WAKEFIELD SPECTRA IN THE PLASMA-DIELECTRIC ACCELERATOR WHEN CHANGING THE PLASMA DENSITY

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The analysis of the changes in the spectra of a wake field excited in a cylindrical plasma-dielectric waveguide by relativistic electron bunch is carried out. Three variants of structures are considered: the parameters of the dielectric structure and bunches are fixed; inner or outer radius of the dielectric tube is changed, so that the frequency of the first radial mode coincides with the frequency of the plasma wave. The latter two options are necessary in the case of a regular sequence of bunches to increase the amplitude of the wakefield. It is shown that in case of changes in the outer radius an increase in amplitude of the dielectric wave is due to the transformation of the multimode dielectric wave in a monochromatic wave, with the first harmonic, synchronous with the sequence of bunches.

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INTRODUCTION

Using plasma as an element of accelerating structure is perspective, rapidly developing direction of wakefield acceleration methods [1]. Plasma provides not only giant accelerating fields, which are unattainable in conventional accelerators, but also give an ability to focus accelerated bunch [2, 3]. If we use another drive bunch of electrons for wakefield excitation, plasma will focus this bunch too. But the area of the focusing phase limited to quarter length of plasma wave. At the boundaries of this interval accelerating field or focusing field turn to zero, so real accelerated bunches must have significantly smaller length than quarter length of excited wave.

Another type of wakefield accelerator structures, dielectric wakefield accelerator (DWA), also allows to receive the accelerating gradients, surpassing those in traditional accelerators [4 - 6]. Although these gradients are inferior than accelerating gradients in plasma wakefield structures, but DWA has it's own advantages, associated with the simplicity of manufacture, stable operating, repeatable results etc. [7]. For focusing accelerated bunches in DWA were suggested to fill drift channel by isotropic plasma with a certain density [8, 9]. In such wakefield accelerator structure acceleration is produced by a longitudinal electric field of dielectric wave, and focusing – by transverse field of a plasma wave.

In case, considered in the papers [8, 9], the plasma density was small, so the spatial period of the plasma wave was bigger than wavelength of any dielectric mode. Possibilities of increasing the amplitude of the accelerating field by increasing longitudinal electric field of plasma wave with increasing density of the plasma had been analyzed by us[1].

Main results of these investigations are the next. A case without frequency adjustment of bunch repetition and the eigen frequencies of the structure was considered. Also were reviewed cases with this frequency adjustment, when changing the plasma density, synchronously changes frequency of the first radial mode of dielectric wave.

It turned out, that the amplitude of the dielectric wave with increasing plasma density behaves differently, depending on the method of the first radial mode dielectric wave frequency adjustment, by varying the outer radius or by varying the inner radius of dielectric tube.

The observed behavior of the total field and dielectric wave field is qualitatively different from the behavior of wakefield of the plasma density in the case of a dielectric waveguide with magnetized plasma in the drift channel [10, 11], where the amplitude of the wake field increases with increasing of plasma density.

In order to understand the behavior of the amplitude of total Wakefield and its components from plasma density we continue research begun before.

Research results of depending spectral characteristics of accelerating field, excited in the plasma wakefield dielectric structure, from the plasma density are, presented in this paper.

1. STATEMENT OF THE PROBLEM

Let there beam et a waveguide with radius b, in which is inserted the dielectric tube with inner radius a, and its outer radius coincides with the radius of the metal waveguide. The dielectric constant of the tube material is ε. Dielectric tube channel (drift channel) is completely filled with an isotropic plasma with density n_p. In drift channel travels regular sequence of N_b electron bunches of cylindrical shape with uniform distribution of the charge density inside each bunch. The length of a single bunch is L_b, radius the total charge for each bunch is Q_0. Leading bunches move uniform rectilinear, and they excite wake field, which then accelerates test bunches (accelerated bunches). All such system we will call plasma-dielectric wakefield accelerator (PDWA).

2. EXPRESSIONS FOR THE FIELDS

Current density, produced by electron bunches with uniform density distribution inside them, looks as:

$$\vec{j} = \vec{z}Q_0\Theta(r_b - r)\sum_{i=1}^{N_b}\left\{\Theta[\tau-(i-1)T]-\Theta[\tau-(i-1)T-\tau_b]\right\}, \qquad (1)$$

where $\tau = t - z/v_0$; $\tau_b = L_b/v_0$; $\Theta(\tau)$ is Heaviside function; T is the repetition period of bunches; v_0 is

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[1] The results will be published somewhere: R.R. Kniaziev, I.N. Onishchenko, G.V. Sotnikov. Wakefield generation when filling dielectric structure with a plasma.

longitudinal velocity of bunch electrons; $\vec{z}$ is the unit vector along the axis of the waveguide.

Solving Maxwell's equations with the source like (1), we obtain expressions for wakefield in plasma and dielectric. Further we are interested only in the longitudinal component of wakefield E_z in drift channel. It can be shown as [8, 9]:

$$E_z = -\frac{4Q_0}{a^2}\sum_{i=1}^{N_b}\sum_{s} R_s(r_b)e_z^s(r)\Psi_{\parallel}^s\left[\tau-(i-1)T\right]- -\frac{4Q_0}{r_b L_b}e_z^p(r)\sum_{i=1}^{N_b}\Psi_{\parallel}^p\left[\tau-(i-1)T\right]. \qquad (2)$$

In equation (2) functions $e_z^{p,s}(r)$, describe the transverse field structure, and function $\Psi_{\parallel}^{p,s}(\tau)$, describe the longitudinal structure of a field:

$$e_z^s(r) = \left(\frac{a}{\omega_s D'(\omega_s)}\right)^{1/2}\frac{I_0(\kappa_p^s r)}{I_0(\kappa_p^s a)}, \qquad (3)$$

$$e_z^p(r) = \left\{\begin{array}{l} \dfrac{1}{k_p r_b} - \dfrac{I_0(k_p r)}{I_0(k_p a)}\Delta_1(k_p r_b, k_p a), r < r_b \\ \dfrac{I_1(k_p r)}{I_0(k_p a)}\Delta_0(k_p a, k_p r_b), r_b < r < a \end{array}\right\}, \qquad (4)$$

$$\Psi_{\parallel}^{p,s}(\tau) = \frac{1}{\omega_{p,s}\tau_b}\left[\sin(\omega_{p,s}\tau)\Theta(\tau) - -\sin[\omega_{p,s}(\tau-\tau_b)]\Theta(\tau-\tau_b)\right], \qquad (5)$$

designation left in (2)-(5) are:

$$R_s(r_b) = \frac{2}{\kappa_p^s r_b}e_z^s(r_b)\ ,\ k_p = \omega_p / v_0\ ,\ \beta_0 = v_0 / c\ ,$$

$$\kappa_p^s = \left[1-\beta_0^2\varepsilon_p(\omega_s)\right]^{1/2}\omega_s / v_0\ ,\ \varepsilon_p(\omega) = 1-\omega_p^2/\omega^2\ ,$$

$$\omega_p^2 = 4\pi e^2 n_p \big/ m\ ,\ D'(\omega) = dD(\omega)/d\omega\ ,$$

$$\Delta_n(x,y) = I_n(x)K_0(y) - (-1)^n K_n(x)I_0(y)\ ,$$

where -*e*, m are charge, mass of electrons; I_n and K_n - modified Bessel and Macdonald functions *n*-th order.

Eigen frequencies ω_s of dielectric waves are determined by solving the dispersion equation:

$$D(\omega_s) \equiv \frac{\varepsilon_p(\omega_s)}{\kappa_p^s}\frac{I_1(\kappa_p^s a)}{I_0(\kappa_p^s a)} + \frac{\varepsilon}{\kappa_d^s}\frac{F_1(\kappa_d^s a, \kappa_d^s b)}{F_0\left(\kappa_d^s a, \kappa_d^s b\right)} = 0\ , \qquad (6)$$

where $\kappa_d = \left(\beta_0^2\varepsilon - 1\right)^{1/2}\omega_s \big/ v_0$,

$F_n(x,y) = (-1)^n\left[J_n(x)Y_0(y) - Y_n(x)J_0(y)\right]$; J_n and Y_n - Bessel and Weber functions *n*-th order.

As we can see from (2), the longitudinal electric field consists of two parts. The first amount describes the wake field of dielectric waves, another – wake field of plasma wave. They were both excited by a relativistic electron bunch. By varying the density of the plasma the ratio between the amplitudes of plasma and dielectric waves can be changed. The excitation the two types of waves in isotropic plasma is the significant difference from case of wakefield excitation in a magnetized plasma [10, 11], where for excitation of plasma wave by electron bunch an upper limit on its energy exists. But such bunches are not of interest for the given accelerator scheme, so we can assume, that relativistic electron bunches do not excite plasma wave in a magnetized plasma waveguide.

3. NUMERICAL ANALYSIS OF WAKEFIELD SPECTRUM

To investigate the dependence of the amplitude of the longitudinal electric field in PDWA as the initial parameters of dielectric structure and bunches parameters, typical for the experimental apparatus “Almaz-2”were taken [12]: $a = 1.1$ cm, $b = 4.3$ cm, $\varepsilon = 2.1$, $r_b = 1.0$ cm, $L_b = 1.7$ cm, $Q_0 = -0.32$ nC, energy electron bunches $U_b = 5$ MeV. For such parameters the eigen frequency of the first radial mode of vacuum structure, defined by the equation (3), $f_1 = \omega_1 / 2\pi = 2710$ MHz.

With the increase of the plasma density the resonant frequency of the first radial mode dielectric wave increases, and if fix the frequency of bunch repetition, then synchronicity of bunches effect on structure will deteriorate, and the amplitude of the total wakefield will be reduced. Under total field we understand the sum of plasma wakefield and the dielectric wakefield. To prevent violations of synchronicity it was proposed to change the dimensions of the structure so the resonant frequency of the Cherenkov the first radial mode dielectric wave was equal to the plasma frequency. The repetition frequency of bunches adjust to these frequencies.

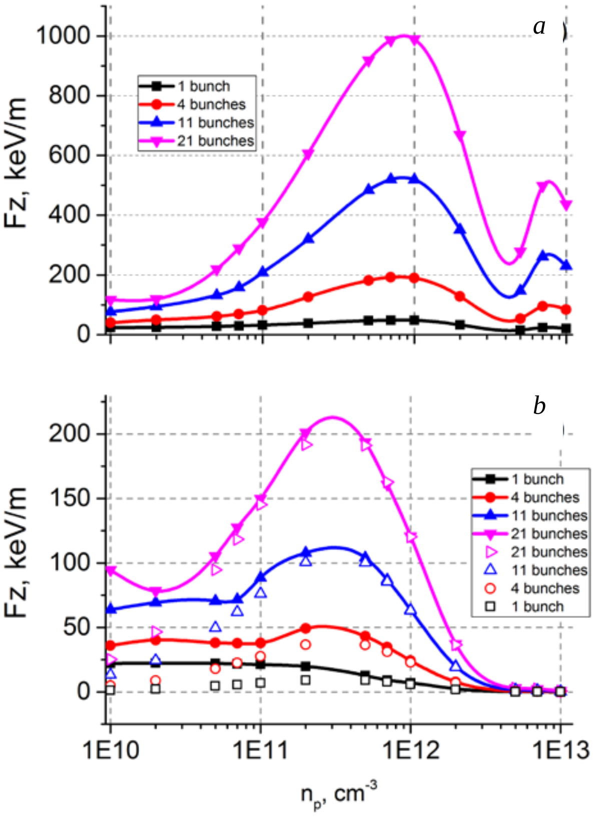

Fig. 1. Dependence of wakefield amplitude in PDWA with changing outer radius from plasma density for different numbers of bunches in a sequence ($N_b = 1,4,11,21$): total field (a); dielectric wave field (b). For comparison open symbols show the field of the first radial mode of dielectric wave

Fig. 1 shows the dependence of the amplitude of the axial force (the maximum value of the longitudinal force behind bunch on the axis of the drift channel) $F_z = -eE_z$, acting on a test electron, from the density of the plasma for different numbers of bunches in the sequence: one bunch, 4, 11 and 21. Fig. 1,a corresponds to the total wake field, Fig. 1,b – dielectric wave (first double sum in equation (2)). The values of the outer radius of the waveguide at different plasma densities are given in Table.

Outer radius b of dielectric tube in case adjusting the first radial mode dielectric wave to frequency of plasma wave

n_p, cm^{-3}	$\omega_p/2\pi$, MHz	b, cm
10^{10}	897.9	12.406
$5\cdot10^{10}$	2008	5.815
10^{11}	2839	4.31
$2\cdot10^{11}$	4015	3.281
$5\cdot10^{11}$	6349	2.4115
10^{12}	8979	1.997
$5\cdot10^{12}$	20080	1.479
10^{13}	28390	1.3638

According to Fig. 1,b, for bunch sequences the total field increases with increasing of plasma density, reaches its maximum, and then decreases. Maximum of accelerating field if $n_p \sim 10^{12}$ cm^{-3} is due to plasma wave field maximum [9]. The increase of the total field over a range of plasma densities $n_p < 3\cdot10^{11}$ cm^{-3} related as with increasing plasma wave field, and also with increasing dielectric wave field (Fig. 2,b). In same time, for a single bunch increasing of dielectric wave amplitude on the entire range of plasma densities is not seen (see black solid line on Fig. 1,b). Dielectric wave amplitude is almost constant at low plasma densities $n_p \le 2\cdot10^{11}$ cm^{-3}, then decreases with further increase of the plasma density. As shown by numerical analysis so different behavior of the amplitude of the dielectric wave in the case of structure excitation by a single bunch or by bunches sequence is due to changing spectral characteristics of excited wakefield. On Fig. 1,b amplitude value of the first radial mode of dielectric wave at different plasma densities are shown by open symbols. For a single bunch the dielectric wakefield increases with increasing of plasma density from zero and reaches its maximum in range $n_p=(2\ldots5)\cdot10^{11}$ cm^{-3}. Bunches, injected in *PDWA*, with a repetition rate equal the frequency of a first radial mode, will only reinforce this resonance mode. So the more bunches are in the chain, the more precisely the amplitude of the total field dielectric wave will approach the resonant radial mode field. Open symbols on Fig. 1,b demonstrate this clearly.

Fig. 2 shows the spectral characteristics of a total wakefield at different plasma densities. For the low plasma density, $n_p = 10^{10}$ cm^{-3} (which corresponds to the outer radius of the waveguide 12.4 cm), spectrum of wakefield is multimode. 5-th harmonic dielectric wave has maximum amplitude, amplitude of 1st harmonic is small and comparable in magnitude with the amplitude of the plasma wave.

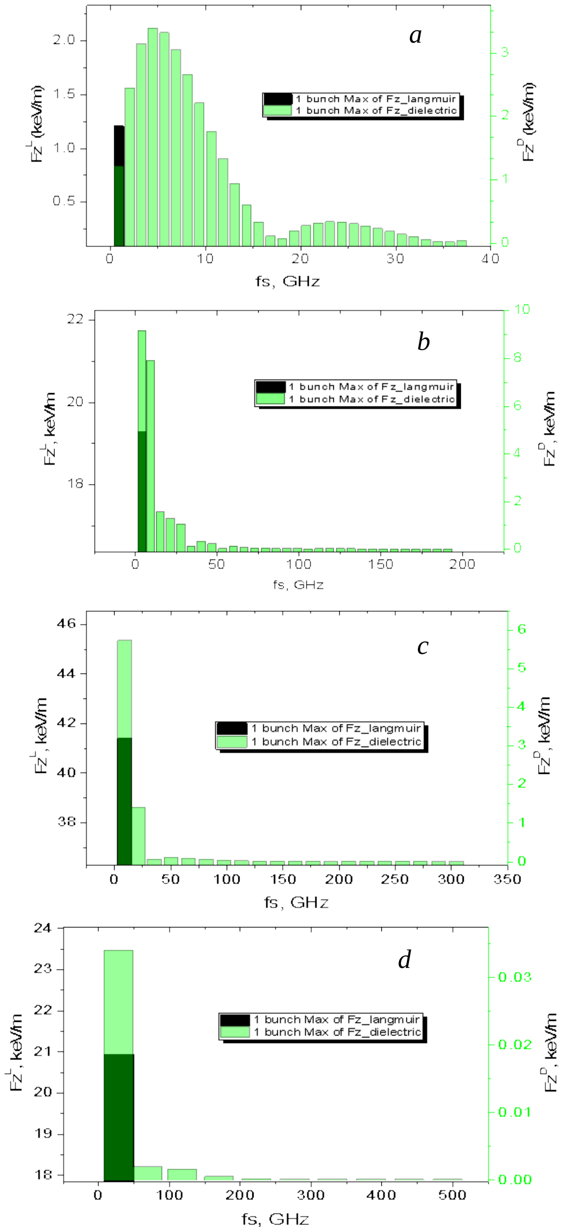

Fig. 2. Amplitudes of the wakefield harmonics for different plasma densities: $n_p=10^{10}$ cm^{-3} (a); $n_p=2\cdot10^{11}$ cm^{-3} (b); $n_p=10^{12}$cm^{-3} (c); $n_p=10^{13}$ cm^{-3} (d). Light green squares mark dielectric waves amplitudes (right scale), black – plasma wave amplitude (left scale). The outer radius of the dielectric tube is changed in accordance with the Table

Note that due to the slow decay of the harmonic amplitudes with high number in the calculation of the total field we used 30 harmonic of dielectric wave. With the increase of the plasma density the spectrum of the excited oscillations is narrowed (in a relative meaning) and shifted to lower harmonic numbers. For example, for the density $n_p = 2\cdot10^{11}$ cm^{-3} (see Fig. 2,b) the first harmonic becomes dominant and major contribution to the total field make plasma wave and first and second radial modes of dielectric wave. When the plasma density is

$n_p = 10^{12}$ cm^{-3} (see Fig. 2,c), when the maximum of the total of the accelerating field, plasma wave becomes the predominant, in dielectric modes notable importance has only the first mode of dielectric wave. And if the plasma density is $n_p = 10^{13}$ cm^{-3} (see Fig. 2,d) in the spectrum of the total field is presented only plasma wave.

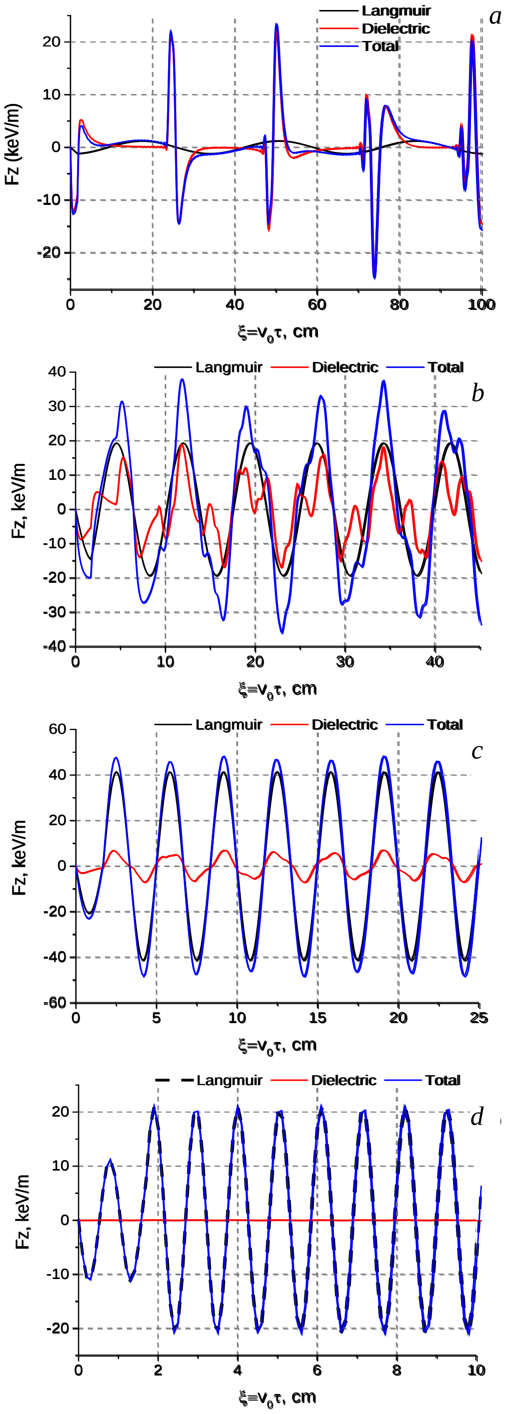

Fig. 3. The time dependence of wakefield for the same plasma densities, that on Fig. 2. Line ‘Total’ notes the total field, ‘Dielectric’ - dielectric wave field, ‘Langmuir’ - plasma wave field

The axial wakefield structure for the same plasma densities, as for Fig. 2 the spectral characteristics are given, is shown in Fig. 3. When the plasma density $n_p = 10^{10}$ cm^{-3} distribution has “spiking” character, the amplitude of plasma wave is very small. But when the density reaches $n_p = 2 \cdot 10^{11}$ cm^{-3} the plasma wave amplitude coincides with dielectric wave amplitude. When the plasma density $n_p = 10^{12}$ cm^{-3} amplitude of plasma wave much higher than dielectric waves amplitude, axial distribution of which is already close to monochromatic. When the plasma density $n_p = 10^{13}$ cm^{-3} only plasma wave is excited.

Here is the spectra wakefield for the other two options *PDWA*, wakefield amplitude are investigated in [10]. In Fig. 4 is shown the spectra wakefield for case when frequency of the first radial mode is adjusted to the repetition frequency of bunches by changing inner radius of dielectric tube. With increasing plasma density the radius of the drift of the channel should be increased, so with increasing plasma density dielectric wave amplitude decreases rapidly and the total field is mainly determined by the field of a plasma wave.

This trend is clearly confirmed by comparing the spectra of wakefield, shown on Figs. 4,a,b for two characteristic plasma densities: $n_p = 10^{11}$ and $5 \cdot 10^{11}$ cm^{-3}.

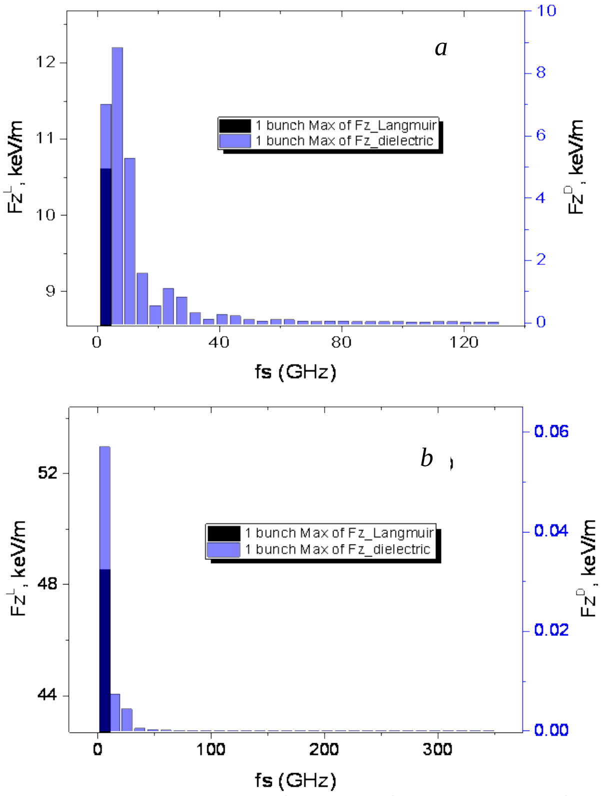

Fig. 4. The amplitudes of the wakefield harmonics for plasma densities $n_p = 10^{11}$ cm^{-3} (a) and $n_p = 5 \cdot 10^{11}$ cm^{-3} (b) in the case of changing the inner radius of dielectric sleeve; inner radius a=1.085 and 3.094 cm. Blue rectangles mark the dielectric wave amplitude (right scale); black – plasma wave amplitude (left scale)

Distribution of wake field behind a bunch of two plasma densities, which oscillation spectrum is shown in Fig. 4, is shown in Fig. 5. For relatively low plasma density $n_p = 10^{11}$ cm^{-3} plasma wave amplitude and dielectric wave field amplitude same order. Although the total field has irregular behavior, but the period of plasma wave is approximately equal to period of the dielectric wavefield. When using a sequence of bunches they will be amplified synchronous.

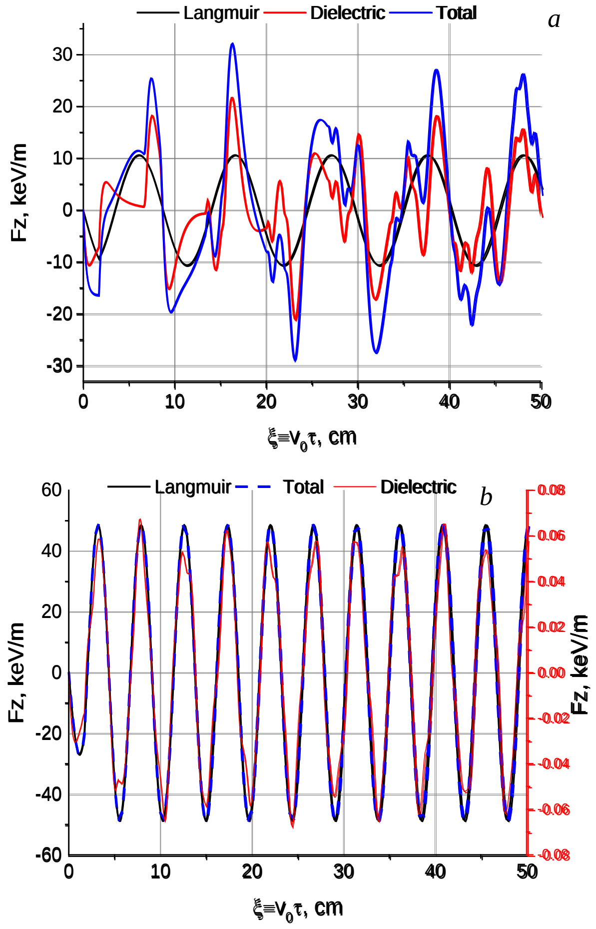

Fig. 5. The time dependence of wakefield for the case of change in the radius of the drift channel. The plasma density and radius of the channel as shown in caption of Fig. 4

Third variant of *PDWA* – without the use of any method of adjusting the eigen frequencies. The parameters of the dielectric structure and the bunch are fixed, only plasma density is changed. In Fig. 6 are given axial distributions of the total field and its components (see designation as in Fig. 3) for three values of the density of the plasma. As it follows from Fig. 6,b for plasma density $n_p = 10^{11}\,\mathrm{cm}^3$ plasma and dielectric waves are same, that increases the amplitude of the total field. When the plasma density is $n_p = 10^{12}\,\mathrm{cm}^3$ the longitudinal structure of the field is almost entirely determined by the plasma wave.

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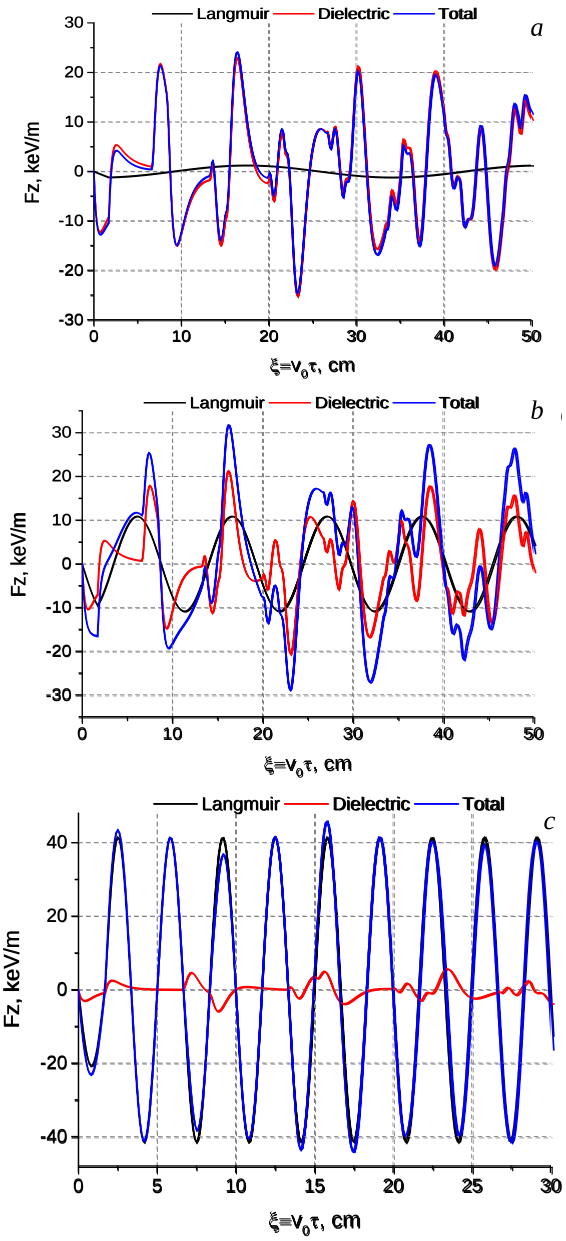

Fig. 6. The time dependence of wakefield in the case with fixed parameters PDWA except plasma density: $n_p=10^{10}$ *cm*$^{-3}$ *(a);* $n_p=10^{11}$ *cm*$^{-3}$ *(b);* $n_p=10^{12}$ *cm*$^{-3}$ *(c)*

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СПЕКТРЫ КИЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМЕННО-ДИЭЛЕКТРИЧЕСКОМ УСКОРИТЕЛЕ ПРИ ИЗМЕНЕНИИ ПЛОТНОСТИ ПЛАЗМЫ

Р.Р. Князев, П.И. Марков, И.Н. Онищенко, Г.В. Сотников

Проведен анализ изменения спектров кильватерного поля, возбуждаемого в цилиндрическом плазменно-диэлектрическом волноводе релятивистскими электронными сгустками. Рассмотрены три варианта структур: параметры диэлектрической структуры и сгустков фиксированы; внутренний или внешний радиусы диэлектрической трубки изменяются так, что частота первой радиальной моды совпадает с частотой плазменной волны. Последние два варианта необходимы в случае использования регулярной последовательности сгустков для увеличения амплитуды кильватерного поля. Показано, что в случае изменения внешнего радиуса рост амплитуды диэлектрической волны связан с трансформацией многомодового спектра диэлектрической волны в одномодовый с первой гармоникой, синхронной с последовательностью сгустков.

СПЕКТРИ КІЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМОВО-ДІЕЛЕКТРИЧНОМУ ПРИСКОРЮВАЧІ ПРИ ЗМІНІ ГУСТИНИ ПЛАЗМИ

Р.Р. Князєв, П.І. Марков, І.М. Оніщенко, Г.В. Сотніков

Проведений аналіз зміни спектрів кільватерного поля, збуджуваного в циліндричному плазмово-діелектричному хвилеводі релятивістськими електронними згустками. Розглянуто три варіанти структур: параметри діелектричної структури й згустків фіксовані; внутрішній або зовнішній радіуси діелектричної трубки змінюються так, що частота першої радіальної моди збігається із частотою плазмової хвилі. Останні два варіанти необхідні у випадку використання регулярної послідовності згустків для збільшення амплітуди кільватерного поля. Показано, що у випадку зміни зовнішнього радіуса ріст амплітуди діелектричної хвилі пов'язаний із трансформацією багатомодового спектра діелектричної хвилі в одномодовий з першою гармонікою, яка синхронна з послідовністю згустків.