ENERGY SPECTRA OF ACCELERATED ELECTRONS OF CHAIN RELATIVISTIC ELECTRON BUNCHES AT EXCITATION OF WAKEFIELD IN DIELECTRIC RESONATOR

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The energy spectra of a long sequence of relativistic electron bunches at the output of the dielectric resonator taking into account initial energy spread of the relativistic electron bunches are investigated. It is considered also the finite value of the Q-factor of the dielectric cavity.

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INTRODUCTION

A characteristic feature of the experimental setup "Almaz 2" is the presence of energy spread in a relativistic electron bunches. Energy spread will influence on the process of wakefield excitation in the dielectric cavity and thus on the acceleration of electrons by these fields. Not less important is the account of the finite value of the Q-factor of the dielectric cavity, as it limits the wakefield excited by a long sequence of relativistic electron bunches and the maximum energy of the accelerated electrons [1].

Let’s consider the process of self-acceleration of relativistic electron bunches which is realized under conditions when there is a detuning between the repetition frequency of the relativistic electron bunches and the frequency of the excited oscillations, which is in cherenkov synchronism with electron bunches. In this case, electric field beat is excited. Wherein the first half of the sequence of electron bunches is in the decelerating phase and thus losses energy in the excitation of the electric field. The second half of the sequence of bunches is in the accelerating phase and therefore gaining energy.

1. STATEMENT OF THE PROBLEM. BASIC RELATIONS

The dielectric cavity is formed by a metallic cylindrical cavity which volume is completely filled a homogeneous dielectric with permittivity *ε*. The length of the cavity is *L*, its radius is *a*. The cavity has a finite value of the Q-factor. On the left end of the dielectric cavity in the axial region periodic sequence of *N* relativistic electron bunches with finite longitudinal and transverse dimensions is injected. In the radial direction of the charge density is distributed according to a Gaussian law and in longitudinal direction charge density has a rectangular profile. In addition, each relativistic electron bunch has an initial energy spread, which is described by the model distribution function

$$f_0 = \frac{1}{\sqrt{\pi}\Delta\gamma} e^{-\frac{(\gamma-\gamma_0)^2}{\Delta\gamma^2}}, \tag{1}$$

where $\Delta\gamma$ is characteristic spread of relativistic factor, γ_0 is the average value of the relativistic factor.

Energy *i*-th particle o f bunch with number α and having the initial value of the relativistic factor γ_l at the input of the dielectric cavity is described by expression

$$\gamma_{\alpha jl} = \gamma_l + \Delta\gamma_{\alpha jl}, \tag{2}$$

where

$$\Delta\gamma_{\alpha jl} = -\varepsilon_0 \left[\frac{4v_0^2}{L^2} \sum_{n=1}^{\infty} F_n(r) \sum_{m=0}^{\infty} \alpha_m \omega_{mn}^2 S_{mn} \Lambda_{mn\alpha} G_{mn} \right],$$

$$G_{mn} = \frac{\sin\left(\frac{\omega_{mn} t_b}{2}\right)}{\frac{\omega_{mn} t_b}{2}},$$

ω_{mn} are frequencies of dielectric cavity oscillations, t_b is the bunch duration, $\alpha_m = 1/2$ for $m = 0$ and $\alpha_m = 1$ for $m > 0$,

$$F_n(r) = \frac{J_0(\lambda_n r/a)}{J_n^2(\lambda_n)} \cdot e^{-\frac{1}{2}\left(\frac{r_b \lambda_n}{a}\right)^2},$$

λ_n are roots of Bessel’s function $\left(J_0(\lambda_n) = 0\right)$, r_b is characteristic bunch radius,

$$\Lambda_{m,n,\alpha} = e^{-\gamma_{mn}\left(t_j - t_b/2\right)} \cdot \mathrm{Re}\left[e^{i\omega_{mn}\left(t_j - t_b/2\right)} \frac{e^{-\alpha(\gamma_{mn}T - i\omega_{mn}T)} - 1}{1 - e^{(\gamma_{mn}T - i\omega_{mn}T)}} \right],$$

$\gamma_{mn} = \frac{\omega_{mn}}{2Q}$ is damping decrement of the cavity oscillations, T is the repetition period of the bunches, $t_L = \alpha T + t_j + z/v_0$ is the Lagrangian time of *j*-th particle of the bunch with number α;

$$\varepsilon_0 = \frac{4eQ_b L}{a^2 mc^2 \varepsilon^2 \beta_0{}^2} \cdot \left(\beta_0{}^2 \varepsilon - 1\right),$$

Q_b is charge of bunch, $\beta_0 = v_0/c$, v_0 is velocity of bunches, e, m are charge and mass of electron, c is light velocity,

$$S_{mn} = \frac{1}{\sqrt{\pi}\Delta\gamma} \int_1^{\infty} d\gamma\, e^{-\frac{(\gamma-\gamma_0)^2}{\Delta\gamma^2}} \frac{\left[1 - (-1)^m \cos\frac{\omega_{mn} L}{v_0(\gamma)}\right]}{k_m^2 v_0^2(\gamma) - \omega_{mn}^2},$$

$$k_m = \pi m / L \, .$$

Taking into account initial distribution function (1) and the expression for the relativistic factor (2) the distribution function of sequence of N bunches at the exit of the dielectric cavity can be written as

$$f = \frac{1}{\sqrt{\pi}\Delta\gamma}\sum_{\alpha=1}^{N} e^{-\frac{(\gamma_l + \Delta\gamma_{lj\alpha} - \gamma_0)^2}{\Delta\gamma^2}} \, . \qquad (3)$$

2. NUMERICAL RESULTS

Numerical calculations of the energy distribution function of sequence relativistic electron bunches at the output dielectric cavity (3) were performed for the following parameters of the dielectric cavity and the sequence of relativistic electron bunches: $a = 4.025$ cm is the radius of the cavity, $L = 31.925$ cm is the cavity length, the dielectric constant is $\varepsilon = 2.045$, the initial value of the relativistic factor is $\gamma_0 = 9.8064$, the energy spread is $\Delta\gamma / \gamma_0 = 3.6\%$, the charge of the electron bunch is $Q_b = 0.32\,nC$, longitudinal phase size of bunch is 60°, $r_b / a = 1/8$ is the radius of the bunch. The number of bunches in the sequence N = 1000, 1500 and the quality factor of the cavity Q are varied. Simple estimates show that Q-factor has no effect on the process of self-acceleration of electron bunches sequence when the condition is satisfied

$$Q >> \pi \frac{\omega}{\delta\omega} = \pi N \, , \qquad (4)$$

where $\delta\omega = \omega_{res} - \omega_{\scriptscriptstyle M}$ is the detuning between frequency of resonant oscillation of the cavity ω_{res} and the repetition frequency of bunches $\omega_{\scriptscriptstyle M}$, and was chosen from the condition $\delta\omega / \omega_{res} = 1/N$. Note that for the chosen parameters of the dielectric cavity and the energy of bunches the resonant oscillation have indexes $m = 6$, $n = 1$. Under condition (4) beat waves is formed before stabilization of field growth due to the finite Q of the resonator. In opposite case beat of wakefield is not formed because in this case the field is stabilized at low level.

Fig. 1 shows the distribution of energy (relativistic factor) chains of bunches at the input and output of the dielectric cavity for without ohmic losses. Noticeable broadening of distribution function takes place.

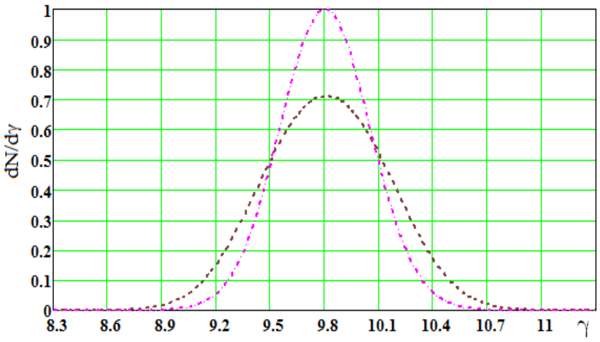

Fig. 1. Initial distribution function (magenta line), distribution function $(\Delta\gamma / \gamma_0 = 0.36\%)$ *(brown line),* $N = 1000$

Thus its maximum value slightly decreased since the total area (total number of particles) is always conserved. At half height of distribution function we observe increasing of energy equal to 34 keV, on 0.1 height increasing of energy is 73 keV.

For a sequence of $N = 1000$ bunches inequality (4) takes the form $Q >> 1000\pi$. Fig. 2 illustrates the case $Q = 9000$ when Q-factor has little influence on the process of self-acceleration of bunches chain.

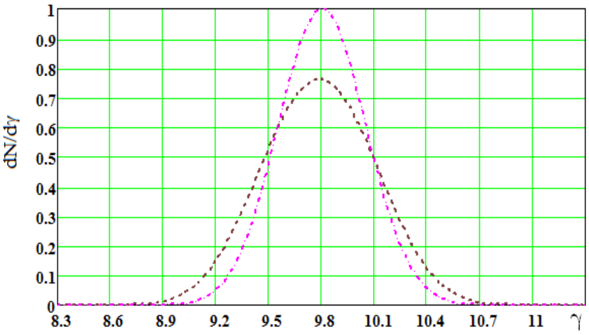

Fig. 2. Initial distribution function (magenta line), distribution function $(\Delta\gamma / \gamma_0 = 0.36\%)$ *(brown line),* $N = 1000$, $Q = 9000$

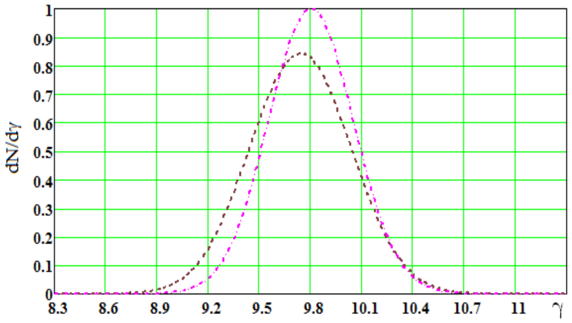

Fig. 3. Initial distribution function (magenta line), distribution function $(\Delta\gamma / \gamma_0 = 0.36\%)$ *(brown line),* $N = 1000$, $Q = 3000$

Energy distribution has decreased, though not significantly. The picture radically changes when the inequality (4) is not satisfied. Fig. 3 shows the distribution function for $Q = 3000$. It can be seen that the accelerated particles are absent and there are only slow particles. The maximum of the distribution function is shifted somewhat to the left.

Let us now discuss effects due to increasing of the number of bunches ($N = 1500$). Fig. 4 shows the distribution function at the output of the dielectric cavity is lossless $Q = \infty$. Increasing the number of bunches leads to a marked broadening of the distribution function. At half height of distribution function we observe increasing of energy equal to 84 keV, on 0.1 height increasing of energy is 136 keV.

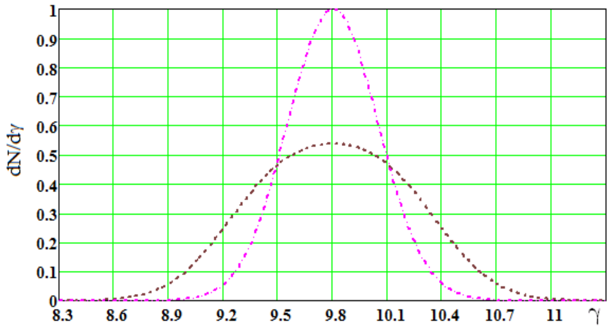

Fig. 4. Initial distribution function (magenta line), distribution function $(\Delta\gamma/\gamma_0 = 0.36\%)$ *(brown line),* $N = 1500$

Thus, increasing of bunches number in the sequence requirements on Q-factor ($Q >> \pi N$) of the dielectric cavity are increased. If this condition is not satisfied, the proportion of accelerated particles is decreased and the number of deceleration particles is increases.

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ЭНЕРГЕТИЧЕСКИЕ СПЕКТРЫ УСКОРЕННЫХ ЭЛЕКТРОНОВ ЦЕПОЧКИ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОГО ПОЛЯ В ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ

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Исследованы энергетические спектры длинной последовательности релятивистских электронных сгустков на выходе диэлектрического резонатора с учетом энергетического разброса релятивистских электронных сгустков на входном торце резонатора. Учтено также конечное значение добротности диэлектрического резонатора.

ЕНЕРГЕТИЧНІ СПЕКТРИ ПРИСКОРЕНИХ ЕЛЕКТРОНІВ ЛАНЦЮЖКА РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОГО ПОЛЯ В ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ

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Досліджено енергетичні спектри довгої послідовності релятивістських електронних згустків на виході діелектричного резонатора з урахуванням енергетичного розкиду релятивістських електронних згустків на вхідному торці резонатора. Враховано також скінченне значення добротності діелектричного резонатора.