COMPUTER SIMULATION OF THE STOCHASTIC DYNAMICS IN SYSTEM OF TWO COUPLED NONLINEAR OSCILLATORS

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Dynamics of two coupled nonlinear oscillators has been studied. Oscillators are represented by conservative LC circuits with nonlinear capacitors. The system exhibits chaotic behavior in some range of initial conditions. Chaotic behavior is observed in one circuit if high amplitude oscillations are performed in the other circuit due to the appropriate initial conditions. Stochasticity of the oscillations is proved by the continuity of their spectrum and by the Poincare mapping.

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INTRODUCTION

Electrical noise generators are widely used in modern radio electronics so further study of this problem is actual. On the other hand investigation of chaotic processes in low dimension systems remains crucial interdisciplinary problem since their appearance is studied insufficiently.

In present paper we discuss computer simulation of chaotic behavior of two coupled nonlinear oscillators presented by LC circuits with nonlinear capacitors. The study of such system anticipates investigation of a similar coupled system of self-generators.

This approach demonstrates the transition from conservative to dissipative systems with chaotic behavior.

1. SYSTEM UNDER STUDY

Let us consider the system of two coupled LC circuits. Coupling is performed by linear capacitor. Capacitors in LC circuits are nonlinear. Considered system is shown on Fig. 1.

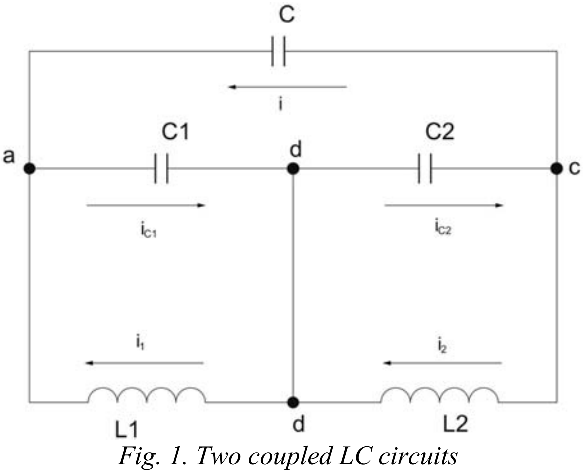


Fig. 1. Two coupled LC circuits

Capacitor's nonlinearity is defined as:

$$C_{1,2}(U) = C_{10,20}\left(1+\alpha U^2\right). \qquad (1)$$

U is voltage drop on capacitor. System behavior is described by a system of equations obtained from Kirchhoff`s circuit laws. Performing simple transformations and leave out current trough the coupling capacitor C and voltage drops on capacitors C1 and C2 we obtain system of differential equations for currents trough the inductors L1 and L2:

$$\begin{cases} i_1 + L\left\{C + C_{10}\left[1+3\alpha L^2\left(\frac{d i_1}{d t}\right)^2\right]\right\}\frac{d^2 i_1}{d t^2} + LC\frac{d^2 i_2}{d t^2} = 0; \\ i_2 + L\left\{C + C_{20}\left[1+3\alpha L^2\left(\frac{d i_2}{d t}\right)^2\right]\right\}\frac{d^2 i_2}{d t^2} + LC\frac{d^2 i_1}{d t^2} = 0 \end{cases}, \qquad (2)$$

we assume $C_{10} = C_{20} = C_0$, $L_1 = L_2 = L$. Since currents trough inductors and their time derivatives are phase variables (2) describes behavior of the studied system.

2. NUMERICAL RESULTS

System (2) was numerically integrated with Wolfram Mathematica program. Initial conditions were set to nonzero voltage drop on capacitor and zero current trough inductor in the leading circuit. Electrical circuit was also simulated with OrCAD package. In thre last case oscillations were started by step voltage generator. If difference in initial conditions is taken into consideration results of simulation types are consistent.

There are no nonlinear effects in the system if initial voltage is small (0.1 V). One can see from Fig. 2 that there are two different frequencies in the spectrum of oscillations. This is explained by the eigenfrequency repulsion phenomenon.

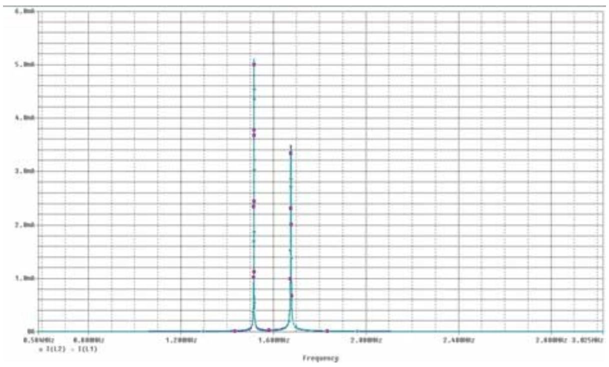


Fig. 2. The oscillations' spectrum for the linear case

Quasi-periodic oscillations are observed for 5…9.8 V initial voltage (Figs. 3-5). For 25 V oscillations in first circuit stays periodic. These oscillations play the role of the external force applied to a second driven oscillator. Driven oscillator`s behavior appears chaotic, its oscillations spectrum is continuous.

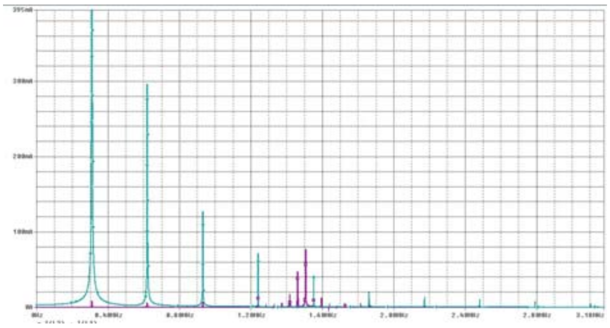


Fig. 3. Spectra of oscillations *in the leading and driven circuits for the quasiperiodic mode*

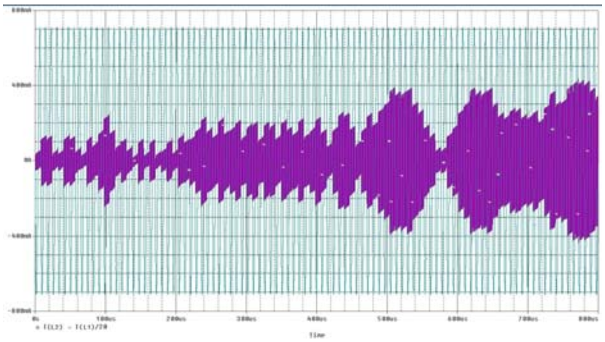
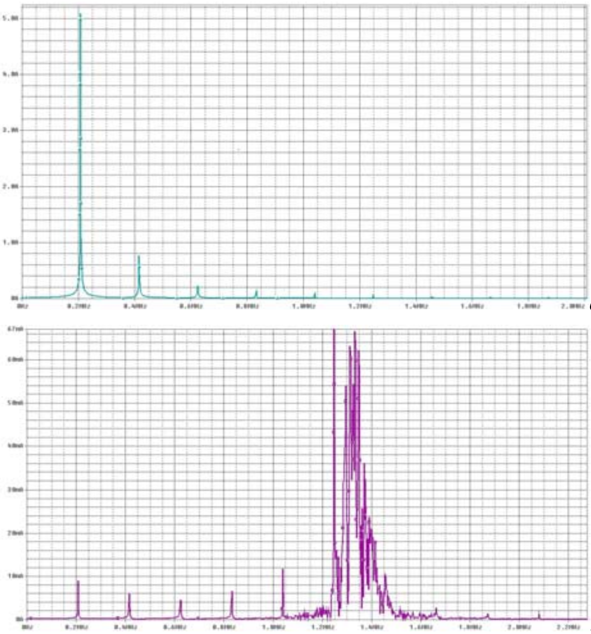


Fig. 4.Currents trough inductors versus time



a

b

Fig. 5. Spectra of currents trough inductors in the leading (a) and driven (b) circuits for the stochastic mode

Furthermore Poincare map can also confirm the system stochasticity (Fig. 6). For 5 V the points lay regularly, but for 25 V they are situated accidentally.

Let us consider two initial conditions that slightly differ from each other (in the seventh order of value). Projections of their phase trajectories on the phase variables of the driven circuit are shown on Fig. 7. So the small divergence increases during the evolution of the system. It`s turn out that behavior of the system is unpredictable.

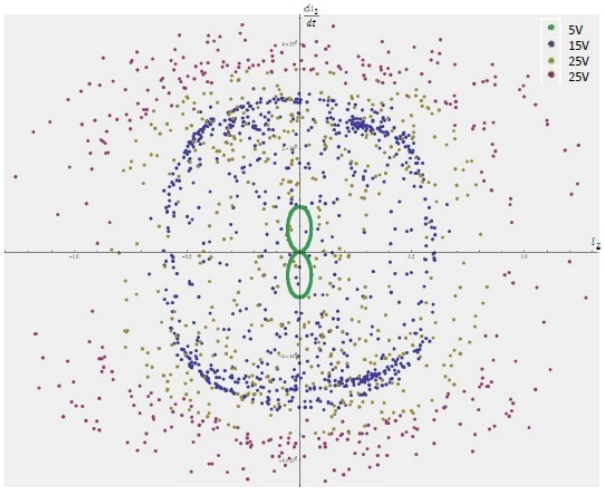


Fig. 6. Poincare map for different initial conditions

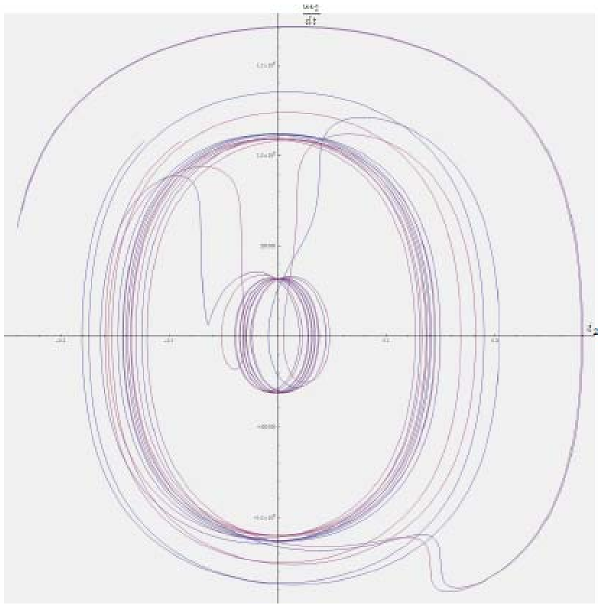


Fig. 7. Divergence of two projections of phase trajectories

3. DISCUSSION OF OBTAINED RESULTS

The problem of nonlinear oscillator`s dynamic with applied outer force is well-known. In the considered system the leading circuit becomes a source of such force that is applied to the driven oscillator. As evident from (2) this force appears to be a second time derivative of the leading circuit inductor`s current. Time dependence of leading circuit variables are shown on Fig. 8 (brown, red and blue represent current trough inductor, time derivative of current trough inductor in the power of two and second time derivative of current trough inductor correspondingly).

It turns out that second time derivative of the leading circuit inductor`s current is a periodic sequence of short narrow impulses. So the dynamic of driven circuit is governed by such sequence. Oscillations of driven circuit shift under influence of the sequence of short impulses from one mode of stable amplitude to another. Such dynamic still could be quasi-periodical. But in case of high amplitude impulses nonlinearity of the driven circuit start to affect oscillations of phase variables of the driven circuit become chaotic.

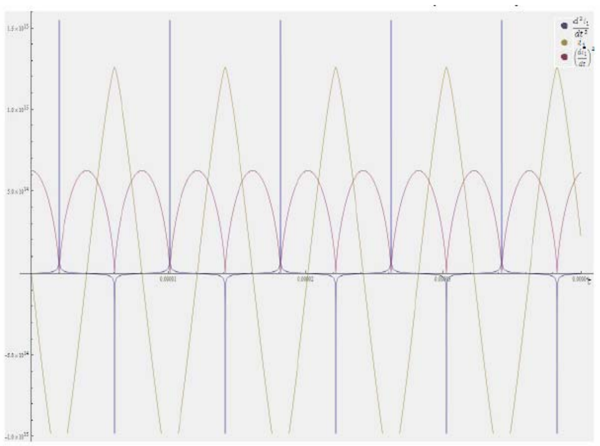


Fig. 8. Variables of the leading circuit

CONCLUSIONS

Obtained results are in well consistency with a well-known problem of nonlinear oscillator`s dynamics with applied periodical outer force. In this paper outer force is represented by nonlinear oscillations in first circuit. This force has the form of narrow impulses which is due to the current maximums in first circuit.

Considered system manifest different dynamic behavior: beat`s mode (linear case), quasi-periodic case, chaotic behavior. If energy in initial moment focused mostly in one circuit its oscillations appear nonlinear and periodic. It`s come out that energy delivers into other circuit by means of the current impulses trough the coupling capacitor. If some critical amplitude is surpassed behavior of the driven circuit become chaotic.

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КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ СТОХАСТИЧЕСКИХ КОЛЕБАНИЙ В СИСТЕМЕ ДВУХ СВЯЗАННЫХ НЕЛИНЕЙНЫХ КОНТУРОВ

С.С. Сябер, И.А. Анисимов

Путем численного моделирования исследованы стохастические колебания, которые возникают в системе двух связанных консервативных колебательных контуров с нелинейными конденсаторами. Такие колебания наблюдаются во втором контуре, если в первом происходят регулярные собственные колебания большой амплитуды, обусловленные соответствующими начальными условиями. Стохастичность колебаний подтверждается непрерывностью их спектров и построением отображения Пуанкаре.

КОМП’ЮТЕРНЕ МОДЕЛЮВАННЯ СТОХАСТИЧНИХ КОЛИВАНЬ У СИСТЕМІ ДВОХ ЗВ’ЯЗАНИХ НЕЛІНІЙНИХ КОНТУРІВ

С.С. Сябер, І.О. Анісімов

Шляхом числового моделювання досліджені стохастичні коливання, що виникають у системі двох зв’язаних консервативних коливних контурів з нелінійними конденсаторами. Такі коливання спостерігаються в другому контурі, якщо в першому відбуваються регулярні власні коливання великої амплітуди, спричинені відповідними початковими умовами. Стохастичність коливань підтверджується неперервністю їхніх спектрів та побудовою відображення Пуанкаре.