METHODS OF CALCULATION OF CROSS SECTION OF REACTION $^{115}In(\gamma,\, n)^{114m}In$

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The cross section of reaction $^{115}In(\gamma,\, n)^{114m}In$ is expected by different methods. Results of the got cross section it is well comported inter se the Penfold-Leiss and Tikhonov's methods. The calculation of cross section is conducted the Penfold-Leiss method with smoothing out by the method of iterations. Number of iterations $n = 1;\, 3;\, 5$. In the programmatic package of TALYS-1.4 got cross section for five models of closeness of levels. Theoretical and experimental results well coincide in a maximum.

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1. INTRODUCTION

The purpose of the majority of physical researches that are carried spent on brake beams electronic accelerators,- studying of power dependence of cross sections of different photonuclear processes. As the spectrum received from the accelerator γ-quantum's has continuous character, therefore in experiment not cross section is measure of reaction, it is so-called output. The output is intensity photonuclear to the process, the doze attributed to unit γ-quantums that have passed through a target with researched substance at various values of the top border of a brake spectrum [1].

The output is directly connected with effective cross section of reaction by integrated equation Fredholm's (or Voltera's) the first sort:

$$Y(E_{\gamma max}) = \int_{E_{nop}}^{E_{\gamma max}} \sigma(E)\Phi(E,\, E_{\gamma max})dE\,, \quad (1)$$

where E_m - a threshold of reaction, $E_{\gamma max}$ - the maximal energy of a spectrum brake γ- quantum's, $\sigma(E)$ - cross section of reaction, $\Phi(E,\, E_{\gamma max})$ - a spectrum of brake radiation (Shiff's spectrum [2]). So, the cross section of reaction can be received from experimental data about output as a result of the decision of a return task (1). For the numerical decision of this task mathematical methods have been developed. The most widespread methods are "a difference of photons", "the least structure" Cook's [3], "a return matrix" (Penfold-Leiss method) [4], "regularizations" (Tikhonov's method) [5-8]. Penfold-Leiss and Tikhonov's methods differ the form of the effective spectrum of photons hardware function of the method. It is the direct decision of a return task. But other methods of definition of cross section are also possible: a combination of outputs of reaction and a method of a reduction. Conditions of correct statement of Adamar's task [6,7] is: 1) Existence of the decision in space of possible values for any curve of an output from space of her possible values; 2) The decision should be the only thing; 3) continuous dependence of the decision on the initial data. One more condition is that the cross section should be positive.

2. METHODS OF CALCULATION OF SECTION OF REACTION

In a method of a return matrix [4] required cross section $\sigma(E_j - \Delta E)/2$ is defined on experimental outputs $Y(E_{\gamma max})$:

$$\sigma(E_j - \Delta E_j/2) = \frac{E_j - \Delta E_j/2}{f(E_j - \Delta E_j/2)\Delta E_j} \sum_{i=1}^{j} B_{ji} Y_i\,, \quad (2)$$

where B_{ij} – elements of a return matrix. The standard deviation (2) is defined as

$$\delta\sigma(E_j - \Delta E_j/2) = \frac{E_j - \Delta E_j/2}{f(E_j - \Delta E_j/2)\Delta E_j} \times \sqrt{\sum_{i=1}^{j} (B_{ji})^2 (\Delta Y_i)^2}\,, \quad (3)$$

where ΔY_i - a standard deviation of experimental value Y_i.

One more method of the decision of the integrated equation (1) is three dot method or measurement of photonuclear cross sections with the help "quasi-monochromatic" γ-quantum's of brake radiation [9]. For the fixed values j elements $B_{j,j-1}$, $B_{j,j-3}$, $B_{j,j-5}$ have negative values, and also at $i < j-2$ values B_{ji} are decreased very quickly. Using these properties of

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elements $B_{j,i}$ for the analysis of cross section $\sigma(E_\gamma)$, it is possible to write down

$$\sigma_{j1} = \frac{E_j - \Delta E_j/2}{f(E_j - \Delta E_j/2)\Delta E_j} \times (B_{j,j} - B_{j,j-2}) \frac{Y_j - Y_{j-2}}{2\Delta E}, \quad (4)$$

$$\sigma_{j2} = \frac{E_j - \Delta E_j/2}{f(E_j - \Delta E_j/2)\Delta E_j} \frac{\Delta E}{2} \times (B_{j,j} - B_{j,j-2}) \frac{Y_j - 2Y_{j-1} + Y_{j-2}}{\Delta E^2}. \quad (5)$$

Numerical experiment in the work [9] has shown, that cross section $(E_j - \Delta E/2)$ can be approximated by expression

$$\tilde{\sigma} = \sigma(E_j - \Delta E_j/2) = \sigma_{j,1} + \sigma_{j,2}. \quad (6)$$

According to this method the cross section of photonuclear reaction can be found on three experimental values of output $Y(E_{\gamma max})$, received on a brake beam for three different values of maximal energy $E_{\gamma max}$.

In Cook's method [3] set of σ_j s will be considered as acceptable solution to equation (1) if

$$\chi^2(\sigma_j) = \sum_{i=1}^{n} \frac{\left(\sum B_{i,j}\sigma_j - Y_i\right)^2}{\left(\Delta Y_i\right)^2} \le n. \quad (7)$$

The "most smooth" set of decisions is chosen from physical decisions. For this purpose an auxiliary function $S(\sigma_j)$ called "the structure function" will be defined. Within certain wide limits, the exact definition of $S(\sigma_j)$ is arbitrary. Several definitions of $S(\sigma_j)$ have been extensively explored, namely:

$$S_1(\sigma_j) = \sum_{j=1}^{n-1} (\sigma_{j+1} - \sigma_j)^2, \quad (8)$$

and

$$S_2(\sigma_j) = \sum_{j=2}^{n} (\sigma_{j+1} - 2\sigma_j + \sigma_{j-1})^2. \quad (9)$$

In Tikhonov's method a choice of the approached cross section the principle of smoothness [7] of set of "formal" decisions $\sigma(k)$, satisfying a condition is used

$$\sum_{j=1}^{m} \frac{\left[\int_{E_m}^{E_j} a(E_j,k)\sigma(k)dk - y(E_j)\right]^2}{\left[dy(E_j)\right]^2} \le m \quad (10)$$

is chosen such, for which special functional

$$\Omega[\sigma] = \int_{E_m}^{E_{max}} \left[\sigma^2(k) + \left(\frac{d\sigma(k)}{dk}\right)^2\right] dk \quad (11)$$

if has the minimal value. That is the finding of function $\sigma(k)$ is reduced to search of a minimum functional

$$\overset{\alpha}{M} = \left\| \int_{E_m}^{E} a(E,k)\sigma(k)dk - y(E) \right\| + \alpha\Omega[\sigma], \quad (12)$$

where α – parameter regularizations.

In work [1] research of influence by miscellaneous of methods of smoothing of an experimental curve of an output on power dependence of cross section of reaction (γ, γ') is carried out. By the example of reaction $^{115}In(\gamma, n)^{115m}In$ it is shown, that smoothings by a method of iterations and a method of approximation give three variants of power dependence of differential cross section. In some power area the cross section has negative values, and it testifies about "not physical" results. To remove this discrepancy in calculations, it is necessary to impose a requirement: the cross section should be the positive.

3. CALCULATIONS OF CROSS SECTION OF REACTION $^{115}In(\gamma, n)^{114m}In$ AND CONCLUSIONS

Calculation of differential cross section of reaction was spent on outputs from work [10]. Results of calculation of cross section are resulted by Penfold-Leiss method on Fig.1. Smoothing cross section by a method of iterations was used. Number of iterations $n = 1;\ 3;\ 5$. The received cross section $\sigma(E) > 0$ also is quite comparable to the data in [11,12]. On Fig.1 comparison calculations with the data of operation [11] is made. Nuclear performances of an isomers ^{114m}In is specified in Table 1, where E_m – a threshold (γ, n) – responses that gives in formation of an isomer; $T_{1/2}$ – a half-life period; E_γ – energy γ- quantum's that radiates an isomer; J_m, J_g – the complete moment isomeric and the basic states; "+", "–" – paired relationship of a state.

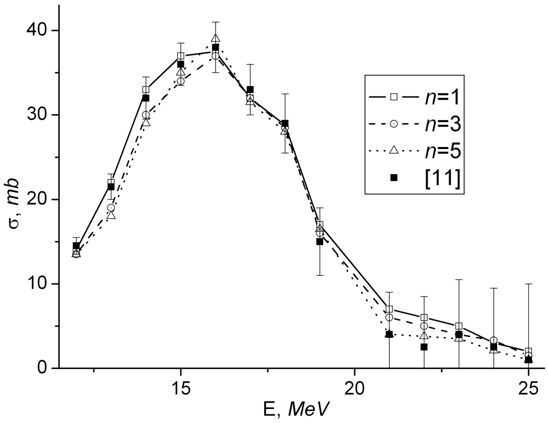

Fig.1. *Cross section of reaction $^{115}In(\gamma, n)^{114m}In$ (Penfold-Leiss method)*

Table 1. *Nuclear characteristics of an isomers In*

Isomer	E_m MeV	$T_{1/2}$	E_γ keV	J_g	J_m
^{113m}In	0.39	1.658 h	392	9/2+	1/2-
^{114m}In	9.23	43 ms	310	1+	8-
^{115m}In	0.33	4.486 h	336	9/2+	1/2-

For Tikhonov's method a nucleus of the equation (1) will be Shiff's spectrum $\Phi(E, E_{\gamma max})$. Thus it is necessary to take into account function of response of the absolute chamber $f(E)$. On Fig.2 the cross section received by a method of Tikhonov's is resulted. The difference between calculations by Penfold-Leiss and Tikhonov's methods makes 5...7%.

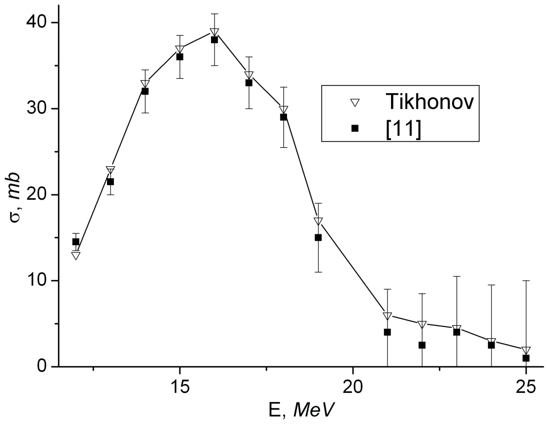

Fig.2. *Cross section of reaction* $^{115}In(\gamma,\, n)^{114m}In$ *(Tikhonov's method)*

For calculation of cross section of reaction $^{115}In(\gamma,\, n)^{114m}In$ is possible to use programmatic package TALYS-1.4 [13,14]. At calculations of cross section in TALYS-1.4 is possible to choose a level density model per nuclide considered in the reaction (parameter ldmodel). There are 3 phenomenological level density models and 2 options for microscopic level densities: ldmodel 1 – constant temperature + Fermi gas model; ldmodel 2 – back-shifted Fermi gas model; ldmodel 3 – generalised superfluid model; ldmodel 4 – microscopic level densities from Goriely's table; ldmodel 5 – microscopic level densities from Hilaire's table. In package TALYS-1.4 the cross section of reaction $^{115}In(\gamma,\, n)^{114m}In$ in the interval of energies 10...25 MeV with step 0.1 MeV is numerically designed. In Fig.3 is resulted the received results for five models of density of levels nuclide (ldmodel 1-5). For ldmodel 1-3 value of cross section is relatives, and a maximum $\sim$ 58...59 mb. Also differ from experimental in 1.52 times. A maximum thus displaced to the left on 0.1 MeV. The results of treatment of peaks of power dependences of section of reaction are driven to the Table 2, where next denotations are used: χ^2 – value of function of selection; R^2 – coefficient of determination. As an approximating function the Gauss function was chosen:

$$y = y_0 + \frac{A}{w\sqrt{\pi/2}} e^{-(x-x_c)^2/w^2}, \qquad (13)$$

where A, w, x_c - parameters of Gauss function.

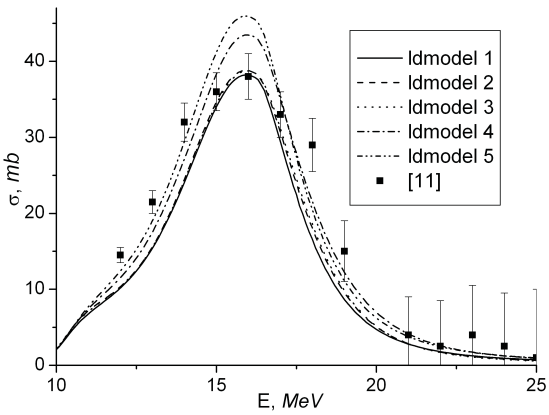

Fig.3. *Cross section of reaction* $^{115}In(\gamma,\, n)^{114m}In$ *(TALYS-1.4)*

Consequently, the cross section of reaction is expected by the Penfold-Leiss method, by the Tikhonov's method and in the package of TALYS - 1.4. The got results well coincide in a maximum.

Table 2. *Results of treatment of peaks of section of reaction*

Model	χ^2	R^2	Area	E_{max} MeV	Width	Offset	σ_{max} mb
ldmodel 1	2.21121	0.98574	168.40	15.632	3.8456	1.7953	34.941
ldmodel 2	2.14085	0.98681	175.97	15.672	3.9378	1.7112	35.656
ldmodel 3	2.09347	0.98714	177.93	15.698	3.9794	1.6275	35.674
ldmodel 4	2.19684	0.98902	199.53	15.742	4.0273	2.1202	39.530
ldmodel 5	2.67697	0.98810	206.03	15.633	3.9095	2.1692	42.048
[11]	2.37070	0.98893	232.66	15.715	5.0989	1.6392	36.407

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МЕТОДЫ РАСЧЕТА СЕЧЕНИЯ РЕАКЦИИ $^{115}In(\gamma, n)^{114m}In$

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Сечение реакции $^{115}In(\gamma, n)^{114m}In$ рассчитано разными методами. Результаты полученного сечения методами Пенфольда-Лейсса и Тихонова хорошо согласуются между собой. Расчет сечения методом Пенфольда-Лейсса проведен со сглаживанием методом итераций. Число итераций $n = 1$; 3; 5. В программном пакете TALYS-1.4 получены сечения для пяти моделей плотности уровней. Теоретические и экспериментальные результаты хорошо совпадают в максимуме.

МЕТОДИ РОЗРАХУНКУ ПЕРЕРІЗУ РЕАКЦІЇ $^{115}In(\gamma, n)^{114m}In$

В.І. Жаба, О.М. Парлаг

Переріз реакції $^{115}In(\gamma, n)^{114m}In$ розраховано різними методами. Результати отриманого перерізу методами Пенфольда-Лейсса і Тіхонова добре узгоджуються між собою. Розрахунок перерізу методом Пенфольда-Лейсса проведено зі згладжуванням методом ітерацій. Число ітерацій $n = 1$; 3; 5. У програмному пакеті TALYS-1.4 отримано переріз для п'яти моделей густини рівнів. Теоретичні та експериментальні результати добре співпадають у максимумі.