

HE- AND EH-HYBRID WAVES IN A CIRCULAR DIELECTRIC WAVEGUIDE WITH AN ANISOTROPIC IMPEDANCE SURFACE

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The boundary-value problem for a circular dielectric waveguide with an anisotropic impedance boundary is reduced to two independent problems. Thus the hybrid waves of different types are shown to be uncoupled in such waveguide. Analytical expression is obtained for the parameter defining relative contributions of TE- and TM-polarizations to the field of a hybrid wave. Its analysis shows that hybrid waves of the one type are nearly always TE-like waves. By this criterion, they are classified as HE-waves. In contrast, waves of another type are shown to have TM-like polarization and represent EH-waves. Dispersion equation and some of its properties are presented for hybrid waves of each type.

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INTRODUCTION

Waveguide with impedance surface is a widespread approximation effectively used to describe a broad class of waveguides. Among them are the waveguides with imperfectly conducting walls [1], the waveguides with artificial hard and soft electromagnetic boundaries [2], the waveguides with the walls made of artificial magnetic conductor [3], dielectric-coated metallic waveguides [4], helical waveguides [5], the waveguides with tensor surface impedance [6], and others. In the general case the waves in such waveguides are the hybrid waves of two types. Traditionally, they are called HE and EH waves.

Classification of hybrid waves, as a rule, is based on their behavior in one or more limiting cases. Often it uses the fact that hybrid waves of the first type (e.g. HE waves) in some limit transform to TE waves, while waves of the second type (e.g. EH waves) become TM waves. However, firstly, such limits not always exist and, secondly, behavior of HE and EH waves could be diametrically opposite [4] in different limiting cases.

Therefore, up to now there has been no strict classification of hybrid waves in waveguides with impedance walls. The main objective of the study is to identify a criterion for such classification in a circular dielectric waveguide with an anisotropic impedance boundary

1. DIFFERENT APPROACHES TO CLASSIFICATION OF HYBRID WAVES IN A WAVEGUIDE WITH IMPEDANCE SURFACE

In this section we consider waves in a waveguide of arbitrary cross-section for generality. The waveguide is assumed to be uniform along z -axis and filled completely with isotropic medium characterized by permittivity ε (dielectric, metamaterial, plasma in zero magnetic field, etc.). Its wall impedance is anisotropic.

The waveguide fields are taken as $A(\mathbf{r}, t) = A(\mathbf{r}_\perp) \exp(-i\omega t + ik_z z)$. Then Maxwell's equations yield the wave equations

$$(\Delta_\perp + k_\perp^2) B'_z = 0, \quad (1a)$$

$$(\Delta_\perp + k_\perp^2) E'_z = 0 \quad (1b)$$

for the longitudinal components of magnetic B_z and electric E_z fields coupled together through the boundary conditions on the contour C of the waveguide cross-section [7]

$$\left. \frac{\partial B'_z}{\partial n} + a B'_z + i \frac{k_z}{\sqrt{\varepsilon k}} \frac{\partial E'_z}{\partial s} \right|_C = 0, \quad (2a)$$

$$\left. \frac{\partial E'_z}{\partial n} + c E'_z + i \frac{k_z}{\sqrt{\varepsilon k}} \frac{\partial B'_z}{\partial s} \right|_C = 0, \quad (2b)$$

where $E'_z = iE_z$, $B'_z = B_z/\sqrt{\varepsilon}$; Δ_\perp is the transverse part of Laplace operator; ω is the wave frequency; k_z and $k_\perp = \sqrt{\varepsilon k^2 - k_z^2}$ are the longitudinal and the transverse wavenumbers; k is the wave vector in free space; $a = c\eta_s\eta_z\varepsilon = -ik_\perp^2\eta_s k^{-1}$; η_z and η_s are the wall impedances in \mathbf{z} and \mathbf{s} directions, respectively, vector \mathbf{s} is directed along the contour C , vector \mathbf{n} is the outward normal to C (i.e. directed deep into the walls), vectors \mathbf{n} , \mathbf{s} and \mathbf{z} form a right-hand triple.

From (2) it follows that the waves under study are hybrid ($E_z \neq 0$ and $B_z \neq 0$) in the general case. In principle, pure TE ($E_z = 0, B_z \neq 0$) and TM ($B_z = 0, E_z \neq 0$) waves are also possible if their fields are independent of variable s (see (2)).

Let us introduce parameter $P = E'_z/B'_z$ [8] to define relative contributions of E'_z (TM polarization) and B'_z (TE polarization) to the total field of a hybrid wave. Here E'_z and B'_z mean their peak values in the case when P depends on \mathbf{r}_\perp . Obviously $P = 0$ and $P = \infty$ for pure TE and TM waves, respectively. Therefore, hybrid waves will be called TE-like (TM-like) waves, if their fields satisfy inequality $|P| < 1$ ($|P| > 1$). In the borderline case $|P| = 1$ ($|E_z/B_z| = 1/\sqrt{\varepsilon}$) polarization of a hybrid wave represent equally-weighted mixture of TE and TM polarizations. Therefore, such waves will be called maximally hybrid.

Traditionally, hybrid waves are classified as HE and EH waves. To identify whether a hybrid wave is HE or EH wave, it is necessary to know its behavior in some

limiting cases, when equations (1) and (2) reduce to separate boundary-value problems. Among such cases are limits

- (a) $k_z \rightarrow 0$,
- (b) $\eta_z \rightarrow 0$,
- (c) $\eta_s \rightarrow \infty$.

In these cases, hybrid HE (EH) waves must transform to TE (TM) waves by the prevalent definition [9].

Besides, in the limiting cases

- (d) $\eta_s \rightarrow 0, \eta_z \rightarrow \infty$,
- (e) $k_z \rightarrow \infty$, when $\lim_{k_z \rightarrow \infty} \sqrt{\epsilon k}/k_z = 1$,

hybrid waves of different types also satisfy independent boundary-value problems

$$(\Delta_{\perp} + k_{\perp}^2)\Psi = 0, \quad \left. \frac{\partial \Psi}{\partial n} - i \frac{k_z}{\sqrt{\epsilon k}} \frac{\partial \Psi}{\partial s} \right|_C = 0, \quad (3a)$$

$$(\Delta_{\perp} + k_{\perp}^2)\Phi = 0, \quad \left. \frac{\partial \Phi}{\partial n} + i \frac{k_z}{\sqrt{\epsilon k}} \frac{\partial \Phi}{\partial s} \right|_C = 0, \quad (3b)$$

where $\Psi = B'_z - E'_z$, $\Phi = B'_z + E'_z$.

Hence it follows that such hybrid waves differ in fields according to the conditions $\Phi = 0$ ($P = -1$) and $\Psi = 0$ ($P = +1$). It is possible to identify these waves as EH or HE waves depending on the sign of parameter P . Such mode classification is often used, for example, in dielectric waveguides [8]. However, in this case a contradiction between (a)-(c) and (d), (e) appears. As shown in [4], hybrid waves satisfying conditions $P = -1$ ($P = +1$) in the case (d) (or (e)) can transit to either TE (TM) or TM (TE) waves in the cases (a)-(c) depending on the path of such transition. This will be additionally shown below. Therefore, sign of P in extreme case (d) (or (e)) is inappropriate criterion for mode classification in a waveguide with impedance boundary.

The rest of limits (a)-(c) sometimes give no information about the type of hybrid waves. Such a situation will be also demonstrated below. In particular, limiting process (b) (or (c)) are useless for waves with no analog in the waveguide under condition $\eta_z = 0$ (or $\eta_s^{-1} = 0$). Besides, the problem (1) and (2) may not have solutions at $k_z = 0$ or such solutions may be absent for a number of hybrid waves. This renders the limit (a) unsuitable for revealing the type of such waves.

Thus, there is no appropriate criterion suitable for revealing the type of hybrid waves regardless their frequencies $\omega(k_z)$ and waveguide parameters. For a circular dielectric waveguide with impedance wall such a criterion will be presented below.

2. FIELDS OF HE- AND EH-WAVES IN A CIRCULAR WAVEGUIDE WITH AN ANISOTROPIC IMPEDANCE BOUNDARY

We will show that the problem (1) and (2) reduces to independent boundary-value problems for HE and EH waves in the case of a circular waveguide with an anisotropic impedance boundary. In this case $A(\mathbf{r}_{\perp}) = A(r) \exp(i l \phi)$ and operator $\partial/\partial s$ in (2) could be eliminated by the change $\partial/\partial s \rightarrow il/R$, where

$\mathbf{r}_{\perp} = \{r, \phi\}$ are the polar coordinates, l is the azimuth wavenumber, R is the waveguide radius.

It is essential that parameter P in a circular waveguide is independent of \mathbf{r}_{\perp} . Thus, to satisfy both boundary conditions (2) simultaneously, this parameter must take one of the following values:

$$P_{1,2} = \alpha \left(1 \mp \sqrt{1 + \alpha^{-2}} \right) = (\lambda_{1,2} - a)/b = b/(\lambda_{1,2} - c), \quad (4)$$

where $\alpha = (c - a)/(2b)$, $b = -lk_z/(\sqrt{\epsilon k R})$,

$\lambda_{1,2} = (a + c)/2 \pm (a - c)/2\sqrt{1 + \alpha^{-2}}$. It is notable that equality $P_1 P_2 = -1$ holds for the fixed values of a , b and c .

The coupled conditions (2) for B'_z , and E'_z with (4) reduce to independent boundary conditions

$$\left[\frac{\partial B'_z}{\partial r} + \lambda_1 B'_z \right]_{r=R} = 0, \quad (5a)$$

$$\left[\frac{\partial E'_z}{\partial r} + \lambda_2 E'_z \right]_{r=R} = 0, \quad (5b)$$

and corresponding relations

$$E'_z = P_1 B'_z, \quad (6a)$$

$$B'_z = P_2^{-1} E'_z. \quad (6b)$$

Uncoupled wave equations (1) and boundary conditions (5) represent independent boundary-value problems for two types of hybrid waves. Waves of the first and the second type satisfy conditions $P = P_1$ ($\lambda = \lambda_1$) and $P = P_2$ ($\lambda = \lambda_2$), respectively, and transform to TE ($P_1 = 0$) and TM ($P_2 = \infty$) waves at $\lambda_1 = a$ and $\lambda_2 = c$. Such transformation takes place in the cases (a)-(c) and dictates the choice of roots for λ and P in (5) and (6), respectively. Therefore, in accordance with mode classification in [9], waves of the first (second) type may be called HE (EH) waves. Hereinafter, their characteristics are specified by the subscript "1" ("2").

It is remarkable that parameter P for a hybrid wave depends solely on the quantity α . This quantity incorporates wave characteristics as well as waveguide parameters. Thus it is possible to examine P (Fig. 1) with respect to all possible values of wave frequency $\omega(k_z)$, azimuth index l , waveguide radius R , permittivity ϵ , wall impedances η_z and $\eta_s = \eta_{\phi}$.

As is seen from Fig. 1, HE waves in a circular waveguide with impedance boundary are nearly always TE-like waves ($|P| < 1$). By contrast, EH waves nearly always have TM-like polarization ($|P| > 1$). The only exceptions are the extreme cases (see, for example, (d) and (e)) when α takes imaginary values within the range $[-i, i]$. In these cases the HE and EH waves become maximally hybrid.

Thus fields always satisfy condition $|E_z/B_z| \leq 1/\sqrt{\epsilon}$ for HE waves and condition $|E_z/B_z| \geq 1/\sqrt{\epsilon}$ for EH waves. Therefore, the field ratio $|E_z/B_z|$ for a hybrid

wave in a circular waveguide with impedance boundary always indicates its type.

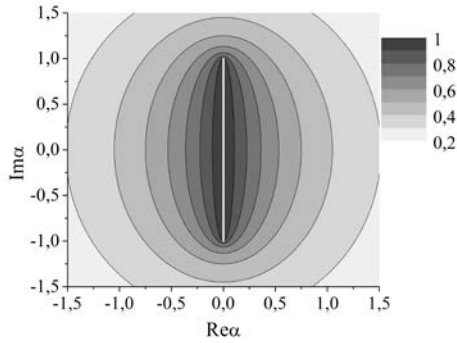


Fig. 1. Contours of $|P_1|$ and $|P_2^{-1}|$ in the plane $(\text{Re } \alpha, \text{Im } \alpha)$. Unit values are shown by the white line

3. FREQUENCIES OF HE- AND EH-HYBRID WAVES

To derive the dispersion equations for HE- and EH-hybrid waves, we substitute the general solutions of equations (1)

$$B'_z = A_1 J_l(k_\perp r), \quad (7a)$$

$$E'_z = A_2 J_l(k_\perp r), \quad (7b)$$

with independent amplitudes A_1 and A_2 into boundary conditions (5), respectively. This gives

$$k_1 J'_l(k_1 R) + \lambda_1 J_l(k_1 R) = 0, \quad (8a)$$

$$k_2 J'_l(k_2 R) + \lambda_2 J_l(k_2 R) = 0, \quad (8b)$$

where $J_l(x)$ is the l -th order Bessel function, $J'_l(x) = dJ_l(x)/dx$, k_1 and k_2 mean the values of the transverse wavenumber k_\perp for HE- and EH-waves, respectively.

It should be noted that separate equations (8a) and (8b) could be derived directly [4] from the general dispersion equation for the hybrid waves of both types. The last-named equation is well-known and can be found in the literature [10 - 12], including recent papers [13, 14].

Meanwhile, separate dispersion equations (8a) and (8b) for HE- and EH- hybrid waves and their analysis are much easier. From this analysis it follows:

(a*) with the change of surface impedances $\eta_\varphi \rightleftharpoons (\varepsilon \eta_z)^{-1}$ ($a \rightleftharpoons c$), accompanied by the change of coefficients $\lambda_1 \rightleftharpoons \lambda_2$ in (8), the dispersion curves of HE- and EH-waves exchange places with one another. Thus frequencies $\omega(k_z)$ of HE- (EH-) waves at some $\eta_\varphi = \eta_1$ and $\eta_z = \eta_2$ coincide with frequencies of EH- (HE-) waves in the same waveguide with modified wall impedances $\eta_\varphi = (\varepsilon \eta_2)^{-1}$, and $\eta_z = (\varepsilon \eta_1)^{-1}$. Such a conclusion can also be drawn directly from (1) and (2).

(b*) in the absence of any losses, the dispersion equations (8) with real coefficients λ_1 and λ_2 have only real roots [15, 16]. The exception is the case, when condition $\lambda_1 R + |l| < 0$ (or $\lambda_2 R + |l| < 0$) holds true. In this case the first (or the second) equation in (8) has a single purely imaginary root [15] associated with a surface HE- (or EH-) wave.

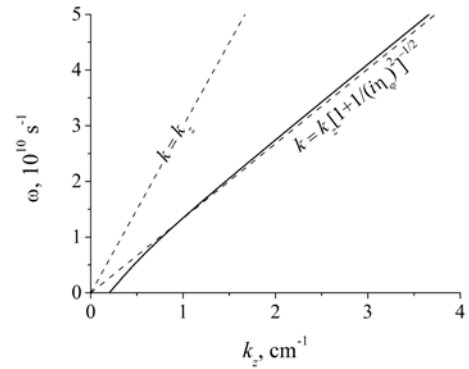


Fig. 2. Dispersion curve of the surface HE_1 ($l=1$) wave at $\varepsilon=1$, $i\eta_\varphi = -0.5$, $i\eta_z = 0.5$, $R=5$ cm

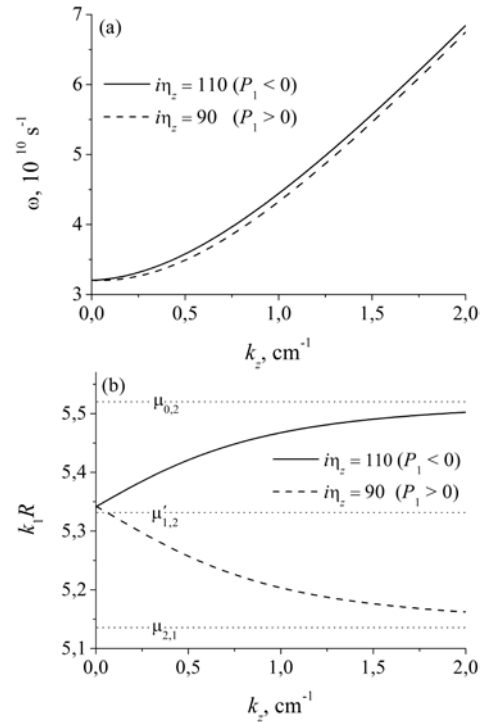


Fig. 3. Frequencies $\omega(k_z)$ of $HE_{1,2}$ ($l=1$, $s=2$) wave at $\varepsilon=1$, $R=5$ cm, $i\eta_\varphi = -0.01$ and different values of $i\eta_z$ (a), and their respective values of $k_1 R$ (b). Here $\mu_{l,s}$ and $\mu'_{l,s}$ are the s -th zeroes of the functions $J_l(x)$ and $J'_l(x)$, respectively

(c*) the surface HE wave exists under condition $i\eta_\varphi < 0$ [7]. In the limit $k_z \rightarrow \infty$ it transforms to purely TE wave with asymptotic frequency $k = k_z / \sqrt{\varepsilon + (i\eta_\varphi)^{-2}}$. Hence it follows that the dispersion curve of this wave degenerates into line $k=0$ or $k = k_z / \sqrt{\varepsilon}$ (so-called quasi-TEM wave [17]) in the limiting case $\eta_\varphi \rightarrow 0$ or $\eta_\varphi \rightarrow \infty$, respectively. As a consequence surface HE wave has no analog among TE- and TM-eigenwaves of a circular waveguide with $\eta_\varphi = 0$ or $\eta_\varphi = \infty$. Besides, this wave lacks cutoff frequency at some values of η_φ and η_z (Fig. 2). This exemplifies the case when limiting processes (a)-(c) (as well as (d) and (e)) give no information about the type of hybrid wave. Similar conclusions can be drawn for a

surface EH-wave using change $\eta_\phi \rightarrow (\epsilon\eta_z)^{-1}$ and results of Item (a*).

(d*) the sign of real (or imaginary) part of P does not indicate the type of a hybrid wave. This is shown in Fig. 3, where the dispersion curve of the chosen HE wave is presented in two cases. In the first case $\alpha > 0$. This provides negative value of P_1 , which varies from 0 to -1 with increasing k_z . The value of k_1R therewith tends to the zero of function $J_{l-1}(x)$. In the second case $\alpha < 0$ and thus $P_1 > 0$. As a result, increase in k_z changes P_1 in the range from 0 to +1. Coincidentally with this change, the value of k_1R tends to the zero of function $J_{l+1}(x)$. It should be noted that the sign of P also changes with substitution $b \rightarrow -b$ (e.g. $l \rightarrow -l$). As seen from (1) and (2) such a substitution is equivalent to the change $E_z \rightarrow -E_z$.

(e*) independent dispersion equations (8) can have joint solutions. This requires the relation $\lambda_1 = \lambda_2$ ($\alpha = \pm i$) to be valid. Thus the condition $P = \pm i$ also holds true. Therefore, intersection points for the dispersion curves of HE and EH-waves, once they exist, must correspond to maximally hybrid waves.

CONCLUSIONS

Two independent boundary-value problems have been obtained for a circular waveguide with an anisotropic impedance boundary and dielectric filling. This shows that HE and EH hybrid waves are always uncoupled in such waveguide. Such situation is the exception rather than rule for waveguides supporting hybrid waves. As an example, the reverse is generally true for hybrid waves in a circular waveguide with gyroelectric filling [18, 19] or corrugated wall [20, 21].

An analytical expression has been obtained for parameter $P = i\sqrt{\epsilon} E_z/B_z$ defining relative amounts of the axial field components. Its value has been studied for all possible characteristics of hybrid waves and waveguide. Regardless of these characteristics, HE- and EH-waves have been demonstrated to be mostly TE-like ($|P| < 1$) and TM-like ($|P| > 1$) waves, respectively. Hence it follows that the field ratio $|E_z/B_z|$ always indicates the type of hybrid wave and thus can be applied for mode classification in a dielectric circular waveguide with impedance boundary. The exceptions are some extreme cases when both HE- and EH-waves become maximally hybrid ($|P| = 1$). In the strict sense, such waves are indistinguishable without additional information about their behavior in the vicinities of the exceptional points.

Dispersion equations have been found separately for HE and EH waves. Some of their dispersion properties have been studied in details.

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НЕ- И ЕН-ГИБРИДНЫЕ ВОЛНЫ В КРУГЛОМ ДИЭЛЕКТРИЧЕСКОМ ВОЛНОВОДЕ С АНИЗОТРОПНОЙ ИМПЕДАНСНОЙ ПОВЕРХНОСТЬЮ

В.И. Щербинин, Г.И. Загинайлов, В.И. Ткаченко

Краевая задача для круглого диэлектрического волновода с анизотропной импедансной границей сведена к двум независимым задачам. Тем самым показано, что в рассматриваемом волноводе гибридные волны различных типов являются несвязанными. Получено аналитическое выражение для параметра, определяющего относительный вклад ТЕ- и ТМ-поляризации в поле гибридной волны. Из его анализа следует, что гибридные волны одного типа практически всегда являются преимущественно ТЕ-волнами. По данному признаку они относятся к НЕ-волны. В противоположность этому, волны другого типа обладают преимущественно ТМ-поляризацией и представляют собой ЕН-волны. Дисперсионное уравнение и некоторые его свойства представлены для гибридных волн каждого типа.

НЕ- ТА ЕН-ГИБРИДНІ ХВИЛІ В КРУГЛОМУ ДІЕЛЕКТРИЧНОМУ ХВИЛЕВОДІ З АНІЗОТРОПНОЮ ІМПЕДАНСНОЮ ПОВЕРХНЕЮ

В.І. Щербінін, Г.І. Загинайлов, В.І. Ткаченко

Крайову задачу для круглого діелектричного хвилеводу з анізотропною імпедансною границею було зведено до двох незалежних задач. Тим самим було показано, що в розглядуваному хвилеводі гібридні хвилі різних типів є нез'язаними. Було отримано аналітичний вираз для параметра, що визначає відносний внесок ТЕ- та ТМ-поляризацій в полі гібридної хвилі. З його аналізу виходить, що гібридні хвилі одного типу практично завжди є переважно ТЕ-хвилями. За цією ознакою вони належать до НЕ-хвиль. На противагу цьому, хвилі іншого типу мають переважно ТМ-поляризацію та являють собою ЕН-хвилі. Дисперсійне рівняння та деякі його властивості наведені для гібридних хвиль кожного типу.