DYNAMICS OF EXPLOSIVE INSTABILITY

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It was shown that in general case explosive instability dynamics should be described as four wave interaction. The main difference from three wave interaction is that this dynamics may not contain explosive instability. Besides it may by irregular. If the characteristics of one of the wave is closed to one of the interacting wave and they are connected linearly then explosive instability may be suppressed.

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INTRODUCTION

Conception of wave with negative energy was found enough successful. Introduction of such waves essentially simplified understanding of many processes taking place in moving and inverted matters. Existence of these waves allows to consider in another way on such processes as beam instabilities [1], in particular plasma-beam instabilities, superradiation (see, for example, [2]). Essential interest is when decay takes place of negative energy wave into ones with positive energy. This process may occur as explosive instability (see, for example, [3, 4]). The theoretical studying of explosive instability processes in many cases is limited by three wave interaction. In this case it is supposed that other waves are far from synchronism conditions with waves taking part in decay processes. But really in many cases besides negative energy waves the waves with positive energy with closed characteristic may exist. Such waves may influence on the dynamics synchronously interacting waves.

The goal of this work is to investigate positive energy wave influence which is enough closed to negative energy wave on its characteristic (frequency, wave vector) on explosive instability process. It will be shown that existence of such waves may essentially change dynamics of explosive instability. The time of it arising may increase. It may be possible that it will not realize. The dynamics of such four wave interaction may be chaotic.

Arising of the explosive instability may be useful process, for example, to excite oscillations. Besides, this process may be undesirable, for example, to transport flow of charged particles across plasma. In this case this process is needed to remove. Latter we will show that using whirligig principle [5], it is possible to suppress arising of explosive instability.

In the section 1 the problem definition and basis equations that describe the linear and nonlinear interaction of five waves have been formulated. This set of equations is transformed in particular cases in famous ones that describe the processes of ordinary decay, explosive instability and process of linear energy exchange between waves. In the section 2 the some analytical results of investigation of obtained set are presented. The numerical results are presented in section 3. In the section 4 the conditions of explosive instability suppression by means external electromagnetic wave that characteristics (frequency and wave vector) are closed to characteristics of one wave that takes part in nonlinear wave interaction are formulated.

1. PROBLEM DEFINITION AND BASIC EQUATIONS

Explosive instability may be realized in the physical systems of different types. The equations for complex slowly varying amplitudes are similar in these cases. We suppose that in the investigated system (this may be electrodynamics system filled with plasma) there are two closed waves one of them has negative energy. It is supposed that wave frequencies of the interacted waves obey such relations

$$\Omega_{11,12} = \Omega_1 \pm \delta\omega \,, \qquad (1)$$

where $\Omega_{11,12}$ – nearly located natural frequencies of the investigated system ($\Omega_1 \gg \delta\omega$) such that wave 11 has positive energy and wave 12 has negative energy. Both waves have identical wave number k_1. Besides we suppose that in this system there are two natural waves that frequencies are less than $\Omega_{11,12}$. We will consider interaction the first pair of wave (11 and 12) with mode 2 and 3 that is realized in the next way:

$$1 \to 2 + 3, \quad \Omega_{12} = \Omega_2 + \Omega_3, \quad k_1 = k_2 + k_3, \qquad (2)$$

where $\Omega_{2,3}$ – frequencies of third and fourth waves (the natural waves of system) taking part in the interaction, $k_{2,3}$ – their wave numbers. Such nonlinear interaction usually causes excitation of the explosive instability. If in the expression (2) index 12 to replace on 11 this way will be correspond to decay process. Diagram of interacting waves is presented in the Fig. 1. We will consider case when the frequencies satisfy for following inequality:

$$\Omega_{12} \le \Omega_2 + \Omega_3 \le \Omega_{11} \,. \qquad (3)$$

Besides, we will suppose that there is one wave also (with index 4) that has the frequency and wave vector closed to one wave taking part in nonlinear interaction. These waves are connected linearly. As it will be seen latter existence of this wave allows to suppress explosive instability arising.

The set of shortened equations for dimensionless complex slowly varying amplitudes of all interaction waves was obtained in the ordinary way from the Maxwell equations and hydrodynamics ones and look like:

$$\frac{dE_{11}}{d\tau} = -\mu E_2 E_3 \exp\left(i(\Delta + \delta\omega)\tau\right),$$

$$\frac{dE_{12}}{d\tau} = \mu E_2 E_3 \exp\left(i(\Delta - \delta\omega)\tau\right),$$

$$\frac{dE_2}{d\tau} = \left[E_{11} \exp\left(-i(\Delta + \delta\omega)\tau\right) + \right.$$
$$\left. + E_{12} \exp\left(-i(\Delta - \delta\omega)\tau\right) \right] E_3^*,$$
$$\frac{dE_3}{d\tau} = \left[E_{11} \exp\left(-i(\Delta + \delta\omega)\tau\right) + \right. \qquad (4)$$
$$\left. + E_{12} \exp\left(-i(\Delta - \delta\omega)\tau\right) \right] E_2^*,$$
$$\frac{dE_4}{dt} = \frac{\mu_L}{2i} E_{12},$$

where E_{11}, E_{12}, E_2, E_3, E_4 ($E \rightarrow eE/(mc\omega)$ m – electron mass, c – light velocity) dimensionless complex slowly varying amplitudes of the interacting waves, μ – dimensionless coefficient. Dimensionless time τ is measured in the period of Ω_1 frequency. Further the dimensionless frequencies are used. $\omega_2 = \Omega_2/\Omega_1$, $\omega_3 = \Omega_3/\Omega_1$, Δ – characterizes synchronism conditions of waves 2 and 3 with modes 11 and 12 which are defined by correlation

$$\omega_2 + \omega_3 = 1 - \Delta . \qquad (5)$$

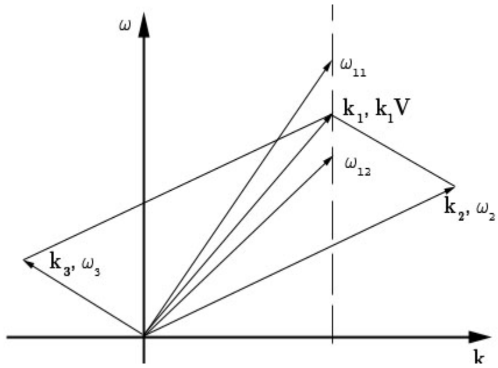

Fig. 1. Diagram of the wave interaction with positive (11) and negative (12) energy with any other waves of the physical system (2 and 3)

When there is synchronism second and third wave with 12 mode the condition $\Delta = \delta\omega$ is satisfied. If there is synchronism with wave 11 then $\Delta = -\delta\omega$. E_4 – complex amplitude of wave that may be linearly connected with one of modes taking part in nonlinear interaction. In this case this wave interacts with wave 12. As it will be shown latter the role of this wave is such as if it interacts with any other wave that taking part in nonlinear interaction. μ_L – coefficient of linear connection.

2. RESULTS OF ANALYTICAL INVESTIGATION

First of all we will consider case when wave 4 is absent ($\mu_L = 0$). In this case some important results may be obtained analytically from set of equation (4). First of all it has following integral:

$$|E_2|^2 - |E_3|^2 = const . \qquad (6)$$

There is analogous integral in the set of equations describing three wave explosive process. It is following from this condition that amplitudes of the second and third waves may infinitely increase but their difference is constant. Thus taking in account of fast wave 11 does not cause breakdown of explosive instability.

When condition $\Delta = \delta\omega = 0$ is satisfied there is also integral in the set (4)

$$E_{11} + E_{12} = C_0 , \qquad (7)$$

and for slowly varying complex amplitudes of waves 2 and 3 it is obtained following expressions:

$$E_2 = C_{21} \exp\left(|C_0|\tau\right) + C_{22} \exp\left(-|C_0|\tau\right),$$
$$E_3 = C_{31} \exp\left(|C_0|\tau\right) + C_{32} \exp\left(-|C_0|\tau\right). \qquad (8)$$

It is following from expressions (8) that modes 2 and 3 exponentially growth when $\Delta = \delta\omega = 0$. It may obtain analogous expressions for amplitudes of waves 11 and 12. The difference is that coefficient in exponent is equal $2|C_0|$. In general case the condition $\Delta = \delta\omega = 0$ does not satisfy. But it is approximately correct in the time intervals $\tau << 1/\delta\omega$, that may be large for small values of detuning $\delta\omega$. In this case the integral (7) and correlations (8) are satisfied approximately. This growth is pure nonlinear and is not connected with linear instabilities that may exist in the investigated system. This is confirmed by numerical results.

The limiting cases may be obtained from set (4), i.e. decay instability of fast wave 11 and explosive instability of slow wave 12. We will consider only cases when waves 2 and 3 are in synchronism with either fast mode 11 or slow mode 12 (see correlations (3) and (5) and comment after (5)). In each of these cases there are exponential oscillating multipliers and terms in the equations (4). Averaging on the time intervals $\tau \gg 1/(2\delta\omega)$ these terms will be equal to zero.

Thus if there is synchronism waves 2 and 3 with slow mode 12 ($\Delta = \delta\omega$) the oscillating term is in the right part of the equation for fast wave amplitude (E_{11}) and first addends in the equations for waves 2 and 3. After averaging the set (4) is transformed in the one describing explosive instability. If there is synchronism waves 2 and 3 with fast mode 11 ($\Delta = -\delta\omega$) oscillating addend will be contained in the right part equation for slow wave (E_{12}). The second addends in the equations for 2 and 3 modes will be oscillating. After averaging we will obtain set of equations describing decay process of fast mode 11.

3. RESULTS OF NUMERICAL INVESTIGATION

The main goal of numerical investigation was definition of features of four wave interaction dynamics when there is synchronism of natural waves 2 and 3 as with slow wave 12 as with fast one 11. As it was noted above in the first case the condition $\Delta = \delta\omega$ is satisfied and in the second case $\Delta = -\delta\omega$ is satisfied. For following values $\delta\omega = 1.0 \cdot 10^{-6}$, 0.001, 0.01, 0.1, 0.2; $\mu = 1$ the numerical calculation was performed for each of these cases. The following initial conditions were selected $E_{110} = E_{120} = 0.1$, $E_{20} = 0.003$, $E_{30} = 0.001$. Here the digit 0 in index points on amplitude initial value of corresponding wave. Practically in all cases initial values of mode 11 and 12 were selected more larger than ones of wave 2 and 3. It is convenient graphically to present numerical investigations results in logarithm scale. Temporal dependence of logarithm of amplitude module of wave 12 is presented on Fig. 2 for case when waves 2 and 3 is synchronized with them and $\Delta = \delta\omega = 1.0 \cdot 10^{-6}$.

As it is seen from this figure depending from initial conditions there is moment when in the wave dynamics appears exponential growth that is corresponding with analytical conclusion presented in section 2. Later this growth is changed by explosive growth of amplitude. The dynamics of others modes is similar that is pre-

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sented on Fig. 2. Qualitatively similar dynamics is observed for other values of $\delta\omega$ at synchronism wave 2 and 3 with slow wave 12. When $\delta\omega$ increase time corresponding explosive growth at the beginning decreases after slightly increase and stop on value τ ~48.

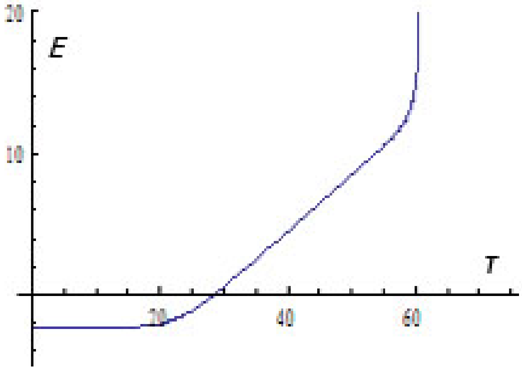

Fig. 2. Dynamic of modules of amplitude wave 12.
$\Delta = \delta\omega = 1.0{\cdot}10^{-6}$, $\mu = 1.0$, $E_{110} = E_{120} = 0.1$, $E_{20} = 0.003$, $E_{30} = 0.001$

Explosive instability arises at synchronism natural modes 2 and 3 with fast wave 11 for values $\delta\omega = 1.0{\cdot}10^{-6}$, 0.001, 0.01, 0.1 too. Besides visually the process is seen identical that is observed for same values of $\delta\omega$ for synchronism with slow wave 12. The picture qualitatively is changed for $\delta\omega = 0.2$ and presented in Fig. 3.

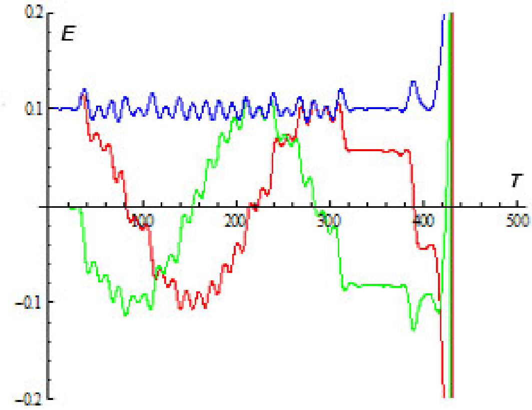

Fig. 3. Dynamics of wave 12 with negative energy at $\Delta = -\delta\omega$, $\delta\omega = 0.2$. Red curve corresponds to real part of amplitude, green corresponds to imaginary part of one and blue corresponds to module

The exponential growth does not observe here. This is conditioned that it duration is $\tau \sim 1/\delta\omega \sim 10$ time units. Explosive instability is observed more latter than at synchronism with explosive mode 12. On the time interval from beginning to explosive instability process is oscillating and irregular. There is energy exchange between waves that is typical for interaction fast wave 11 having positive energy with 2 and 3 waves. Irregularity of process is confirmed by spectrum and autocorrelation analysis. Spectrum and autocorrelation function for slowly varying complex amplitude of wave 2 are presented in Figs. 4 and 5. As it is seen from this figures spectrum is enough wide and autocorrelation function decreasing.

Time of explosive instability beginning in this case is very sensitive to initial conditions. The initial amplitude of fast wave 11 in the process presented in Fig. 3 was equal 0.1 of dimensionless units. If this value was 0.099 time of explosive instability beginning increased to 700 time units. Oneself process in this case qualitatively is similar that is presented in Fig. 3. Spectrum and autocorrelation function are similar that presented in Figs. 4 and 5. This point out that there is parameters and initial conditions region where four wave interaction dynamics will be unstable.

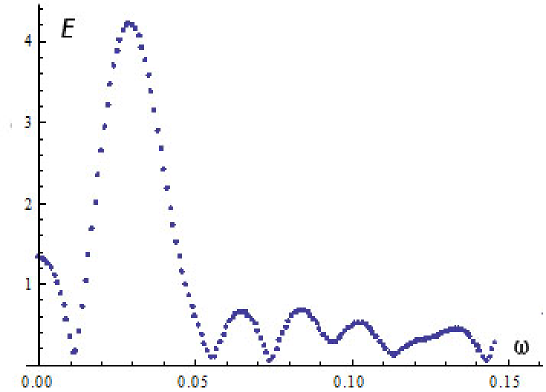

Fig. 4. Spectrum of real part of wave 2 ($Re(E_2)$) amplitude for realization presented on Fig. 3

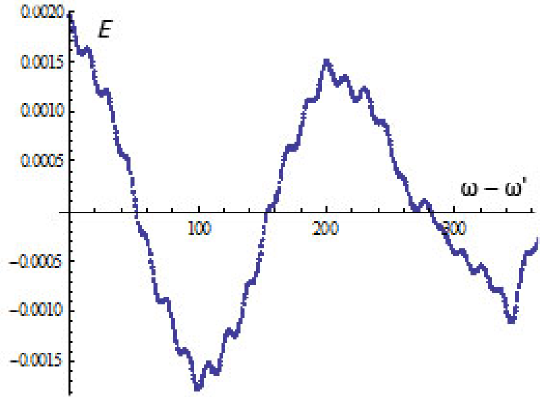

Fig. 5. Autocorrelation function of the real part of wave 2 ($Re(E_2)$) amplitude for realization presented on Fig. 3

Numerical simulation was performed for case when wave 2 and 3 are in synchronism with negative energy wave 12 which initial value is equal zero. Numerical simulation was carried out for $\delta\omega = 1.0{\cdot}10^{-6}$, 0.001, 0.01, 0.1. In the first three cases explosive instability arose. At the beginning on the exponential growth stage amplitudes of wave 12 with negative energy and modes 2 and 3 are increasing. Latter the wave 11 having large initial value is included in the growth process. Latter exponential growth transfers into explosive instability. When detuning $\delta\omega$ increaseы to value 0.1 interaction between waves at selected initial conditions is stopped and explosive instability in this case does not excite. Amplitudes of all waves in this case weakly oscillate. When initial value of wave 12 with negative energy is equal zero and $\delta\omega$ <0.1, at the beginning energy from wave with positive energy 11 transfers to other modes of system. Latter when contribution of wave with negative energy is essential the explosive instability is excited. From set (4) it follows that at selected initial conditions the right parts of equations are quadraticly small and time of exponential growth is not enough for essential increasing of amplitudes of interacting waves.

The dispurtion of explosive instability at increasing of $\delta\omega$ does not occur discontinuously. When $\delta\omega$ come up to 0.1 on the left, time interval from the process beginning to arising explosive instability increases to infinite. The initial value of slow wave 12 was increased to 0.0451. In the range from 0.0 to 0.0451 wave interaction was absent. It appear when initial values was E_{120}=0.045155 and is completed by explosive instability.

The role of approximate integral (7) that is correct in the beginning stage of process before influence of exponential multipliers was noted above. May be occur that modules of initial values of complex amplitudes of wave 11 and 12 are equals and phases will different on π. In this case initial exponential growth is absent. To define influence of initial phases of complex amplitudes of wave 11 and 12 on investigated four wave interaction the following parameters were selected: $\Delta = \delta\omega = 1.0\cdot10^{-6}$, μ=1.0, $|E_{110}|=|E_{120}|$ =0.1, $|E_{20}|$=0.003, $|E_{30}|$ = 0.001. Waves 2 and 3 are synchronized with negative energy wave 12. The initial phase of complex amplitude of fast wave 11 changes in range from 0 to π. The initial phases of complex amplitudes of other waves were zero. Numerical results are presented in Table 1. In the first and third rows the initial phases are presented. In the second and fourth rows the time of explosive instability beginning is presented. When $\phi_{110} = \pi$ excitation time of explosive instability is 6700 time units.

Table 1

Excitation time of explosive instability versus initial phase of complex amplitude of fast wave 11

ϕ_{110}	0	0.1π	0.2π	0.3π	0.4π
τ expl	60.6	61.3	63.4	67.2	73.3
ϕ_{110}	0.5π	0.6π	0.7π	0.7π	0.9π
τ expl	82.5	97.1	121	170	305

In the Table 2 the numerical results for two cases of synchronism are presented. Here the initial phase of fast wave 11 is equal π and detuning δω is changed. The following parameters were used μ = 1.0, $|E_{110}| = |E_{120}|$ =0.1, $|E_{20}|$ =0.003, $|E_{30}|$ = 0.001, $\varphi_{10} = \pi$, $\varphi_{20} = \varphi_{30} = 0$, $\varphi_{20}, \varphi_{30}$ initial phases of 2 and 3 waves correspondingly.

Table 2

Excitation time of explosive instability versus detunin δω

$\delta\omega$	0.000001	0.001	0.01	0.1	0.2
τ expl, $\Delta=\delta\omega$	6700	260	87	47	47
τ expl, $\Delta=-\delta\omega$	6700	260	87	61	62

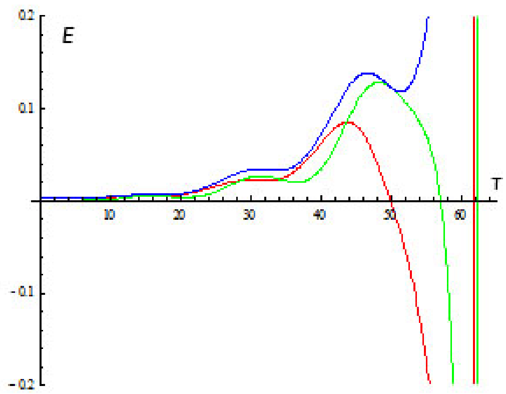

Fig. 6. Dynamics of process described by equations (4) (wave 2) for following parameters: $\Delta = -\delta\omega=0.2$, μ = 1.0, $|E_{110}| = |E_{120}|$ =0.1, $|E_{20}|$ =0.003, $|E_{30}|$ = 0.001, $\varphi_{110} = \pi$, $\varphi_{20} = \varphi_{30}$ =0.0. Red curve corresponds to real part of complex amplitude, green – to imaginary and blue – to module

As it seen from this table excitation time of explosive instability is same for two synchronization variants in the detuning range from 0 to ~0.01. The slowly changing amplitudes dynamics of all waves for these two synchronization variants is practically identical. Modules of amplitudes monotonously growth. Latter for more values of detuning differences appear. Modules become oscillating. Essentially this is seen for detuning $\delta\omega$ = 0.2, that is shown on the Fig. 6.

4. SUPPRESSION OF EXPLOSIVE INSTABILITY

4.1. GENERAL CONDITIONS

In this section we will show that existence of additional wave that is linearly connected with one of the nonlinearly interacting modes may cause suppression as decay instability as explosive one. To prove this fact we rewrite the set of equations (4) more simply:

$$\begin{aligned} \frac{dE_{12}}{d\tau} &= \mu E_2 E_3 + \frac{\mu_L}{2i} E_4, \\ \frac{dE_2}{d\tau} &= \mu E_{12} E_3^*, \\ \frac{dE_3}{d\tau} &= E_{12} E_2^*, \\ \frac{dE_4}{d\tau} &= \frac{\mu_L}{2i} E_{12}. \end{aligned} \qquad (9)$$

In this set we retained only that waves that is in the exact synchronism one with each other. Besides we suppose that negative energy wave decay takes place. Let notice that if we change the sign before first item in the right part of the first equation in (9) then such system at $\mu_L = 0$ will describe decay instability.

Below we will show that addition of the fourth wave (E_4) can suppress both decay processes, and process of explosive instability. It is necessary to notice that in set (9) we have considered connection only a first wave with a stabilization wave (fourth). The same results turn out and when any other wave (the first or the second waves) will be involved in process of stabilization interaction. We assume, according to the general ideology that decay instability will be suppressed as soon as there will be fulfilled condition $\mu_L / 2 > |\mu E_{12}(0)|$. The left part of this inequality is frequency of exchange energy between the stabilization wave and one of the waves participating in three-wave interaction. The right part is increment of decay instability. We will analyze system (9) by numerical methods. For this purpose it is convenient to enter following parameters and new real variables:

$$\begin{aligned} E_{12} &= x_0 + ix_1, \\ E_2 &= x_2 + ix_3, \\ E_3 &= x_4 + ix_5, \\ E_4 &= x_6 + ix_7, \\ \varepsilon &\equiv \mu_L / 2\mu, \\ \tau_1 &\equiv \mu t. \end{aligned},$$

The usual decay process is observed if the stabilizing wave is absent ($\varepsilon = 0$). The stabilization process of decay instability was observed in all cases when we introduce in dynamics the wave E_4 (stabilization wave) and when the condition $\mu_L / 2 > |\mu E_{12}(0)|$ was fulfilled.

4.2. STABILIZATION OF EXPLOSIVE INSTABILITY

It is interesting to notice that stabilization can be realize and for explosive instability. Really, in Figs. 7-9 the dynamics of wave amplitudes is presented ($x_0(0) = 0.1$, $x_2(0) = 0.001$, $x_6(0) = 0.01$,) at explosive instability in absence stabilization wave (see Fig. 7), and also dynamics of these amplitudes in the presence of a stabilization wave (Figs. 8, 9). It is seen from these figures that already at values of parameter $\varepsilon = 0.09$ full stabilization of explosive шnstability have been observed. Only the basic wave (E_{12}) and the stabilization wave (E_4) have periodic dynamics. Other waves practically don't change. However already at $\varepsilon = 0.08$ the explosion appears. However time of its occurrence became significantly large (more 400).

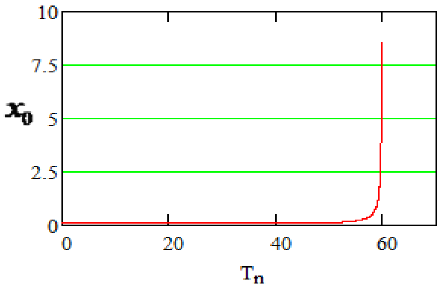

Fig. 7. Explosive instability at $\varepsilon = 0$

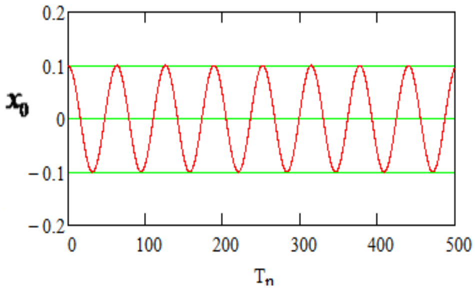

Fig. 8. Suppression of explosive at $\varepsilon = 0.09$

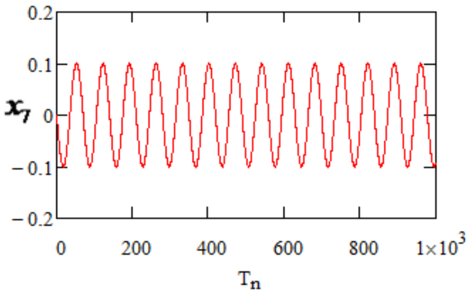

Fig. 9. Suppression of explosive at $\varepsilon = 0.09$

CONCLUSIONS

Thus considering usual process of nonlinear three wave interaction it is necessary to draw attention to possible additional wave that characteristics may be closed to ones of the wave taking parts in the nonlinear interaction. Taking in account of this wave may essentially change usual dynamics of wave interaction. It may say that in the common case nonlinear wave interaction, for example, in beam systems must consider as four wave process.

The obtained above results show also that using whirligig principles allows lightly to suppress processes of nonlinear instabilities. This simplicity of suppression is lightly explained that fact that characteristic times of nonlinear instabilities are more larger as rule than characteristic times of linear process. Really, in our case character times of arising of nonlinear instabilities are inversely proportional to initial amplitudes of decaying wave. This value practically in all real cases is more less than coefficient of linear connection between waves. In this case to suppress instabilities it is lightly to realize conditions when time of energy exchange between waves conditioned by linear connection is more less than time of arising of nonlinear instabilities. As it known [5] this is main criterion of stabilization mechanism at using whirligig principle.

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ДИНАМИКА ВЗРЫВНОЙ НЕУСТОЙЧИВОСТИ

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Показано, что, в общем случае, динамика взрывной неустойчивости должна описываться в рамках четырехволнового взаимодействия. В отличие от трехволнового взаимодействия эта динамика может не содержать взрывного нарастания амплитуд взаимодействующих волн. Более того, она может быть нерегулярной. Если одна из четырех волн близка по своим характеристикам к одной из взаимодействующих волн и связана с ней линейной связью, то взрывная неустойчивость может быть подавлена.

ДИНАМІКА ВИБУХОВОЇ НЕСТІЙКОСТІ

В.О. Буц, І.К. Ковальчук

Показано, що, в загальному випадку, динаміка вибухової нестійкості повинна описуватися в межах чотирьох хвилевої взаємодії. На відміну від трихвильової взаємодії ця динаміка може не містити вибухового зростання амплітуд хвиль, що взаємодіють. Більш того, вона може бути нерегулярною. Якщо одна з хвиль близька по своїм характеристикам до однієї з тих, що взаємодіють, та зв'язана з нею лінійно, то вибухова нестійкість може бути подавлена.