TRAPPING OF NONMONOENERGETIC ELECTRON BUNCHES INTO A WAKE WAVE

S. V. Kuznetsov*

Joint Institute for High Temperatures of RAS, Moscow, Russia

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The trapping of nonmonoenergetic electron bunches in a wake field wave excited by a laser pulse in a plasma channel is studied analytically. Electrons are injected into the region of the wake wave potential maximum at a ve-locity lower than the phase velocity of the wave. The formula for length of a bunch in the accelerating stage as func-tions of its initial energy spread and initial sizes (length and cross-section radius) is derived.

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INTRODUCTION

A number of sufficient successful experiments on laser-plasma acceleration of electron bunches to energies of a few GeV while in relative energy spread of the accelerated monoenergetic bunch $\sim 5...10\%$ [1, 2] was carried out in recent years. However, for many practical applications, the energy spread of the electron bunch after acceleration must be tenths of a percent [3, 4]. In this regard, the theoretical study of the influence of various factors on the energy spread of the accelerated electron bunch is very important to identify ways to get in the experiment bunches of high energy electrons with a small energy spread between them.

It is known that the energy spread of the accelerated electron bunch largely depends from its length in the accelerating stage [5, 6]. In laser-plasma accelerators using an external electron injector the length of accelerated electron bunch is determined by the regimen of the injection bunch into a wake field, its parameters at the instant of injection and the parameters of the wake field [7, 9]. In this paper we theoretically investigate the influence of the transverse dimensions of the injected bunch on its length in the accelerating stage in the scheme of injection, when a bunch is injected into the vicinity of the wake wave potential maximum with a velocity less than the phase velocity of the wake wave [9, 10]. Grouping phenomenon is observed in this injection scheme, owing to which electrons are arranged in the trapping area along the longitudinal axis more densely than at the instant of injection. The phenomenon of electron grouping allows to relax the requirements to the original length of the injected bunch. In previous studies the phenomenon of electron bunching by wake wave was studied in one dimensional formulation [11, 12]. The aim of this work is an analytical study of this phenomenon in two-dimensional formulation and definition of the corrections to the trapped electrons bunch length, resulting from transverse dimensions of the injected bunch.

*Corresponding author E-mail address: shenau@rambler.ru

1. TRAPPING OF A SINGLE ELECTRON BY A WAKE WAVE

To describe the motion of relativistic electrons of a bunch accelerated in the wake field generated by an axisymmetric laser pulse propagating along the OZ axis, we use the motion equations in the form:

$$\frac{dp_z}{d\tau} = F_z(\xi, \rho), \tag{1}$$

$$\frac{d\boldsymbol{p}_r}{d\tau} = \boldsymbol{F}_r(\xi, \boldsymbol{\rho}), \tag{2}$$

$$\frac{d\xi}{d\tau} = \frac{p_z}{\sqrt{1 + p_z^2 + p_r^2}} - \beta, \tag{3}$$

$$\frac{d\boldsymbol{\rho}}{d\tau} = \frac{\boldsymbol{p}_r}{\sqrt{1 + p_z^2 + p_r^2}}, \tag{4}$$

where p_z and $\boldsymbol{p}_r = \{p_x, p_y\}$ are normalized to mc the longitudinal and transverse components of the momentum of a bunch electron, perpendicular to the OZ axis; $k_p(z - V_{ph}t)$, $\boldsymbol{\rho} = k_p\boldsymbol{r} = k_p\{x, y\} = k_p r\{\cos\phi, \sin\phi\}$, $\phi = \arctan(y/x)$ are its dimensionless coordinates; $\tau = \omega_p t$; electron plasma frequency $\omega_p = \sqrt{4\pi e^2 n_0/m}$ is determined from the background plasma density n_0; $k_p = \omega_p/c$; V_{ph} is the phase velocity of the wake wave; and $\beta = V_{ph}/c$. The axial and radial components of the normalized force acting on an electron moving along the OZ axis with a velocity close to the velocity of light c can be expressed in terms of potential φ of the wake field as ($\boldsymbol{F}_r = \{F_x, F_y\} = F_r\{\cos\phi, \sin\phi\}$:

$$F_z \equiv \frac{eE_z}{mc\omega_p} = \frac{|e|}{mc^2}\frac{\partial\varphi}{\partial\xi}, \tag{5}$$

$$F_r \equiv \frac{eE_r}{mc\omega_p} - \frac{eB_\phi}{mc\omega_p} = \frac{|e|}{mc^2}\frac{\partial\varphi}{\partial\rho}, \tag{6}$$

where E_r, E_z, B_ϕ - electric and magnetic fields of the wake wave.

To simplify theoretical analysis of the motion of electrons in the wake field, we assume here that the wake field potential depends, apart from radius ρ, only on the concomitant variable ξ (and it does not depend explicitly on time); this implies steady-state waveguide propagation of the laser pulse in the plasma channel. It should also be noted that expression (6) for the radial force taking into account magnetic field B_ϕ is correct and quite accurate only for electrons with radial velocities much smaller than the velocity of light c. However, in the case of not very strong nonlinearity of the wake field ($|e\varphi|/mc^2 \leq 1$) and its characteristic transverse scale much exceeding skin depth k_p^{-1}, the magnetic field of the wake wave is much weaker than its electric field, and both force components, (5) and (6), are independent of the radial velocities of electrons being accelerated.

The effect of the laser pulse on the subsequent slow (low-frequency) motion of electrons can be disregarded because the ponderomotive potential of the laser field (which is proportional to $\sim a_0^2$) in the coordinate system associated with the wave is small as compared to the wake field potential proportional to $\sim \gamma_{ph}\varphi$ ($a_0 = |e|E_{L,0}/mc\omega$, $E_{L,0}$ being the amplitude of the high-frequency laser pulse and ω being its frequency, $\gamma_{ph} = 1/\sqrt{1-\beta^2}$) provided that $a_0 \leq 1$.

The effect of the intrinsic charge of the bunch on the process of its acceleration is also disregarded; this is justified for not very narrow bunches with a characteristic transverse size of $10^{-4} < k_p R_{b0}$, where R_{b0} is the radius of the injected electron bunch, which satisfy the condition [13]:

$$N_e << n_0(c/\omega_p)^3 = 4 \cdot 10^6 \lambda_p, \qquad (7)$$

where N_e is the number of electrons in the bunch and λ_p is the wavelength of plasma oscillations. Condition (7) indicates that the fields induced by the charge of the bunch are much weaker than the wake field in which electrons are accelerated.

We assume that a short relativistic nonmonoenergetic electron bunch is injected into a stationary wake wave propagating without changing its shape so that the region of bunch injection includes coordinate (or phase) ξ_m at which the potential of the wake wave has the maximal value $\varphi_{max} = \varphi\ (\xi_m,\ \rho = 0)$. The electron bunch injection scheme is shown in Fig.1, where along the axis OZ dotted line shows the laser pulse generating wake field, the solid line is the wake potential, dash-dot line shows the boundary of focusing and defocusing areas of wake field on the axis.

The direction of bunch injection coincides with the direction of phase velocity V_{ph} of the wake wave, the energy of electron injection being $E_{inj} = E_{res} = \gamma_{ph}mc^2$. It is believed that the electron bunch injection conditions are such that all injected electrons are trapped by wake field and grouped during their trapping near the boundary of focusing and defocusing areas.

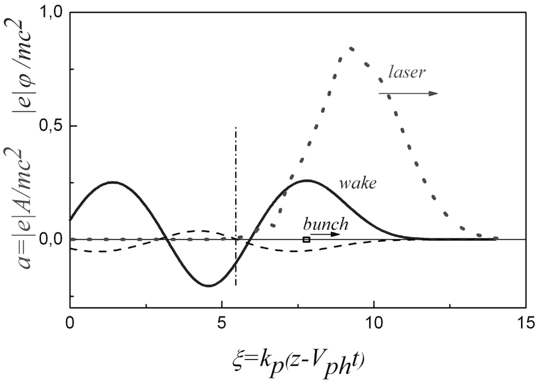

Fig.1. *Injection scheme. Mutual location along the OZ axis of the laser pulse, wake wave and bunch at the instant of injection*

The time-independence of the wake field in the coordinate system associated with the wave (i.e., in the concomitant system of coordinates moving with velocity V_{ph} along the OZ axis) makes it possible to write the energy conservation law in this coordinate system for an arbitrary electron bunch in the 2D formulation in the form

$$E' - |e|\varphi'(\xi, \rho) = E'_{inj} - |e|\varphi'(\xi_{inj}, \rho_{inj}), \qquad (8)$$

here and below, primes indicate that the primed quantities are taken in the wave frame of reference, ξ_{inj}, ρ_{inj} and E_{inj} are the coordinate and energy of electron injection into the wake wave.

We assume that the electron bunch is quite compact at the instant of injection and in the subsequent trapping and acceleration process, and that the electron energies differ insignificantly ($\Delta\xi_{inj} = k_p\Delta z_{inj} << 1$, $\rho_{inj} << 1$, $\Delta E_{inj} << E_{inj}$); i.e., electrons of the bunch in phase space (ξ, E) are quite close to one another at any instant. Then the analysis of the spatial structure of the bunch along the OZ axis in the region of trapping can be done as follows. We choose an electron that is exactly at the potential maximum of the wake field at the instant of injection and has injection energy equal to the mean electron injection energy of the bunch; we will henceforth refer to this electron as the central electron. The energy conservation law (8) at a known potential $\varphi(\xi\ , \rho)$ and known energy injection E_{inj} can determine the phase of the wake wave its trapping ξ_{tr} on the axis OZ, in which the velocity of the central electron is equal to the phase velocity of the wake wave (respectively, the electron energy will be equal $E = \gamma_{ph}mc^2$).

Earlier it was shown [11] that the greatest degree of grouping of 1D bunch along the axis OZ is realized when the trapping area of the injected electrons is located in the vicinity of the phase of the wake wave, in which its accelerating field is maximum. Since for the process of laser-plasma acceleration with minimal energy spread is preferred minimum length of accelerated bunch, then we assume that this condition is satisfied. Since the properties of linear and weakly nonlinear wake waves and look close enough, we can assume that for a weakly nonlinear wake wave the boundary of focusing and defocusing areas of wake field $\rho = \rho_{df}(\xi)$ close to the plane, as well as in the linear wake wave. On this boundary it is executed $\frac{\partial\varphi}{\partial\rho} = 0$ by definition, and the accelerating field near the border is close to the maximum.

All the trapped electrons of the bunch come to trapping area, i.e. acquire energy $\gamma_{ph}mc^2$, at different instants. But as in the trapping area the velocity of electrons close to the speed of light, then later in the process of the acceleration their mutual arrangement practically is unchanged. Therefore, the length of electron bunch in accelerating stage is determined by the length in the trapping region, which depends on the electron distribution along the axis OZ in this area.

Deviation of the phase of trapping along the axis OZ any trapped electron relatively the central electron ξ_{tr} can be found by varying the integral (8) in the small deviations $\delta\xi_{inj}$, ρ_{inj}, $\delta E'_{inj}$ of the electron from the central electron at the instant of injection. Neglecting in the trapping region the terms containing derivatives of wake potential along the radius, which are small due to the fact that the trapping area is located near the boundary of focusing and defocusing wake wave phase, we obtain the relation:

$$\delta\xi_{tr} = \left\{ \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial\xi_m^2}\delta\xi_{inj}^2 + \frac{\partial^2\varphi}{\partial\rho_m^2}\rho_{inj}^2 \right) + \right. \\ \left. + \left(\frac{V_{ph}}{u_{inj}} - 1 \right) \frac{\delta E_{inj}}{|e|} \right\} \left[\frac{\partial\varphi}{\partial\xi_{tr}} \right]^{-1}, \qquad (9)$$

where $\frac{\partial^2\varphi}{\partial\xi_m^2}$ and $\frac{\partial^2\varphi}{\partial\rho_m^2}$ are determined in the phase of wake field ξ_m, in which the wake potential is maximum, $\frac{\partial\varphi}{\partial\xi_m}$ is calculated in the trapping point of central electron.

2. TRAPPING OF ELECTRON BUNCH

The relation (9) makes it possible to analytically evaluate the length of the bunch in the trapping area and in the accelerating stage, if the spatial distribution of electrons in the bunch and their energy distribution in it at the instant of injection are known. If the distributions are normal (Gaussian), in which $\sigma_{z,inj}$, $\sigma_{\gamma,inj}$, $\sigma_{E,inj}$ are the corresponding standard deviations of the bunch electrons from the injection coordinate and energy of the central electron, we obtain the following expression for the bunch length in the accelerating stage:

$$k_pL_b = 2\left\{ \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial\xi_m^2} \right)^2 (k_p\sigma_{z,inj})^4 + \left(\frac{V_{ph}}{u_{inj}} - 1 \right)^2 \times \right. \\ \left. \times \frac{\sigma_{E,inj}^2}{|e|^2} + \left(\frac{\partial^2\varphi}{\partial\rho_m^2} \right)^2 (k_p\sigma_{r,inj})^4 \right\}^{1/2} \left[\frac{\partial\varphi}{\partial\xi_{tr}} \right]^{-1}. \qquad (10)$$

In this formula it is assumed that all the electrons injected into the wake field, were trapped for subsequent acceleration.

To check the formula (10) we numerically simulated the formation by wake field of electron bunch during their trapping. The simulation were performed for wake field that was established at the end of transition process after injecting into a plasma channel of short ($\tau_{FWHM} \approx 100\ fs$) pulse Ti:Sa laser with a wavelength of $\lambda_0 = 0.8\ \mu m$, a power 78 TW and the size of the focal spot $r_L \approx 68\ \mu m$.

Analysis of the nonlinear structure of a wake wave used in our study for simulating the trapping of electrons is based on the equations described in [14-16], where the generation of the wake field excited by a laser pulse propagating in the preliminarily prepared plasma channel was studied. We assume that the density $n(r)$ of plasma electrons in the channel varies in the direction perpendicular to channel axis OZ in accordance with the parabolic law $n(r) = n(r=0)\left[1 + r^2/R_{ch}^2\right]$, where channel radius R_{ch} is related to radius r_L of the focal spot of laser radiation by the condition $R_{ch} = k_pr_L^2/2$. The density $n(r) = n(r=0)$ of the plasma on the channel axis was chosen such that the resonance condition $\omega_p\tau_{FWHM} = 2\sqrt{2\ln 2}$ was satisfied for the excitation of a wake wave by the laser pulse of duration τ_{FWHM}. The envelope of electric field E_L of the laser pulse generating the wake wave and focused at the input of the plasma channel can be written in the form:

$$a = \frac{|e|E_L}{mc\omega} = a_0 \exp\left[-\frac{r^2}{r_L^2} - 2\ln 2\frac{(\xi - \xi_0)^2}{(\omega_p\tau_{FWHM})^2} \right], \qquad (11)$$

where τ_{FWHM} is the total duration of the laser pulse at half the maximum intensity (FWHM).

The resonance density of plasma electrons on the channel axis is $n_0 = n(r=0) \approx 1.75 \cdot 10^{17}\ cm^{-3}$ in this case and the matched channel radius is $R_{ch} \approx 180\ \mu m$, which corresponds to the following values of dimensionless parameters: $k_pr_L = 5.35$, $k_pR_{ch} = 14.3$, $a_0 = 0.71$. The gamma factor determined by the phase velocity of the wake wave being excited is $\gamma_{ph} = 1/\sqrt{1-\beta^2} = 100$.

Injected electron bunch were characterized by an average energy of electron injection $E_{inj} = 1.8\ mc^2$. Distribution of the injected electrons in the bunch, as all spatial axes, and in the energy space was considered Gaussian. Initial bunch length was 30.2 fs, which corresponds to the standard deviation in the distribution of the electron bunch length $\sigma_{z,inj} \approx 3.2\ \mu m$ (or $k_p\sigma_{z,inj} \approx 0.25$). The bunches of such longitudinal dimension can be generated by the best modern injectors [17, 18]. Original size of the bunch in the longitudinal direction was chosen on the basis of that in the case of one-dimensional bunch (bunch of infinitely small radius) all of its electrons do not go during the trapping process in the defocusing region of wake wave. Formula (10) in the limit ($\sigma_{z,inj} \to 0$) gives the result obtained earlier in [9-12], which indicates a significant contraction in the longitudinal direction of the injected bunch. Thus, it is expected that the application of the above expansion in the small parameters in the derivation of (10) is an adequate representation, and the length of the trapped bunch in the form (10) is right for compact injected electron bunch of not too large transverse dimension.

Fig.2 compares the results of numerical simulation (squares) with the results of a calculation, using formula (10) for the electron bunch length (dashed line), trapped from the initial nonmonoenergetic bunch with the initial energy spread between the electrons $\sigma_{E,inj} = 0.00825\ mc^2$ which corresponds to a relative energy spread 1 %. The axis of abscissa shows the characteristic transverse dimension of the injected bunch, on the left y-axis it is shown the bunch length in the trapping region. In the calculations we used the values $\frac{|e|}{mc^2}\frac{d\varphi}{d\xi_{tr}} = 0.22$, $\frac{|e|}{mc^2}\frac{d^2\varphi}{d\xi_m^2} = -0.16$, $\frac{|e|}{mc^2}\frac{d^2\varphi}{d\rho_m^2} = -0.055$, that characterize this wakefield.

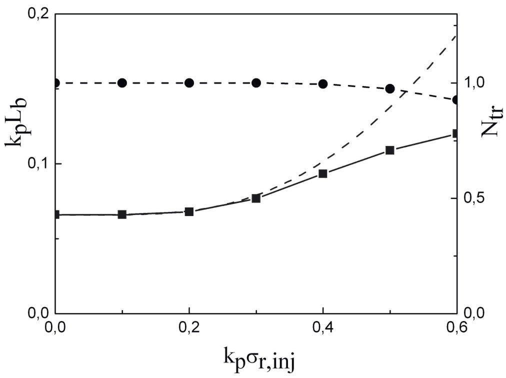

Fig.2. *Length of the trapped bunch and the proportion of trapped electrons (circles) as a function of the transverse dimension of the bunch at the instant of injection. For length: squares -simulation, dash - calculation by formula (10)*

From Fig. 2 it is seen that the bunch length, calculated from the analytical formula (10), sufficiently coincides exactly with the simulation results up to the value of the radius of the injected bunch, comparable to the longitudinal dimension of the injected bunch $k_p\sigma_{z,inj} \approx 0.3$. Then there is a growing difference between the graphs.

The reason is that with the growth of $k_p\sigma_{z,inj}$ an increasing number of electrons is not trapped by wake field, as electrons pass the boundary of focusing and defocusing regions of the wake wave. This clearly shows the graph (dashed line marked by circles) of the proportion of trapped electrons (right y-axis) in Fig. 2. Electrons scattered by wave wake not participate in the formation of a trapped electron bunch and, therefore, does not determine its length. Therefore, the formula (10) for the injected electron bunches of large transversal radius gives an inaccurate result.

CONCLUSIONS

The process of trapping of nonmonoenergetic compact electron bunch injected into the vicinity of the maximum potential of the wake wave generated by the laser pulse is investigated analytically and by numerical simulation. The phenomenon of electron grouping arising in the process of the trapping of electrons in the wake wave, if the initial velocity of the injected electrons is much smaller than its phase velocity, is studied in two-dimensional formulation of the problem. A simple formula, quite accurately predicts the longitudinal size of the bunch in the accelerating stage when all electrons of injected bunch are trapped. It is shown that the trapped electron bunch length in an accelerating stage is determined additively by its initial energy spread and spatial transverse and longitudinal dimensions. It is found that for the wake field in which the characteristic longitudinal and transverse dimensions are approximately equal, the initial transverse characteristic size of the bunch affects the length of the trapped bunch to the same extent as its initial longitudinal characteristic size at the instant of injection. Analytical results are confirmed by the results of simulation.

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ЗАХВАТ НЕМОНОЭНЕРГЕТИЧЕСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ В КИЛЬВАТЕРНОЙ ВОЛНЕ

С. В. Кузнецов

Проведено аналитическое исследование процесса захвата в кильватерной волне, возбуждаемой лазерным импульсом в плазменном канале, компактных немоноэнергетических электронных сгустков. Электроны инжектируются в окрестность максимума потенциала кильватерной волны со скоростью меньше её фазовой скорости. Получена формула, определяющая длину сгустка на ускоряющей стадии, как функция его начальных размеров (его длины и радиуса поперечного сечения) и начального разброса электронов по энергиям в нем.

ЗАХОПЛЕННЯ НЕМОНОЕНЕРГЕТИЧНИХ ЕЛЕКТРОННИХ ЗГУСТКІВ У КІЛЬВАТЕРНІЙ ХВИЛІ

С. В. Кузнецов

Проведено аналітичне дослідження процесу захоплення в кільватерній хвилі, збудженій лазерним імпульсом у плазмовому каналі, компактних немоноенергетичних електронних згустків. Електрони інжектуються біля максимуму потенціалу кільватерної хвилі зі швидкістю менше її фазової швидкості. Отримана формула, що визначає довжину згустка на прискорювальній стадії, як функція його початкових розмірів (його довжини і радіуса поперечного перерізу) і початкового розкиду електронів по енергіям в ньому.