QUASI-STATIONARY STREAMER PROPAGATION

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The numerical simulations are carried out for the quasi-stationary stage of the axially symmetric streamer propagation and for the linear stage of its azimuthal perturbation development. The dependences of the streamer velocity on the average electric field strength are obtained for the different assumptions about photo-ionization intensity. Azimuthal instability is revealed and its connection with the streamer branching is discussed.

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INTRODUCTION

The ozonizers based on corona discharge until now are used and processes in them are investigated [1]. These ozonizers are comparatively simple in manufacture and maintenance. Even if constant voltage is applied the processes usually obtain the pulsed form (streamers in positive corona, Trichel pulses in negative corona), and ion current between the pulses heats gas and decomposes ozone. Replacing of the constant voltage with the pulsed one considerably increases an efficiency of ozone production [2]. In the simple variant of the pulsed supply scheme the great charge is drained to ground after the pulse. Saving up of the part of its energy for further usage may be realized, in particular, through charge exchange with relevant half-cycle oscillatory circuit. Similar way is based on sinusoidal oscillations in a simple high-quality oscillatory circuit with capacitor and inductance coil. Usage of the capacitor volume for gas discharge gives well known capacitive discharge, moving up to which may be considered as improvement (with respect to ozone production efficiency) of the constant voltage corona discharge. Covering of one electrode with dielectric layer helps to diminish breakdown probability in the conditions of intensive operation (another electrode, without dielectric layer, serves as effective electron source during relevant half of the oscillation period, through ion-electron emission). So, moving up to dielectric barrier discharge (DBD) may be considered as further improvement of corona discharge. Both in positive corona, and in DBD the ionization processes are realized through positive (cathode-directed) streamers, naturally, somewhat different in their characteristics. A streamer quickly develops from avalanches, then it may to cover some distance without considerable change of its characteristics, and then, making stop in the gap or coming nearer to electrode, it quickly changes the characteristics. In the present work the numerical simulations of the quasi-stationary stage of streamer propagation are carried out. In particular, they reveal the linear instability of the considered type of streamers with respect to azimuthal perturbations.

1. SIMULATION MODEL

In the simulations, electron drift and diffusion, and the processes of impact ionization, attachment and photo-ionization are taking into account. Ions are assumed to be motionless, which makes it possible to describe them with one variable, the difference N_i between the densities of positive and negative ions. Time evolution of the densities is determined with the following equations:

$$\partial_t N_e - \mathrm{div}(D_e \nabla N_e + N_e \mu_e \vec{E}) = S_{rd} + \nu_{ia} N_e ,$$

$$\partial_t N_i = S_{rd} + \nu_{ia} N_e , \; \varepsilon_0 \nabla^2 \Phi = q(N_e - N_i) , \; \vec{E} = -\nabla \Phi .$$

Here ∂_t is time derivative, q is elementary charge, ε_0 is electric constant, N_e, μ_e and D_e are electron density, mobility, and diffusion coefficient, ν_{ia} is the difference between ionization and attachment frequencies, $\nu_{ia} = \nu_i - \nu_a$,

$$S_{rd}(\vec{r}) = \int dV' N_e(\vec{r}\,') \nu_{rd}(|\vec{E}(\vec{r}\,')|) \kappa g(|\vec{r} - \vec{r}\,'|) ,$$

κ is photon absorption coefficient, $\nu_{rd}(E)$ is the frequency of the photo-ionization acts made in the unbounded space by the photons radiated from the states exited by one electron moving in the field with the strength E, $g(r) = \exp(-\kappa r)/(4\pi r^2)$. The quantities ν_i, ν_a, and μ_e are taken dependent on the electric field strength value, and D_e is constant. The dependences for ν_{rd} and ν_i are taken in the form

$$\nu_{rd}(E) = \alpha_{rd0} \mu_e(E) E \exp(-E_{rd0}/E) ,$$

$$\nu_i(E) = \alpha_{i0} \mu_e(E) E \exp(-E_{i0}/E) .$$

In the dependence for ν_a two-body and three-body attachment are taking into account. The used mesh is homogeneous along the coordinate z of the cylindrical coordinate system (ρ, φ, z). At the boundaries $z = \mathrm{const}$ the conditions $\Phi = \mathrm{const}$ are imposed with the difference $\Delta\Phi$, corresponding to the average applied electric field strength, $E_{av} = \Delta\Phi/L$, where L is distance between the boundaries. The boundary condition at the maximum radius of calculation space $\rho_{\max}$ is described below, together with one for azimuthal perturbations. The change with time of the average value of the coordinate z for the electrons distributed in the calculation space is controlled, and its stay in the same cell is ensured by mesh shifting on one cell (if necessary) with taking of some initial density value for the cells appeared near the relevant boundary. If L is sufficiently large (in comparing with characteristic transverse streamer dimensions) and the streamer front is situated far from the boundaries $z = \mathrm{const}$ then the electric field strength near the front is not considerably different from one near the front of the half-infinite streamer propagating in the unbounded space with the same E_{av}.

In the part of simulations, along with calculations for the axially symmetric distribution, the calculations for the linear stage of development of the perturbations, dependent as $\cos(m\varphi)$ $(m = 1, 2, 3, \ldots)$ on the azimuthal angle φ are carried out. Some linear relationships used in calculations for the perturbations are written in [3]. But there the hyperboloidal coordinates were used, whereas here are used the cylindrical ones, and the variable $s = \rho^2$, and relevant relationships change the form. With $\rho \to 0$ the ratios of the perturbation amplitudes and the quantity ρ^m comes to the bounded values, so, it is worthy to remove the factor $s^{m/2}$ and to rewrite the equations for the coefficients at it. In particular, the equation for the perturbation of potential has the form $\nabla^2 \Phi^{(m)} = -Q^{(m)}$. Having write the expansions:

$$\Phi^{(m)} = \sum\nolimits_{n=1}^{\infty} \Phi_n^m(s) \sin(k_n z) \cos(m\varphi),$$

$$Q^{(m)} = \sum\nolimits_{n=1}^{\infty} Q_n^m(s) \sin(k_n z) \cos(m\varphi),$$

where $k_n = \pi n / L$, and $\Phi_n^m(s)$ are unknown, one comes to the equation

$$4\partial_s \left[s \partial_s \Phi_n^m(s) \right] - \left[k_n^2 + \left(m^2 / s \right) \right] \Phi_n^m(s) = -Q_n^m(s).$$

Having introduce the functions $\bar{\Phi}_n^m(s) = \Phi_n^m(s) / s^{m/2}$ and $\bar{Q}_n^m(s) = Q_n^m(s) / s^{m/2}$, one gets the equation

$$4\partial_s \left[s \partial_s \bar{\Phi}_n^m(s) + m \bar{\Phi}_n^m(s) \right] - \kappa_n^2 \bar{\Phi}_n^m(s) + \bar{Q}_n^m(s) = 0$$

and the boundary condition

$$\lim\nolimits_{s \to 0} \left[4(m+1) \partial_s \bar{\Phi}_n^m(s) - \kappa_n^2 \bar{\Phi}_n^m(s) + \bar{Q}_n^m(s) \right] = 0.$$

At $\rho > \rho_{\max}$ the charge absence is assumed. So, there the proportionality $\Phi_n^m(\rho^2) \propto \mathrm{K}_m(k_n \rho)$ has to take place (where K_m is McDonald function). This proportionality may be kept through the boundary condition $\partial_\rho \left[\Phi_n^m(\rho^2) / \mathrm{K}_m(k_n \rho) \right] = 0$ at $\rho = \rho_{\max}$.

The simulations were carried out for the calculation space width 1 mm.

2. NUMERICAL RESULTS

In the Fig. 1, for the average field strength 50 kV/cm, the distributions of electron density, of the difference between ion and electron density (charge density), of electric field strength, and of impact ionization rate are shown. The scale (from white to black, at the top) is linear for the difference (from 10^{12} to $+10^{12}$ cm^{-3}) and logarithmic for others (from 10^9 to 10^{15} cm^{-3} for the electron density, from 10^4 to 10^6 V/cm for the strength, from 10^9 to 10^{21} cm^{-3}·s^{-1} for the rate). In the Figs. 2 and 3, the dependences of velocity and of characteristic diameter of streamer on E_{av} are shown for different assumptions about the photo-ionization frequency ν_{rd}. Namely, for $1/\alpha_{rd0}$ the values 0.2, 2, and 20 cm are taken (growing from left to right); the Fig. 1 refers to the value 2 cm. In the Fig. 4, the connection between the velocity and characteristic diameter (according to the Figs. 2, 3) is shown. Its dependence on the parameter $1/\alpha_{rd0}$ (growing from top to bottom) is weak. The dependences may be explained with account of photo-ionization role. Photo-ionization is a source of seed electrons for avalanche multiplication. The weaker the source, the stronger external field should be to keep the same streamer characteristic diameter and streamer velocity. The field strength value in front of streamer remains near to E_{i0} even for the considerable change of external field, and for the given characteristic diameter the field strength at the same distance not far from the streamer front remains approximately the same. In such conditions, the weaker the source of seed electrons, the more electron number doubling is required to produce high density typical for streamer and move the streamer front to new position, and so, the streamer propagates slower (for the same characteristic diameter).

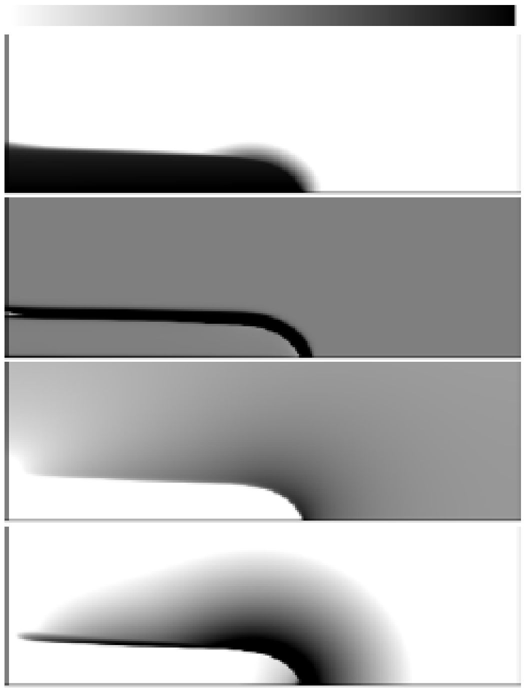

Fig. 1. The distribution of electron and charge density, of electric field strength and ionization rate

It should be noted that in the case of large streamer diameter (when it is not much less than L) the field strength in front of relevant long streamer may be considerably greater than one obtained in the present simulations.

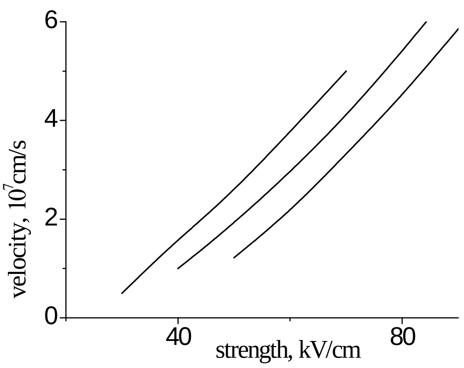

Fig. 2. Streamer velocity

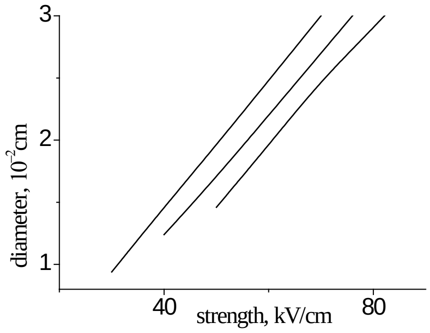

Fig. 3. Characteristic diameter of streamer

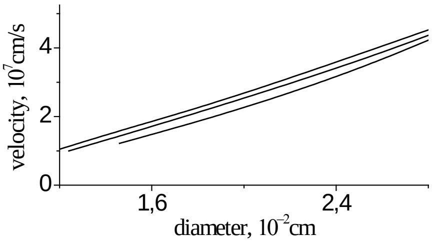

Fig. 4. The velocity related to the diameter

In simulations of azimuthal perturbations, for m, the values 1, 2, and 3 are taken. The values of time increment of perturbation growth in all calculations reveal to be positive, which corresponds to the perturbation instability. In particular, for $m \geq 2$, the instability is connected with the initial stage of streamer branching. But experimental facts witness practical absence of streamer branching for the small excess of applied voltage over its value minimum necessary for the streamer forming. The question arises, is the instability obtained for all calculated cases of the considered model not the consequence of some mistake? In other words, can the instability be somehow deduced in the frames of the model? Another question, how the streamer branching is connected with the linear instability of azimuthal harmonics?

3. INSTABILITY DISCUSSION

Indeed, some arguments may be adduced for the assertion that the azimuthal perturbations are unstable in the wide range of the stationary streamer propagation modes. Let us consider the case of stationary propagation of the axially symmetric streamer, assuming its stability with respect to axially symmetric perturbations.

At first, the main streamer features should be briefly described. A positive streamer trends to become transversely localized even for small excess of E_{av} over its value E_{av0} minimum necessary for the streamer stationary propagation possibility. This trend is connected with opposition of the directions of electron flow and streamer propagation. Avalanche multiplication begins far from the streamer front, electron density there is small, and to increase it from the initial value $10^5 \ldots 10^6$ cm^{-3} given with photo-ionization to the final value $10^{14} \ldots 10^{15}$ cm^{-3} natural for streamer it is necessary approximately 30 successive acts of electron number doubling. The mentioned final value of electron density is determined with the condition $\nu_i \sim \nu_M$, where $\nu_M = qN_e\mu_e/\varepsilon_0$ (Maxwell relaxation frequency), and the strength value for ν_i and μ_e should be taken as one in front of streamer. The difference between the values of ionization coefficient α for the points ahead of streamer at $\rho = 0$ and at nonzero ρ is small, but the proportionality of the obtained electron density to the quantity $\exp\int \alpha dl$ (where integral is taken over the electron drift path) leads to the considerable preference of one direction of avalanche development over other, with relevant consequences for successive ionization results at the different ρ. In the strong field near the streamer front the electron drift path between ionization acts is much less then characteristic transverse streamer dimension, and the streamer front may be considered as sharp. Form of the front is approximately determined with the field strength value distribution (and so, with the ionization frequency value distribution) in the space of intensive ionization near the front at the different radii through the condition $u_\rho = u_0 \cos\theta_\rho$, where u_ρ is the speed of the streamer front propagation along the local normal to it at the radius ρ (in particular, u_0 is streamer velocity), and θ_ρ is the angle between the normal and z axis. Farther from the axis the field strength values are less, the value of u_ρ is less, and the value of θ_ρ is nearer to $\pi/2$.

For the coordinate system moving in z direction with velocity u relative to laboratory frame, introducing the variables (ς, τ) connected with the variables (z, t) through the equalities $\varsigma = z - ut$ and $\tau = t$, one gets the equalities $\partial_\varsigma = \partial_z$,

$$\partial_\tau N_e - u\partial_z N_e - \mathrm{div}(D_e \nabla N_e + N_e \mu_e \vec{E}) = S_{rd} + \nu_{ia} N_e ,$$
$$\partial_\tau N_i - u\partial_z N_i = S_{rd} + \nu_{ia} N_e . \qquad (1)$$

The possibility of stationary streamer propagation with the velocity u_0 is connected with the existence of nonzero solution of the equations

$$-u\partial_z N_e - \mathrm{div}(D_e \nabla N_e + N_e \mu_e \vec{E}) = S_{rd} + \nu_{ia} N_e ,$$
$$-u\partial_z N_i = S_{rd} + \nu_{ia} N_e ,$$

for $u = u_0$. For the fixed ς, in the space of intensive ionization in front of streamer, the electron and ion densities decrease in the case $u > u_0$ (the streamer lags behind the moving frame), and they increase in the case $u < u_0$. The equations, which may be obtained with linearization of the eq. (1), should give the same tendencies. The mentioned linear equations may have the axially symmetric solutions characterized by relevant time increments. As the considered streamer propagation is stable, these increments cannot have positive real part. And assuming the continuous dependence of the increments on the parameters of equations, one obtains that maximum real part of the increments cannot be negative (if it is negative it would have remained negative after a small variation of parameters, in particular, after the small decrease of u value, in contrary to the mentioned densities increase in the case $u < u_0$). So, in the case of the stationary stable axially symmetric streamer propagation, the maximum real part of time increment of the axially symmetric ($m = 0$) perturbation should be equal to zero. The perturbations with $m \geq 1$ should have greater increments, in connection with the additional curvature of their front surface, and so, the additional field enhancement, for the equal change of the front position along the normal to it. So, for $m \geq 1$, the unstable perturbation (with increment having positive real part) should exist.

It should be noted that the word "increment" is used here in somewhat conditionally sense and exponential time dependence is not assumed without fail (especially this refers to the "increments" with zero real part). Also, it should be noted that the word "streamer" here is not defined rigorously, and the adduced arguments do not make the proof of rigorous assertion.

As the azimuthal perturbation linear instability exists at any excess of the average field strength value over one minimum necessary for the streamer stationary propagation possibility, the question remains, why the streamer branching is practically not observed in the experiments at small excess of the applied voltage over one minimum necessary for the streamer forming? One possible cause may be connected with the combined development of the azimuthal perturbations with different m, in the case when their increments have the same order of magnitude, and so, relevant perturbations develop during approximately the same time. The perturbation with $m = 1$, as a rule, strengthens or weakens the perturbation with another m to the different extent in the different radial directions. As a result, the sum of perturbations may to develop in such a way that in some sector the perturbation comes to nonlinear stage faster, the perturbation development in other sectors is slowing down, and instead of streamer branching the small fluctuations of its propagation take place. Also, real experimental conditions usually correspond to decrease of the external field strength with removal of the streamer front from anode (in particular, in connection with anode finite size), and the streamer development in such conditions is not stationary.

CONCLUSIONS

The numerical simulations of streamer propagation in the fixed average external field are carried out for some range of gas parameters, in particular, in different assumption about the intensity of photo-ionization rate, which gives seed electrons for streamer. Their results show that the positive axially symmetric streamer approaches the stationary stable propagation mode. But with respect to the perturbations corresponding to azimuthal harmonics this mode, in the calculated cases, revealed to be unstable. As it follows from the adduced arguments, such situation may be considered as typical for streamers.

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КВАЗИСТАЦИОНАРНОЕ ДВИЖЕНИЕ СТРИМЕРА

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Выполнено численное моделирование квазистационарного этапа движения аксиально-симметричного стримера и линейного этапа развития его азимутального возмущения. Зависимости скорости стримера от средней напряженности электрического поля получены для разных предположений об интенсивности фотоионизации. Выявлена азимутальная неустойчивость и обсуждается ее связь с ветвлением стримера.

КВАЗІСТАЦІОНАРНИЙ РУХ СТРИМЕРА

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Виконано числове моделювання квазістаціонарного етапу руху аксіально-симетричного стримера та лінійного етапу розвитку його азимутального збурення. Залежності швидкості стримера від середньої напруженості електричного поля отримано для різних припущень про інтенсивність фотоіонізації. Виявлено азимутальну нестійкість та обговорюється її зв'язок з розгалуженням стримера.