STABLE RELATIVE EQUILIBRIA IN THE SYSTEM OF SUPERCONDUCTIVE AND PERMANENT MAGNETIC DIPOLES

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(Received January 27, 2015)

This paper analytically proves the existence of stable orbital motions in a system of superconductive and permanent magnetic dipoles. As opposed to a system of two permanent magnetic dipoles, that has been studied in the work of I. Tamm and V. Ginzburg in this system we have no «problem $1/R^3$», because of that the stability of the system becomes possible.

PACS: 45.05.+x, 45.20.-d, 45.20.Jj

1. INTRODUCTION

In the system of permanent dipoles there is no static equilibrium. This result was proved in [1] and that proof is similar to the well known Earnshaw theorem [2]. But the inner and relative rotations of the dipoles can act as stabilizing factors, whereas Earnshaw's theorem not be applicable to the dynamic systems. Therefore, the question of the existence of stable motions of the magnetic dipoles in connection with the hypothesis about magnetic nature of the nuclear forces [3] was considered by I. Tamm and V. Ginzburg in both cases for classical and quantum theory. And they formulate so called «problem $1/R^3$» that relates to the interaction of magnetic dipoles. This result casts doubt on the possibility of stable motions in systems of small bodies that interact by magnetic forces.

In the context of classical electrodynamics the magnetic dipole is an equivalent of a small loop of current. It is assumed that the current in loop is constant. Other hand, the laws of electrodynamics does not forbid us to consider a small superconductive loop. A current of such a loop is not constant, but magnetic flux is constant (or as the phrase goes "frozen"). It is perfectly acceptable to call such an object as a superconductive dipole.

In the paper [1] we derived an expression for the potential energy of interaction in a system consisting of permanent magnets and superconducting circuits.

It develops that interaction of permanent and superconductive magnetic dipoles does not fall under the above-mentioned «problem 1/R3». This raises the question of the possibility of stable orbital motions in such a system.

The modern Hamiltonian formalism based on group-theoretical methods [4, 5, 6, 7, 8] is an effective tool for studying the stability of magnetic systems with symmetries [9, 10].

This approach allows us to analytically prove the existence of stable orbital motions in a system consisting of a superconductive and permanent magnetic dipoles.

2. MATHEMATICAL MODEL

Let's consider the superconducting dipole as a small circular loop that fixed in the origin of coordinates with a normal of $\boldsymbol{\gamma} = \boldsymbol{e}_3$ that is directed along the axis of z. Its radius is denoted by r_s, total "frozen" magnetic flux is Ψ, and its self-inductance is L.

The movable dipole can be described as a circular loop with radius r_p, and a current I that associated with its magnetic moment $\boldsymbol{\mu}$:

$$\boldsymbol{\mu} = \pi r_p^2 I \boldsymbol{\nu} = |\boldsymbol{\mu}|\boldsymbol{\nu}. \tag{1}$$

In the dipole approximation, mutual inductance M of two loops has the form [1]:

$$M = \mu_0 \pi \frac{r_s^2 r_p^2}{4r^5}\left(3\langle\boldsymbol{\gamma},\boldsymbol{r}\rangle\langle\boldsymbol{\nu},\boldsymbol{r}\rangle - r^2\langle\boldsymbol{\gamma},\boldsymbol{\nu}\rangle\right), \tag{2}$$

where r_s, r_p – the corresponding radiuses of the currents loops (s – superconductive current and p – direct current);

$\boldsymbol{\gamma}, \boldsymbol{\nu}$ – the corresponding normals;

$\boldsymbol{r}$ – the radius-vector of a movable dipole.

Potential energy of the interaction of the loops was derived in [1]:

$$V = \frac{1}{2}\frac{(\Psi - MI)^2}{L}, \tag{3}$$

where MI, using (1) can be transformed to

$$MI = \frac{S\mu_0|\boldsymbol{\mu}|}{4\pi r^5}\left(3\langle\boldsymbol{\gamma},\boldsymbol{r}\rangle\langle\boldsymbol{\nu},\boldsymbol{r}\rangle - r^2\langle\boldsymbol{\gamma},\boldsymbol{\nu}\rangle\right), \tag{4}$$

where S – the area of the superconducting dipole, or

$$MI = \frac{S\mu_0|\boldsymbol{\mu}|}{4\pi r^3}(3c'c'' - c'''), \tag{4a}$$

where

$$\begin{cases} r = |\boldsymbol{r}|; \\ \boldsymbol{e}_r = \boldsymbol{r}/|\boldsymbol{r}|; \\ c' = \boldsymbol{e}_3 \cdot \boldsymbol{e}_r = x^3/r; \\ c'' = \boldsymbol{\nu} \cdot \boldsymbol{e}_r; \\ c''' = \boldsymbol{e}_3 \cdot \boldsymbol{\nu} = \nu^3. \end{cases} \tag{5}$$

Then the potential energy has the form

$$V(r, c', c'', c''') = \frac{1}{2}\kappa \left(\psi - \frac{3c'c'' - c'''}{r^3}\right)^2 = \\ = \frac{\kappa}{2}\pi(r, c', c'', c''')^2, \tag{6}$$

where

$$\begin{cases} \kappa = \frac{1}{L}\left(\frac{S\mu_0|\boldsymbol{\mu}|}{4\pi}\right)^2; \\ \psi = \frac{4\pi\Psi}{S\mu_0|\boldsymbol{\mu}|}; \\ \pi(r, c', c'', c''') = \psi - \frac{3c'c'' - c'''}{r^3}. \end{cases} \tag{7}$$

For the first derivatives we have

$$\begin{cases} \partial_r V = \frac{3\kappa\pi_0}{r_0^3}\frac{3c'c'' - c'''}{r_0}; \\ \partial_{c'} V_{|z_e} = -\frac{3\kappa\pi}{r^3}c''; \\ \partial_{c''} V_{|z_e} = -\frac{3\kappa\pi}{r^3}c'; \\ \partial_{c'''} V = \frac{\kappa\pi}{r_0^3}. \end{cases} \tag{8}$$

With regard to the mechanical properties of the movable magnetic dipole it is a small rigid body – symmetric top (two principal moments of inertia $I_1 = I_2 = I_\perp$), and its mechanical symmetry coincides with the magnetic.

The corresponding Hamiltonian of the system will be [9]:

$$H = T + V, \qquad T = \frac{1}{2M}\boldsymbol{p}^2 + \frac{\alpha}{2}\boldsymbol{n}^2, \tag{9}$$

where M – mass of permanent dipole; $\alpha = 1/I_\perp$ – the position of the dipole; $\boldsymbol{p}$ – its momentum; $\boldsymbol{n}$ – the intrinsic angular momentum of the dipole.

Then equations of motion have the form [11, 9]:

$$\begin{cases} \dot{\boldsymbol{r}} = \frac{1}{M}\boldsymbol{p}; \\ \dot{\boldsymbol{p}} = -\partial_r V \boldsymbol{e}_r - \frac{1}{r}(\partial_{c'} V P^e_\perp(\boldsymbol{e}_3) + \partial_{c''} V P^e_\perp(\boldsymbol{\nu})); \\ \dot{\boldsymbol{\nu}} = \alpha(\boldsymbol{n} \times \boldsymbol{\nu}); \\ \dot{\boldsymbol{n}} = \partial_{c''} V(\boldsymbol{e}_r \times \boldsymbol{\nu}) + \partial_{c'''} V(\boldsymbol{e}_3 \times \boldsymbol{\nu}), \end{cases} \tag{10}$$

where the operator $P^e_\perp$ – projection onto the plane that perpendicular to the vector $\vec{e}_r$, i.e. $P^e_\perp(\boldsymbol{e}_3) = \boldsymbol{e}_3 - c'\boldsymbol{e}_r$ and $P^e_\perp(\boldsymbol{\nu}) = \boldsymbol{\nu} - c''\boldsymbol{e}_r$.

3. RELATIVE EQUILIBRIA

Group-theoretic methods Hamiltonian dynamics have proven effective in many problems of mechanics [4, 5, 6, 8] and, in particular, in the study of the stability of the magnetic dynamical systems [9, 10].

There are a number of theorems [7, 8], which give us the conditions of stability of relative equilibria, i.e. such trajectories of the Hamiltonian system that are also the orbits of the one-parameter subgroups of the invariance group of the system under study [4, 7].

Modern Hamiltonian formalism developed in two basic versions: symplectic manifolds and Poisson manifolds. Appropriate tools to investigate the stability of relative equilibria are available in both approaches.

The system has axial symmetry about z axis, i.e. invariant with respect to the subgroup S^1 of the rotation group $SO(3)$ and, additionally, has a mirror symmetry with respect to the plane $z = 0$.

Thus, the system under study has the same set of symmetries, the same set of dynamic variables to describe the state, and the same kinetic energy as for the Orbitron system in work [9].

Therefore, to describe the system under study is suitable Hamiltonian formalism on the basis of the Poisson structure (see [9]), and the difference is in the form of potential energy of the system.

The main tool for studying the stability of relative equilibria in our case, as in the above-mentioned work, will be the Theorem 4.8. [8], which is valid for dynamical systems with symmetry in general case of Poisson manifolds.

An important advantage of this theorem is that investigation of the function space of the trajectories replaced on the investigation of the finite-dimensional vector space of the variations of dynamic variables in the supporting point of the trajectory (relative equilibria). At this case the scheme of stability investigation is broadly similar to the study of conditional extremum of the function using Lagrange multipliers method.

As it was already mentioned, the invariance group of the Orbitron is S^1. Each one-parameter subgroup of this group will be characterized by the own angular velocity $\boldsymbol{\omega}_0 = \omega_0\boldsymbol{e}_3$. The rate of change of any physical quantity $\boldsymbol{v}$ of our problem along the orbit of this subgroup will be given by the formula $\dot{\boldsymbol{v}} = \boldsymbol{\omega}_0 \times \boldsymbol{v}$.

Therefore, for a relative equilibria such relations must be satisfied

$$\begin{cases} \dot{\boldsymbol{r}} = \omega_0(\boldsymbol{e}_3 \times \boldsymbol{r}); \\ \dot{\boldsymbol{p}} = \omega_0(\boldsymbol{e}_3 \times \boldsymbol{p}); \\ \dot{\boldsymbol{\nu}} = \omega_0(\boldsymbol{e}_3 \times \boldsymbol{\nu}); \\ \dot{\boldsymbol{n}} = \omega_0(\boldsymbol{e}_3 \times \boldsymbol{n}). \end{cases} \tag{1}$$

We will show that there is a dynamic orbit, for which performed these relations. Taking into consideration the mirror symmetry of the problem let's consider the orbit that located in the plane $z = 0$. We also assume that $\boldsymbol{\nu} \parallel \boldsymbol{e}_3$ and $\boldsymbol{n} \parallel \boldsymbol{e}_3$. Then along all this trajectory $c' = c'' = 0$, $c''' = \pm 1$ and from (8)§2 followed that $\partial_{c'} V = \partial_{c''} V = 0$.

3rd and 4th equation of system (10)§2 thus performed identically, and the 1st and 2nd are reduced to second-order equation of the form:

$$M\ddot{\boldsymbol{r}} + \left(\frac{\partial_r V}{r}\right)_{|r=r_0} \boldsymbol{r} = 0. \tag{2}$$

For the conditions $(\partial_r V)_{|r=r_0} > 0$ the equation (2) has a solution and corresponds to motion along a circle of radius r_0 at a rate that is determined by the relation

$$\left(\frac{\partial_r V}{r}\right)_{|r=r_0} = \omega_0^2 M. \quad (3)$$

Thus, we can conclude that for this orbit, indeed was proved that it is a relative equilibrium.

4. CHOICE OF THE SUPPORTING POINT

Let's set the point on an orbit of relative equilibrium

$$z_e = \begin{cases} \boldsymbol{x}_0 = r_0 \boldsymbol{e}_1; \\ \boldsymbol{p}_0 = p_0 \boldsymbol{e}_2; \\ \boldsymbol{\nu}_0 = \nu_0 \boldsymbol{e}_3, \quad \nu_0 = \pm 1. \\ \boldsymbol{n}_0 = n_0 \boldsymbol{e}_3; \end{cases} \quad (1)$$

In this point

$$\begin{cases} c' = 0; \\ c'' = 0; \\ c''' = \nu_0; \end{cases} \quad (2)$$

$$\begin{cases} \partial_r V_{|z_e} = -\frac{3\kappa\nu_0\pi_0}{r_0^4}; \\ \partial_{c'} V_{|z_e} = 0; \\ \partial_{c''} V_{|z_e} = 0; \\ \partial_{c'''} V_{|z_e} = \frac{\kappa\pi_0}{r_0^3}; \\ \pi_0 = \psi + \frac{\nu_0}{r_0^3} \end{cases} \quad (3)$$

and the following expressions for the differentials of the arguments of the function V

$$\begin{cases} \boldsymbol{d}r = \boldsymbol{d}x^1; \\ \boldsymbol{d}c' = \boldsymbol{d}\langle \boldsymbol{e}, \boldsymbol{e}_3 \rangle = \frac{1}{r}(\boldsymbol{d}x^3 - c'\boldsymbol{d}x^1); \\ \boldsymbol{d}c'' = \boldsymbol{d}\nu^1 + \frac{1}{r}(\nu_0 \boldsymbol{d}x^3 - c''\boldsymbol{d}x^1); \\ \boldsymbol{d}c''' = \boldsymbol{d}\nu^3 . \end{cases} \quad (4)$$

5. NECESSARY CONDITION OF STABILITY AND LAGRANGIAN COEFFICIENTS

As motion integrals we will take

$$\begin{cases} j_3 = x^1 p^2 - x^2 p^1 + n_3; \\ C_1 = \frac{1}{2}\boldsymbol{\nu}^2; \\ C_2 = \langle \boldsymbol{\nu}, \boldsymbol{n} \rangle; \end{cases} \quad (1)$$

where 1st line represents a 3rd conserved quantity of a body total angular momentum, and the other two are Casimir functions of the system.

Efficiency function (adjoined Hamiltonian) looks like

$$\tilde{H} = T + V - \omega_0 j_3 + \lambda_1 C_1 + \lambda_2 C_2 . \quad (3)$$

The necessary condition of stability in theorem 4.8. [8] requires the differential of efficiency function to be equal to zero in a supporting point.

Write out the correspondent differentials in a supporting point z_e

$$\begin{cases} (\boldsymbol{d}T)_{|z_e} = \frac{p_0}{M}\boldsymbol{d}p_2 + \alpha n_0 \boldsymbol{d}n_3; \\ (\boldsymbol{d}V)_{|z_e} = \partial_r V \boldsymbol{d}x^1 + \partial_{c'''} V \boldsymbol{d}\nu_3; \end{cases} \quad (4)$$

$$\begin{cases} (\boldsymbol{d}j_3)_{|z_e} = p_0 \boldsymbol{d}x^1 + r_0 \boldsymbol{d}p_2 + \boldsymbol{d}n_3; \\ (\boldsymbol{d}C_1)_{|z_e} = \nu_0 \boldsymbol{d}\nu_3; \\ (\boldsymbol{d}C_2)_{|z_e} = (n_0 \boldsymbol{d}\nu_3 + \nu_0 \boldsymbol{d}n_3). \end{cases} \quad (5)$$

Collecting the differentials of efficiency function, we get

$$\boldsymbol{d}\tilde{H}_{|z_e} = (\partial_r V_{|z_e} - \omega_0 p_0)\boldsymbol{d}x^1 + \left(\frac{p_0}{M} - \omega_0 r_0\right)\boldsymbol{d}p_2 \quad (6)$$
$$+(\partial_{c'''} V_{|z_e} + \lambda_1 \nu_0 + \lambda_2 n_0)\boldsymbol{d}\nu^3 + (\alpha n_0 - \omega_0 + \lambda_2 \nu_0)\boldsymbol{d}n_3.$$

Equating $\boldsymbol{d}\tilde{H}_{|z_e} = 0$, we derive the following expression for Lagrange multipliers

$$\begin{cases} p_0/M = \omega_0 r_0; \\ \omega_0 p_0 = \partial_r V_{|z_e}; \\ \lambda_2 = \nu_0 \omega_0 - \alpha \nu_0 n_0; \\ \lambda_1 = -\nu_0 \partial_{c'''} V_{|z_e} - \lambda_2 \nu_0 n_0 = \\ \qquad -\nu_0 \partial_{c'''} V_{|z_e} + n_0(\alpha n_0 - \omega_0). \end{cases} \quad (7)$$

The 1st equation in (7) is an ordinary relationship between linear and angular velocity during circular orbital motion.

The 2nd equation in (7) represents the equality of centrifugal (on the left) and centripetal (on the right) forces.

From this two expressions we get the relationship for angular velocity, namely:

$$M\omega_0^2 = \frac{1}{r_0}\partial_r V_{|z_e} = -\frac{3\kappa\nu_0\pi_0}{r_0^5} = -\frac{3\nu_0}{r_0^2}\partial_{c'''} V_{|z_e}. \quad (8)$$

6. ALLOWABLE VARIATIONS

For the application of the sufficient condition of stability in the theorem 4.8. in [8] it is necessary to extract a linear subspace of allowable variations.

Let's consider the variations of the dynamic variables annihilating the differentials in formula (5)§5.

From the 2nd line in (5)§5 it follows, that $\delta\nu^3 = 0$, then it ensues from the 3rd line, that $\delta n^3 = 0$.

Thus, we obtain

$$\begin{cases} \delta\nu^3 = 0; \\ \delta n_3 = 0; \\ \delta p_2 = -\frac{p_0}{r_0}\delta x_1. \end{cases} \quad (1)$$

Hence it ensues that the variations in the form

$$\delta x^1, \delta x^2, \delta x^3; \quad \delta p_1, \delta p_3; \quad \delta\nu^1, \delta\nu^2; \quad \delta n_1, \delta n_2 \quad (2)$$

can be considered as independent variations, furthermore, we must exclude from this subspace the direction which is tangent to the orbit.

It ensues from formula (1)§3, that this direction (in z_e point) is determined as

$$\begin{cases} \delta\boldsymbol{x} = r_0 \boldsymbol{e}_2; \\ \delta\boldsymbol{p} = -p_0 \boldsymbol{e}_1; \\ \delta\boldsymbol{\nu} = 0; \\ \delta\boldsymbol{n} = 0. \end{cases} \quad (3)$$

In order to eliminate the variation (3), we impose another additional condition on variations, and then we get the constraints

$$\begin{cases} \delta\nu^3 = 0; \\ \delta n_3 = 0; \\ \delta p_1 = \frac{p_0}{r_0}\delta x_2; \\ \delta p_2 = -\frac{p_0}{r_0}\delta x_1, \end{cases} \qquad (4)$$

and an independent set of variations will be

$$\delta x^1, \delta x^2, \delta x^3; \quad \delta p_3; \quad \delta\nu^1, \delta\nu^2; \quad \delta n_1, \delta n_2. \qquad (5)$$

7. BASIC QUADRATIC FORM

Sufficient condition for a minimum consists in positive definiteness of quadratic form of type $\boldsymbol{d}^2\tilde{H}_{|z_e}(\delta z, \delta z)$, where variation vector δz must be expressed through independent variations (5)§6 taking into account the constraints (4)§6. Quadratic form defined in independent variations we denote by Q.

Calculations of the efficiency function hessian (adjoined Hamiltonian) and basic quadratic form in independent variations were performed in Maple.

After insignificant transposition of columns (and corresponding lines with the same number) the matrix of basic quadratic form acquires a form

$$\begin{bmatrix} Q_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Q_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{33} & Q_{35} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{35} & Q_{55} & Q_{57} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{57} & Q_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q_{66} & Q_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & Q_{68} & Q_{88} \end{bmatrix}. \qquad (1)$$

Let's write out diagonal elements from matrix of quadratic form Q:

$$\begin{cases} Q_{11} = \frac{3\kappa}{r_0^5}\left(\nu_0\pi_0 + \frac{3}{r_0^3}\right); \\ Q_{22} = 4M\omega_0^2; \\ Q_{33} = 3M\omega_0^2; \\ Q_{44} = \frac{1}{M}; \\ Q_{55} = n_0\left(\alpha n_0 - \omega_0\right) + \frac{1}{3}Mr_0^2\omega_0^2; \\ Q_{66} = Q_{55}; \\ Q_{77} = Q_{88} = \alpha. \end{cases} \qquad (2)$$

For matrix Q to be positive definite it is foremost necessary that all diagonal elements of the matrix are positive.

The positive definite of Q_{44}, Q_{77}, Q_{88} are scienter positive.

The positive definite of Q_{22} and Q_{33} elements ensured by the physically obvious requirement:

$$M\omega_0^2 = -\frac{3\kappa\nu_0\pi_0}{r_0^5} \longrightarrow \nu_0\pi_0 < 0. \qquad (3)$$

From (2) (i.e. $Q_{11} > 0$, $Q_{55} = Q_{66} > 0$) and (3) we have

$$\begin{cases} -\frac{3}{r_0^3} < \nu_0\pi_0 < 0; \\ n_0\left(\alpha n_0 - \omega_0\right) + \frac{1}{3}Mr_0^2\omega_0^2 > 0. \end{cases} \qquad (4)$$

Let's consider non-diagonal elements.

First consider lower-right-hand block

$$\begin{bmatrix} Q_{66} & Q_{68} \\ Q_{68} & Q_{8,8} \end{bmatrix}, \qquad (5)$$

where

$$Q_{68} = \nu_0\left(\omega_0 - \alpha n_0\right), \qquad (6)$$

then the conditions of positive definiteness of (5) will be

$$Q_{88}Q_{66} - Q_{68}^2 = \omega_0(\alpha n_0 - \omega_0) + \frac{1}{3}Mr_0^2\omega_0^2 > 0. \quad (7)$$

Now consider the central 3×3-block, where

$$Q_{35} = M\nu_0 r_0\omega_0^2, \qquad (8)$$

$$Q_{57} = -\nu_0(\alpha n_0 - \omega_0), \qquad (9)$$

then the condition of positive definiteness of central block are as follows

$$\begin{cases} Q_{33}Q_{55} - Q_{35}^2 = 3Mn_0\omega_0^2(\alpha n_0 - \omega_0) > 0; \\ Q_{33}Q_{55}Q_{77} - Q_{33}Q_{57}^2 - Q_{35}^2 Q_{77} = 3M\omega_0^3(\alpha n_0 - \omega_0) > 0. \end{cases} \qquad (10)$$

So, considering the $\pi_0 = \psi + \frac{\nu_0}{r_0^3}$ we have the following non-trivial conditions for positive definiteness of the matrix Q

$$\begin{cases} -3 < (1 + \nu_0 r_0^3\psi) < 0; \\ n_0(\alpha n_0 - \omega_0) > 0; \\ \omega_0(\alpha n_0 - \omega_0) > 0. \end{cases} \qquad (11)$$

Thus for the stability of a given relative equilibrium in our system the *geometric* conditions must be carried out

$$-3 < (1 + \nu_0 r_0^3\psi) < 0 \qquad (12)$$

and the *dynamical* conditions

$$\begin{cases} n_0(\alpha n_0 - \omega_0) > 0; \\ \omega_0(\alpha n_0 - \omega_0) > 0. \end{cases} \qquad (13)$$

The last conditions can also be written as

$$\begin{cases} \mathrm{sign}(n_0) = \mathrm{sign}(\omega_0); \\ \alpha|n_0| > |\omega_0|; \end{cases} \qquad (14)$$

or, given the $n_0 = I_3\Omega_0$, where Ω_0 — the frequency of self-rotation of the movable body

$$\begin{cases} \mathrm{sign}(\Omega_0) = \mathrm{sign}(\omega_0); \\ |\Omega_0| > \frac{I_\perp}{I_3}|\omega_0|. \end{cases} \qquad (15)$$

8. ESTIMATION OF PHYSICAL PARAMETERS

Let's show that we can choose the system parameters that correspond to the available material and technological capabilities, and in addition does not violate the assumption of dipole nature and justice of quasi-stationary approximation.

Let's consider a small ring Nb_3Sn of radius $R = 0,005$ (m) with the radius of the wire $r = 0,0005$ (m), and disc permanent magnet $NdFeB$ (with density $\rho = 7,4 \cdot 10^3$ (kg/m^3) with residual induction $B_r = 0,25$ (T)) with diameter $D = 0,014$ (m) and height $h = 0,006$ (m) ($M = 0,0068$ (kg) — mass).

These bodies are characterized by a set of magnetic parameters. For the ring this is self-inductance that can be estimated from the expression [12] (5-1) p.207:

$$L = \mu_0 R \left(\ln\left(\frac{8R}{r}\right) - \frac{7}{4} + \frac{1}{8}\frac{r^2 \ln\left(\frac{8R}{r}\right) + \frac{1}{3}}{R^2} \right),$$

for the permanent magnet this is a magnetic moment $\mu = 0,18$ (A m^2).

Let's freeze the flow through the ring (i.e., translate it into a superconducting state) when the permanent magnet located in the equatorial plane at a distance of $r_K = 0,05$ (m) (disc of the magnet located mirror-symmetrically relative to the equatorial plane and with magnetic moment directed down, i.e. $\nu_0 = -1$). Then move the magnet on a distance $r_0 \simeq 0,059$ (m) and provide it initial velocity such that the angular velocity of the orbital motion was $\omega_0 \simeq 0,0152$ (rad/s) and also small self-rotation (such that condition (15)§7 will met).

Then such motion will be stable, because all stability conditions §7 are met.

The smallness of the angular velocity is dictated due smallness of the centripetal magnetic force, because in fact a magnetic dipole interacts with itself through the magnetic flow induced in the superconducting ring (at a relatively large distance). It is easy to deduce that with decreasing of the all size of the system in k times its circular frequency of rotation also increased in the same times.

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УСТОЙЧИВЫЕ ОТНОСИТЕЛЬНЫЕ РАВНОВЕСИЯ В СИСТЕМЕ, СОСТОЯЩЕЙ ИЗ СВЕРХПРОВОДЯЩЕГО И ПОСТОЯННОГО МАГНИТНЫХ ДИПОЛЕЙ

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Аналитически доказано существование устойчивых орбитальных движений в системе, состоящей из сверхпроводящего и постоянного магнитных диполей. В отличие от исследованной И. Таммом и В. Гинзбургом системы из двух постоянных магнитных диполей «проблема $1/R^3$» в данной системе не возникает, и устойчивость становится возможной.

СТІЙКІ ВІДНОСНІ РІВНОВАГИ В СИСТЕМІ, ЩО СКЛАДАЄТЬСЯ З НАДПРОВІДНОГО ТА ПОСТІЙНОГО МАГНІТНИХ ДИПОЛІВ

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Аналітично доведено існування стійких орбітальних рухів у системі, що складається з надпровідного та постійного магнітних диполів. На відміну від системи з двох постійних магнітних диполів, що досліджена І. Таммом та В. Гінзбургом, «проблема $1/R^3$ » в даній системі не виникає, і отже стійкість стає можливою.