

# NUCLEAR STRUCTURE OF $^{110}\text{Pd}$ AND $^{110}\text{Cd}$ ISOBAR BY INTERACTING BOSON MODEL (IBM-1)

*I. Hossain*<sup>1\*</sup>, *Hewa Y. Abdullah*<sup>2</sup>, *I. M. Ahmed*<sup>3</sup>

<sup>1</sup>*Department of Physics, Rabigh College of Science and arts, King Abdulaziz University, Rabigh 21911, Post box 344, Saudi Arabia;*

<sup>2</sup>*Department of Physics, College of Education, Scientific Department, Salahaddin University, Erbil, Krg, Iraq;*

<sup>3</sup>*Department of Physics, College of Education, Mosul University, Mosul, Iraq*

(Received October 28, 2014)

This paper presents a computational study in the field of nuclear structure by interacting boson model (IBM) to represents very important step formed in the description of collective nuclear excitations and the properties of electromagnetic transition. The ground state energy bands and the reduced transition probabilities  $B(E2) \downarrow$  up to  $8_1^+$  level of even-even nuclei  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  have been calculated by interacting boson model (IBM-1) and compared with previous experimental values. The set of parameters used in this calculation is the best approximation that has been carried out so far. The ratio of the excitation energies of the first  $4^+$  and the first  $2^+$  excited states,  $R_{4/2}$ , is also calculated and an achievable degree of agreement has been investigated in transitional symmetry  $U(5) - O(6)$  for  $^{110}\text{Cd}$  and  $O(6)$  for  $^{110}\text{Pd}$  nuclei. We have been compared  $B(E2)$  values of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei with theoretically and experimentally and their systematic studies as a function of angular momentum ( $L$ ). We have been studied systematically the ratios  $R_L = E(L^+)/E(2_1^+)$  and  $R = B(E2 : L^+ \rightarrow (L-2)^+)/B(E2 : 2^+ \rightarrow 0^+)$  of those nuclei in the ground-state band. Moreover, we have compared the attention to the analogy between the rotational frequency in ordinary space and Fermi energy in gauge space between  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei.

PACS: 21.60.Cs, 23.20Lv, 26.30.+k, 27.60+j

## 1. INTRODUCTION

Arima and Iachello have developed the interacting boson model (IBM), which is based on the well-known shell model and on geometrical collective model of the atomic nucleus [1,2]. The IBM-1 is used in the present work to represents very important step formed in the description of collective nuclear excitations and properties of electromagnetic transition. The underlying  $U(6)$  group structure of model basis leads to a simple Hamiltonian which is capable of describing the three specific limits of collective structure vibrational  $U(5)$ , rotational  $SU(3)$  and gamma unstable  $O(6)$ . The  $^{110}\text{Cd}$  and  $^{110}\text{Pd}$  nuclei, with two protons and four protons removed from a strong shell closure, exhibit intriguing aspects of nuclear structure at low excitation energies, namely the coexistence and mixing of vibrational or gamma unstable with other collective degrees of freedom arising from the promotion of a proton pair across shell gap [3,4]. The structure of neutron-rich  $\text{Cd}$  and  $\text{Pd}$  isotopes has been studied the subject of many theoretical and experimental works in recent years. Long et al. explained the low-lying levels and high-spin

states of  $^{116,118,120}\text{Cd}$  in the frame work of interacting boson model [5,6]. The ground state energy band up to  $8^+$  levels and reduced transition probabilities  $B(E2)$  values up to  $6^+$  to  $4^+$  levels in even-even  $^{114-122}\text{Cd}$  isotopes were studied under the framework of IBM-1 [7,8]. The evolution properties of even-even  $^{100-110}\text{Pd}$  nuclei were studied by Ahmed et al. [9]. In this study, we have carried out to compare the nuclear structure of level scheme, reduced transition probabilities, ground state energy band ratio as function of angular momentum between nuclei  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  showing the characteristic  $U(5)$  and  $O(6)$  pattern in those low-lying ground state bands within the frame work of IBM-1.

## 2. THEORY AND METHOD OF CALCULATION

### 2.1. Calculation of Energy levels

The energy levels are calculated using as follows: The Hamiltonian of the interacting bosons in IBM-1 is given by Ref.[10].

\*Corresponding author: hosain196977@yahoo.com, mihossain@kau.edu.sa Tel.: +966-558141319

$$H = \sum_{j=1}^N \varepsilon_j + \sum_{i<j}^N V_{i,j}. \quad (1)$$

Whereas  $\varepsilon$  is the intrinsic boson energy and  $V_{ij}$  is the interaction between bosons  $i$  and  $j$ . The multi-pole form of the IBM-1 the Hamiltonian is given by Ref.10

$$H = \varepsilon n_d + a_0 PP + a_1 LL + a_2 QQ + a_3 T_3 T_3 + a_4 T_4 T_4. \quad (2)$$

The  $n_d$  operator gives the number of d boson,  $p$  is the pairing operator for the  $S$  and d bosons,  $L$  is the angular momentum operator,  $Q$  is the quadrupole operator,  $T_3$  and  $T_4$  are the octupole and hexadecapole operators, respectively. Moreover  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_4$  are strength of pairing, angular momentum and multipole terms. The Hamiltonian as given in Eq.(2) tends to reduce to three limits, the vibration  $U(5)$ ,  $\gamma$ -soft  $O(6)$  and the rotational  $SU(3)$  nuclei [11]. In  $U(5)$  limit, the effective parameter is  $\varepsilon$ , in the  $\gamma$ -soft limit,  $O(6)$ , the effective parameter is the pairing  $a_0$ , and in the  $SU(3)$  limit, the effective parameter is the quadrupole  $a_2$ . The eigenvalues for the three limits are given as follows [1,12]:

$$U(5) : E(n_d, \nu, L) = \varepsilon n_d + K_1 n_d(n_d + 4) + K_4 \nu(\nu + 3) + K_5 L(L + 1). \quad (3)$$

$$O(6) : E(\sigma, \tau, L) = K_3 [N(N + 4) - \sigma(\sigma + 4)] + K_4 \tau(\tau + 3) + K_5 L(L + 1). \quad (4)$$

$$SU(3) : E(\lambda, \mu, L) = K_2 [\lambda^2 + \mu^2 + 3(\lambda + \mu) + \lambda\mu] + K_5 L(L + 1). \quad (5)$$

$K_1$ ,  $K_2$ ,  $K_3$ ,  $K_4$  and  $K_5$  are other forms of strength parameters. Many nuclei have a transition property between two or three of the above limits and their eigenvalues for the yrast-line are given by [12]:

$$U(5) - O(6) : E(n_d, \tau, L) = \varepsilon n_d + K_1 n_d(n_d + 4) + K_4 \tau(\tau + 3) + K_5 L(L + 1), \quad (6)$$

$$U(5) - SU(3) : E(\varepsilon, \lambda, L) = \varepsilon n_d + K_2 [\lambda^2 + 3(\lambda + \mu)] + K_5 L(L + 1), \quad (7)$$

$$O(6) - SU(3) : E(\tau, \lambda, L) = K_2 [\lambda^2 + 3(\lambda + \mu)] + K_4 \tau(\tau + 3) + K_5 L(L + 1). \quad (8)$$

## 2.2. Reduced transition probabilities $B(E2)$

The reduced transition probabilities using interaction boson model (IBM-1) [12] is given by equation (9).

$$B(E2; J + 2 \rightarrow J) \downarrow = \alpha_2^2 \frac{1}{4} (J + 2)(2N - J). \quad (9)$$

Where  $J$  is the state that the nucleus translates to it and  $B$  is the boson number, which is equal half the number of valence nucleons (proton and neutrons). The low-lying levels of even-even nuclei

( $J_i = 2, 4, 6, 8, \dots$ ) usually decay by  $E2$  transition to the lower-lying yrast level with  $J_f = J_i - 2$ . From the given experimental value of the transition ( $2 \rightarrow 0$ ), one can calculate the value the parameter  $\alpha_2^2$  for each isotopes and use this value to calculate the transition ( $8^+ \rightarrow 6^+$ ).

## 2.3. P-factor

The  $P$ -factor is calculated according to Eq.(10).

$$P = \frac{N_n N_p}{N_n + N_p}, \quad (10)$$

where  $N_n$  and  $N_p$  are the numbers of valence protons and neutrons, respectively,  $N_n N_p$  represents the number of  $p-n$  interactions and  $N_n + N_p$  is the number of pairing interactions.

## 2.4. Moment of inertia ( $\vartheta$ ) and gamma energy $E_\gamma$

The relation between the moment of inertia ( $\vartheta$ ) and gamma energy  $E_\gamma$  is given by [9]:

$$\frac{2\vartheta}{\hbar^2} = \frac{4I - 2}{E(I) - E(I - 2)} = \frac{4I - 2}{E_\gamma}. \quad (11)$$

And the relation between  $E_\gamma$  and  $\hbar\omega$  is given by [9,10]:

$$\hbar\omega = \frac{E(I) - E(I - 2)}{\sqrt{I(I + 1)} - \sqrt{(I - 2)(I - 1)}} = \frac{E_\gamma}{\sqrt{I(I + 1)} - \sqrt{(I - 2)(I - 1)}}. \quad (12)$$

## 2.5. Fermi energy (Gauge space)

The Fermi energies are calculated from the following relation [13]:

$$\lambda(N, I) = \frac{1}{2} [E_x(N + 1, I) - E_x(N - 1, I) - S_{2n}^{N+1}], \quad (13)$$

where  $N$  is the neutron number between the two even isotopes which are compared and  $S_{2n}^{N+1}$  is the separation energy.

$$S_{2n}^{N+1} = E_B(Z, N) - E_B(Z, N - 2). \quad (14)$$

## 3. RESULTS AND DISCUSSION

### 3.1. Boson numbers ( $N$ )

A simple correlation exists between the nuclei showing identical spectra and their valence neutron proton ( $N_p$ ), neutron number ( $N_n$ ). The identical of such a correlation scheme provided the clue to understand the identical band phenomena. It was natural to assume that the nuclei with equal total boson number  $N_b = N_p + N_n$  should have the same deformation and identical spectra. The number of valence proton  $N_p$  and neutron  $N_n$  has a total  $N = (N_p + N_n)/2 = n_\pi + n_\nu$  bosons. The boson numbers of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei are 9 and 7 respectively.

### 3.2. $P$ -factor

The pairing interaction between like nucleons drives the nucleons towards a spherical shape. It forms the  $J = 0^+$  coupling of pairs of identical nucleons that have spherical symmetric wave functions. Deformation and collectivity, on the other hand, arise from configuration mixing which corresponds to a non-uniform distribution of magnetic sub-state occupation and hence, of non-spherical shapes. Configuration mixing itself is largely driven by the valence  $p-n$  interaction. Hence it is a pairing  $p-n$  competition that tends to drive the structural evolution of nuclei. This idea was used to estimate the locus of collectivity in nuclei. One accepts significant collectivity and the onset of deformation when the  $P$ -factor given according to Eq. (10) and values were found 1.71 in  $^{110}\text{Cd}$  and 3.11 in  $^{110}\text{Pd}$  nuclei.

### 3.3. The $R_{4/2}$ classification and ground-state bands

In the collective dynamics of energies of the even-even nuclei were grouped into classes, within each class the ratio:

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

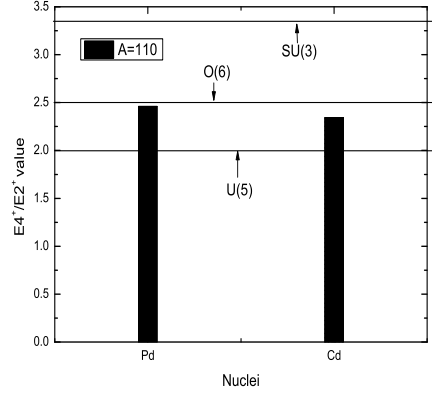
of excitation energies of the first  $4^+$  and the first  $2^+$  excited states. As pointed out by other similar ratios were characteristics of different collective motions of the nucleus.  $R_{4/2}$  has a limit value of 2 for vibrational nuclei  $U(5)$ , 2.5 for  $\gamma$ -unstable nuclei  $O(6)$  and finally 3.33 for rotational nuclei  $SU(3)$ . The suitable parameters for each nucleus at the evolving states are determined using Eq.3 and 4. Table 1 shows the values of these parameters that have been used to calculate the energy of the yrast-line states for the  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei. The experimental  $E(4_1^+)/E(2_1^+)$  for  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  Cadmium are 2.46 and 2.34 respectively. Fig.1 shows  $E(4_1^+)/E(2_1^+)$  values of  $U(5)$ ,  $O(6)$  and  $SU(3)$  limit and experimental values of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei. It is clear that  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei are transitional  $U(5) - O(6)$  symmetry, but  $^{110}\text{Pd}$  is very close to  $O(6)$  symmetry.

**Table 1.** Parameters in (keV) for even-even  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei

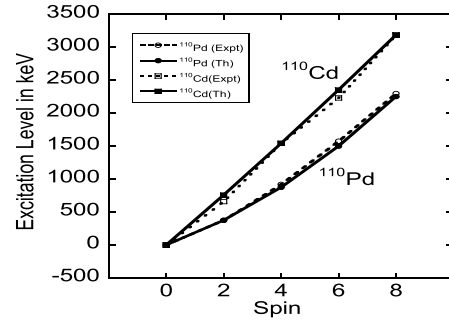
Nucl. $N$	$\epsilon$	$K_1$	$K_2$	$K_4$	$K_5$
$^{110}\text{Pd}$	-	-	-	113.78	-13.55
$^{110}\text{Cd}$	878.79	-46.67	-	-22.63	14.79

In Fig.2 we present the energies of the yrast sequences of ground state band as a function of angular momentum ( $L$ ) using IBM-1 in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei and compared them with previous experimental values [15]. It is shown that theoretical value using IBM-1

in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  are nicely reproduced to experimental values. The set of parameters used in this calculation is the best approximation that has been carried out so far. The excitation levels of ground state band in  $^{110}\text{Cd}$  are greater than those of  $^{110}\text{Pd}$ .



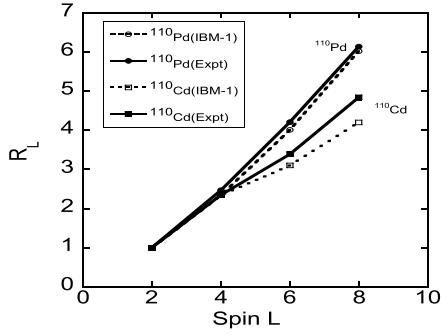
**Fig.1.**  $E(4_1^+)/E(2_1^+)$  in experimental values,  $U(5)$ ,  $O(6)$  and  $SU(3)$  limit of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei



**Fig.2.** Ground-state excitation levels as a function of angular momentum for  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei

### 3.4. Nuclear collectivity $R_L = E(L^+)/E(2_1^+)$ of $^{110}\text{Pd}$ and $^{110}\text{Cd}$

To measure nuclear collectivity, Fig.3 give the comparisons of the ratios  $R_L = E(L^+)/E(2_1^+)$  as a function of angular momentum ( $L$ ) in the ground-state band for  $^{110}\text{Cd}$  and  $^{110}\text{Pd}$  nuclei. The  $E(L^+)$  indicate ground state energy level at angular momentum  $L = 2, 4, 6,$  and  $8$ . The normalizations were taken to the energy of their respective  $2_1^+$  levels. In Fig.3 it is shown that  $R_L$  values up to  $4^+$  levels are overlaps to each other in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei and then diverse to the high spin states. We find that  $R_L$  values of  $^{110}\text{Pd}$  are larger than those of  $^{110}\text{Cd}$  after angular momentum  $L = 4$  and IBM-1 model show better agreement in  $^{110}\text{Pd}$  nucleus than  $^{110}\text{Cd}$  nucleus. The  $R_L$  values for  $^{110}\text{Pd}$  by IBM-1 and experimental results remain same up to spin 8. However, we find that the difference  $R_L$  between the calculation by IBM-1 and experimental results were consistently increases after  $4^+$  level and the  $R_L$  values were consistently smaller in the IBM calculations than those in experimental results [14,15] in  $^{110}\text{Cd}$ .



**Fig.3.** The yrast sequences of ground state band of  $R_L = E(L^+)/E(2_1^+)$  as a function of angular momentum (normalized to the energy of their respective  $2_1^+$  levels) in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei

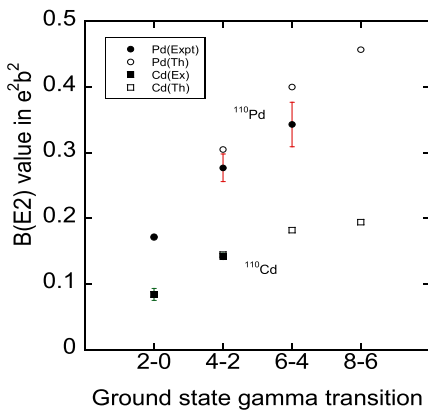
Therefore ground-state band by IBM-1 calculation show better agreement in  $^{110}\text{Pd}$  nucleus comparison to  $^{110}\text{Cd}$  nucleus.

### 3.5. Reduced transition probabilities $B(E2)$

In the principle, the value of the effective charge  $\alpha_2$  of the IBM-1 was determined by normalizing to the experimental data  $B(E2; 2_1^+ \rightarrow 0_1^+)$  of each isotope by using Eq.(1). From the given experimental value of the transitions  $2 \rightarrow 0$ , we have calculated the parameter  $\alpha_2^2$  for both  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei. The parameter  $\alpha_2^2$  is useful in order to calculate the transitions strength ( $4^+ \rightarrow 2^+$ ), ( $6^+ \rightarrow 4^+$ ) and ( $8^+ \rightarrow 6^+$ ). The  $B(E2)$  values were presented in Table 2, where the previous experimental results [14,15] are compared with the present calculations.

**Table 2.** Reduced transition probability  $B(E2) \downarrow$  in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  isobars

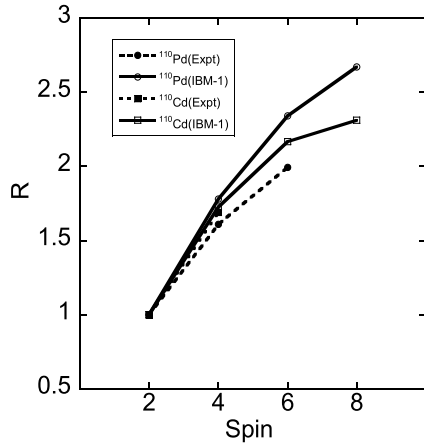
Nuclei	Boson #	Transition level	$B(E2)_{Ref[14,15]}$ W.U.	$e^2b^2$	$B(E2)_{IBM-1}$ $e^2b^2$
$^{110}\text{Pd}$	9	$2^+ \rightarrow 0^+$	55.5(9)	0.171(2)	0.171
		$4^+ \rightarrow 2^+$	90(7)	0.277(21)	0.305
		$6^+ \rightarrow 4^+$	108(11)	0.333(34)	0.400
		$8^+ \rightarrow 6^+$			0.457
$^{110}\text{Cd}$	7	$2^+ \rightarrow 0^+$	27.4(3)	0.084(9)	0.084
		$4^+ \rightarrow 2^+$			0.145
		$6^+ \rightarrow 4^+$			0.182
		$8^+ \rightarrow 6^+$			0.194



**Fig.4.** Reduced transition probabilities  $B(E2 : 2^+ \rightarrow 0^+, 4^+ \rightarrow 2^+, 6^+ \rightarrow 4^+$  and  $8^+ \rightarrow 6^+)$  of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei

The theoretical and experimental results of  $B(E2)$  values were plotted as a function of transition levels are shown in Fig.4 and it is observed that they are good agreement within experimental error. The values of reduced transitional probabilities are greater

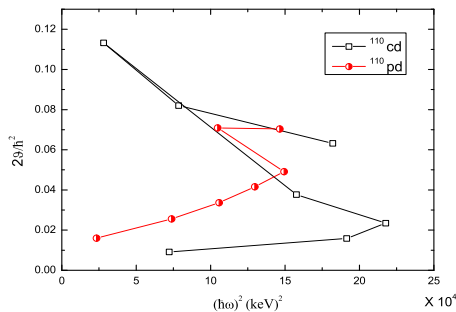
in  $^{110}\text{Pd}$  than those of  $^{110}\text{Cd}$  nucleus. It indicates that the equivalent effective charge in  $^{110}\text{Pd}$  is larger than that of  $^{110}\text{Cd}$  nucleus. The even-even nuclei  $^{110}\text{Cd}$  and  $^{110}\text{Pd}$  were nicely reproduced by the experimental results and their fits are satisfactory. In Fig.5 we Compare the ratio  $R = B(E2 : L^+ \rightarrow (L-2)^+)/B(E2 : 2^+ \rightarrow 0^+)$  of IBM-1 and previous experimental values in the ground state bands (normalized to the  $B(E2 : 2^+ \rightarrow 0^+)$ ) as a function of angular momentum  $L$ . It is shown that the results of  $R$  values are increased with increasing of high spin states. We have found calculated data overlap to experimental data in  $^{110}\text{Cd}$  nucleus. The results of  $R$  values of  $^{110}\text{Pd}$  nucleus are consistently smaller by experimental values than IBM-1 model. However, it is clear that the calculated results in the present work are the best agreement with previous results [14,15]. Actually, in IBM-1 the proton and neutron bosons are not distinguishable as long as valence protons and neutrons are both hole-like or both particle-like [2]. The large  $B(E2)$  values in  $^{110}\text{Pd}$  were the main indicator of gamma soft characters and  $^{110}\text{Cd}$  nuclei indicate the vibration to gamma soft character.



**Fig.5.**  $R$  values of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei using IBM-1 and experiment [14,15]. The ratio  $R = B(E2 : L^+ \rightarrow (L-2)^+) / B(E2 : 2^+ \rightarrow 0^+)$  in the ground state bands (normalized to the  $B(E2 : 2^+ \rightarrow 0^+)$ ) in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei

### 3.6. Moment of inertia of $^{110}\text{Pd}$ and $^{110}\text{Cd}$ nuclei

The moment of inertia  $2\vartheta/\hbar^2$  and rotational frequency  $\hbar\omega$  have been calculated from Eq.(11) and (12) respectively. The ground state bands up to 14 units of angular momentum are investigated for moment of inertia in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei. The moments of inertia as a function of square of rotational energy in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei are plotted in Fig.6. In the lowest order according to variable moment of inertia (VMI) model this should give a straight line in the plot of inertia  $2\vartheta/\hbar^2$  as a function of  $\omega^2$ . It is shown that the value of moment of inertia are greater in  $^{110}\text{Pd}$  nucleus than  $^{110}\text{Cd}$  in the lowest order of angular momentum and back-bending phenomena appear clearly after angular momentum  $L = 10$  and  $L = 8$  in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  respectively.

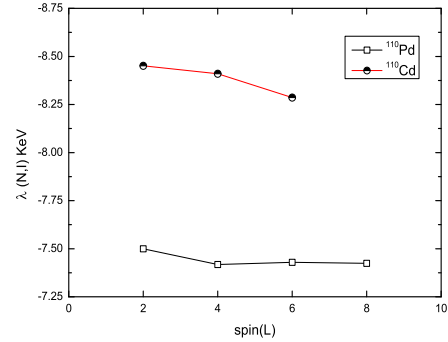


**Fig.6.** Collective moment of inertia vs. square of rotational energy in  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei

### 3.7. Fermi energies of $^{110}\text{Pd}$ and $^{110}\text{Cd}$

The Fermi energies  $\lambda(N, I)$  were calculated from equation (13). The comparisons of Fermi energies of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei in gauge space for

different spin are presented in Fig.7. The Fermi energy at  $2^+$ ,  $4^+$ ,  $6^+$  and  $8^+$  levels of  $^{110}\text{Pd}$  nucleus are  $-7.500$ ,  $-7.418$ ,  $-7.424$  and  $-7.424 \text{ keV}$  respectively. On the other hand the Fermi energy at  $2^+$ ,  $4^+$  and  $6^+$  levels of  $^{110}\text{Cd}$  are  $-8.451$ ,  $-8.410$ , and  $-8.286 \text{ keV}$  respectively. It is shown that Fermi energy as a function of spin for both nuclei is similar up to spin 4. The Fermi energy of  $^{110}\text{Pd}$  nuclei remains constant up to spin 8.



**Fig.7.** Fermi energy vs spin of  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  nuclei

## 4. CONCLUSIONS

The nuclear structure of ground state band up to positive parity states  $8^+$  of even-even  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  have been investigated within the frame works of interacting boson model. It was found that the ground state energy band and electric quadrupole reduced transition probability by IBM-1 are in good agreement with the previous experimental results [14,15]. The even-even  $^{110}\text{Cd}$  and  $^{110}\text{Pd}$  nuclei are  $U(5) - O(6)$  and  $O(6)$  symmetry respectively. The yrast levels of ground state band are greater in  $^{110}\text{Cd}$  than  $^{110}\text{Pd}$  nuclei. The reduced transition probabilities  $B(E2 : 2^+ \rightarrow 0^+, 4^+ \rightarrow 2^+, 6^+ \rightarrow 4^+ \text{ and } 8^+ \rightarrow 6^+)$  for  $^{110}\text{Pd}$  are stronger than  $^{110}\text{Cd}$  nucleus. Moreover, the investigation of the back bending phenomena in ordinary space for even-even  $^{110}\text{Pd}$  and  $^{110}\text{Cd}$  isobars were observed and compared with gauge space for the Fermi energies at different levels.

## ACKNOWLEDGEMENTS

The authors are thanks to king Abdulaziz University.

## References

1. F. Iachello and A. Arima. *The interacting boson model*, Cambridge Univ. Press, Cambridge, England, 1987, p.32-36.
2. A. A. Ridha, E. Abdulrasaq, and A. Jabbar // *J. Thi-Qar Univ.* 2009, v.5(2), p.2.

3. R. Kumar, A. Sharma, and J.B. Gupta // *Armenian J. Phys.* 2010, v.3(3), p.150.
4. A. Aphahamian, D. S. Brenner, R. F. Casten and K. Heyda // *Phys. Lett.* 1984, v.B.140, p.22.
5. G. L. Long, S. J. Zhut and H. Z. Sun // *J. Phys. G; Nucl. Part. Phys.* 1995, v.21, p.331.
6. M. Gorska, H. Grawe, D. Foltescu, D. Fossan, R. Grzywacz, J. Heese, K. Maier, M. Rejmund, H. Roth, R. Schubart // *Zeitschrift für Phys. A Hadrons and Nucl.* 1995, v.353(3), p.233.
7. I. Hossain, H. Y. Abdullah, I. M. Ahmed, M. A. Saeed and S. T. Ahmed // *Int. J. Modern Phys.* 2012, v.E.21(8), p.1250072-(1).
8. I. Hossain, H. Y. Abdullah, I. M. Ahmed, M. A. Saeed and S. T. Ahmed // *Armenian J. Phys.* 2012, v.5(3), p.101.
9. I. M. Ahmed, H.Y. Abdullah, S. T. Ahmed, I. Hossain, M. K. Kasmin, M. A. Saeed, N. Ibrahim // *Int. J. Modern Phys.* 2012, v.E.21(12), p.1250101-(1).
10. O. Scholten et al. // *Ann. Phys.* 1978, v.115, p.325.
11. F. Iachello // *Group Theory and Nuclear Spectroscopy, Lecture notes in Physics*, Nuclear Spectroscopy, "Springer", Berlin, 1981.
12. R. F. Casten, D. D. Warner // *Rev. Mod. Phys.* 1988, v.60, p.389.
13. M. Hussien // *Bull. Fac. Sci., Zagazig Univ.*, 1993, v.15(2), p.536.
14. D. D. Frenne, E. Jacobs // *Nucl. Data Sheets*, 2009, v.110, p.1745.
15. G. Gurdal, F. G. Kondev // *Nucl. Data Sheets*, 2012, v.113, p.1315.

### ЯДЕРНАЯ СТРУКТУРА ИЗОБАР $^{110}\text{Pd}$ И $^{110}\text{Cd}$ В МОДЕЛИ ВЗАИМОДЕЙСТВУЮЩИХ БОЗОНОВ ( $IBM - 1$ )

*И. Хоссайн, Ева Я. Абдуллах, И. М. Ахмед*

Представлено компьютерное исследование в области ядерной структуры с помощью модели взаимодействующих бозонов ( $IBM - 1$ ), представляющей очень важный шаг в направлении описания коллективных ядерных возбуждений и свойств электромагнитных переходов. Уровни энергий основных состояний и соответствующие вероятности переходов  $B(E2) \downarrow$  на уровень  $8_1^+$  четно-четного ядра  $^{110}\text{Pd}$  и  $^{110}\text{Cd}$  были рассчитаны с помощью модели взаимодействующих бозонов ( $IBM - 1$ ) и сравнены с полученными ранее экспериментальными данными. Набор используемых в данной работе параметров является наилучшим приближением в сравнении с полученными ранее. Отношение энергий возбуждения первого  $4^+$  и первого  $2^+$  возбужденных состояний  $R_{4/2}$  также вычислены, и достижимая степень согласия была исследована в переходной симметрии  $U(5) - O(6)$  для ядра  $^{110}\text{Cd}$  и  $O(6)$  для ядра  $^{110}\text{Pd}$ . Мы сравнили  $B(E2)$  величины ядер  $^{110}\text{Pd}$  и  $^{110}\text{Cd}$  с теоретическими и экспериментальными и ихними систематическими исследованиями как функций углового момента ( $L$ ). Мы изучили систематически отношения  $R_L = E(L^+)/E(2_1^+)$  и  $R = B(E2 : L^+ \rightarrow (L - 2)^+)/B(E2 : 2^+ \rightarrow 0^+)$  этих ядер в основном состоянии. Кроме того, мы обратили внимание на аналогию между вращательной частотой в обычном пространстве и энергией Ферми в калибровочном пространстве между ядрами  $^{110}\text{Pd}$  и  $^{110}\text{Cd}$ .

### ЯДЕРНА СТРУКТУРА ИЗОБАР $^{110}\text{Pd}$ І $^{110}\text{Cd}$ У МОДЕЛІ ВЗАЄМОДІЮЧИХ БОЗОНІВ ( $IBM - 1$ )

*І. Хоссайн, Єва Я. Абдуллах, І. М. Ахмед*

Представлено комп'ютерне дослідження в області ядерної структури за допомогою моделі взаємодіючих бозонів ( $IBM - 1$ ), яка є дуже важливим кроком у напрямку опису колективних ядерних збуджень і властивості електромагнітних переходів. Рівні енергій основних станів і відповідні вірогідності переходів  $B(E2) \downarrow$  на рівень  $8_1^+$  парно-парного ядра  $^{110}\text{Pd}$  і  $^{110}\text{Cd}$  були розраховані за допомогою моделі взаємодіючих бозонів ( $IBM - 1$ ) і порівняні з отриманими раніше експериментальними даними. набір використаних у даній роботі параметрів є найкращим наближенням у порівнянні з отриманими раніше. Відношення енергій збудження першого  $4^+$  і першого  $2^+$  збуджених станів  $R_{4/2}$  також вираховані, і доступна ступінь узгодження були досліджені в переходній симетрії  $U(5) - O(6)$  для ядра  $^{110}\text{Cd}$  і  $O(6)$  для ядра  $^{110}\text{Pd}$ . Ми порівняли  $B(E2)$  величини ядер  $^{110}\text{Pd}$  і  $^{110}\text{Cd}$  з теоретичними і експериментальними і їхніми систематичними дослідженнями як функцій кутового момента ( $L$ ). Ми вивчили систематично відношення  $R_L = E(L^+)/E(2_1^+)$  і  $R = B(E2 : L^+ \rightarrow (L - 2)^+)/B(E2 : 2^+ \rightarrow 0^+)$  цих ядер в основному стані. Крім того, ми звернули увагу на аналогию між обертовою частотою в звичайному просторі з енергією Фермі в калібровочному просторі між ядрами  $^{110}\text{Pd}$  і  $^{110}\text{Cd}$ .