandotti. Wavelength scaling of terahertz generation by gas ionization // *Physical Review Letters.* 2013, v. 110, p. 253901-1-5.
10. N.V. Vvedenskii, A.I. Korytin, V.A. Kostin, A.A. Murzanev, A.A. Silaev, A.N. Stepanov. Two-Color Laser-Plasma Generation of Terahertz Radiation Using a Frequency-Tunable Half Harmonic of a Femtosecond Pulse // *Physical Review Letters*. 2014, v.112, p. 055004-1-5.
11. L.N. Alexandrov, M.Y. Emelin, M.Y. Ryabikin. Unidirectional current excitation in tunneling ionization of asymmetric molecules // *Physical Review A*. 2013, v. 87, p. 013414-1-6.
12. A.A. Silaev, N.V. Vvedenskii. Residual-current excitation in plasmas produced by few-cycle laser pulses // *Physical Review Letters.* 2009, v. 102, p. 115005-1-4.
13. A.A. Silaev, N.V. Vvedenskii. Quantum-mechanical approach for calculating the residual quasi-dc current in a plasma produced by a few-cycle laser pulse // *Physica Scripta*. 2009, v. T135, p. 014024-1-5.
14. A.A. Silaev, M.Y. Ryabikin, N.V. Vvedenskii. Strong-field phenomena caused by ultrashort laser pulses: Effective one- and two-dimensional quantum-mechanical descriptions // *Physical Review A*. 2010, v. 82, p. 033416-1-14.
15. A.A. Silaev, N.V. Vvedenskii. Analytical description of generation of the residual current density in the plasma produced by a few-cycle laser pulse // *Physics of Plasmas*. 2015, v. 22, p. 053103-1-14.
16. L.V. Keldysh. Ionization in the field of a strong electromagnetic wave // *Soviet Physics JETP*. 1965, v. 20, p. 1307-1314.
17. B.M. Karnakov, V.D. Mur, V. S. Popov, S.V. Popruzhenko. Ionization of atoms and ions by intense laser radiation // *JETP Letters*. 2011, v. 93, p. 238-249.
18. A.A. Silaev, V.A. Kostin, I.D. Laryushin, N.V. Vvedenskii. Analytical study of residual-current excitation during gas ionization by two-color laser pulse // *Journal of Physics: Conference Series.* 2015, v. 594, № 1, p. 12020-12026.

Article received 01.06.2015

ПРИБЛИЖЕНИЕ СИЛЬНОГО ПОЛЯ ДЛЯ АНАЛИТИЧЕСКОГО РАСЧЁТА ОСТАТОЧНОЙ ПЛОТНОСТИ ТОКА, ВОЗБУЖДАЕМОГО ПРИ ИОНИЗАЦИИ ГАЗА ИНТЕНСИВНЫМ БИХРОМАТИЧЕСКИМ ЛАЗЕРНЫМ ИМПУЛЬСОМ

А.А. Романов, А.А. Силаев, Н.В. Введенский

На основе аналитического решения нестационарного уравнения Шрёдингера исследуется возбуждение остаточной плотности тока в газе, ионизируемом интенсивным бихроматическим лазерным импульсом. Найдено общее аналитическое выражение остаточной плотности тока для произвольных значений параметра Келдыша, которое совпадает с результатами полуклассических расчетов при туннельном режиме ионизации.

НАБЛИЖЕННЯ СИЛЬНОГО ПОЛЯ ДЛЯ АНАЛІТИЧНОГО РОЗРАХУНКУ ЗАЛИШКОВОЇ ГУСТИНИ СТРУМУ, ЗБУДЖУВАНОГО ПРИ ІОНІЗАЦІЇ ГАЗУ ІНТЕНСИВНИМ БІХРОМАТИЧНИМ ЛАЗЕРНИМ ІМПУЛЬСОМ

А.А. Романов, А.А. Силаєв, Н.В. Введенський

На основі аналітичного розв’язку нестаціонарного рівняння Шредингера досліджується збудження залишкової густини струму в газі, що іонізований інтенсивним біхроматичним лазерним імпульсом. Знайдено загальний аналітичний вираз залишкової густини струму для довільних значень параметру Келдиша, який співпадає з результатами напівкласичних розрахунків при тунельному режимі іонізації.