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Locally Imperfect Conical Shells under Uniform External Pressure

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Расчет конических оболочек с локальными геометрическими несовершенствами, подвергнутых внешнему равномерному давлению

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Во многих конструкциях используются конические оболочки с различного рода геометрическими несовершенствами. Однако известно очень мало работ, посвященных исследованию влияния геометрических несовершенств на критическое значение внешнего давления, при котором происходит потеря устойчивости конических оболочек. В настоящей работе исследуется нарушение геометрической формы конической оболочки в виде впадины в зоне продольного сварного шва, описываются разные геометрические модели, включая конечноэлементные, а также обсуждается влияние геометрического несовершенства формы оболочки на величину критической нагрузки, при которой имеет место потеря устойчивости таких конструкций. Полученные результаты сравниваются с ранее приведенными экспериментальными данными, некоторыми аналитическими результатами, а также сопоставляются с рекомендациями и стандартами.

Ключевые слова: конические оболочки, несовершенство, внешнее давление, потеря устойчивости.

Notation

α	–	semi-vertex angle
SL/r_e and t/r_e	–	functions of $\beta_{\min}$
d	–	mean diameter of the cone
D	–	diameter of the wide end of the cone
$L/(2R_{mean}\cos\alpha)$ and $(2R_{mean}\cos\alpha/t)$	–	functions of ε
E	–	the Young modulus
p	–	uniform external pressure
P_{cr}	–	critical external pressure
P_{mc}	–	elastic instability pressure for conical shells
r	–	radius of the small end of the cone

R – radius of the large end of the cone

R_{mean} – mean radius of the cone

SL – slant length

t – thickness of the shell

$$L_e = \frac{SL\cos\alpha}{2}\left(1+\frac{r}{R}\right),\ r_e = \frac{R+r}{2\cos\alpha},\ r_m = \frac{R+r}{2},\ t_e = t\cos\alpha,\ w = r/40$$

Introduction. Conical shells appear in various geometries and in a number of branches of engineering structures such as cooling towers, silos, submarine pressure hulls, off-shore structural components, tanks and their roofs, fluid reservoirs, etc. Thus, such structures deserve much attention of the researchers. On the other hand, locally imperfect truncated conical shells were not thoroughly undertaken in the previous studies, and this issue has remained almost untouched. Therefore, it is a matter of concern to study the buckling behavior of such shells considering the local imperfections developed during fabrication process due to such structures, high imperfection sensitivity.

There is a vast scope of literature available on the effect of the imperfections on the structural behavior of shells of revolution. Amongst those, some investigations are briefly outlined in this section.

Studies based on shell theory which address the issues of dent imperfection stress response are reported by Calladine [1] and Croll et al. [2]. However, these studies are primarily devoted to the application of an approximate method of imperfection analysis called the equivalent load method on the subject. Note that the shell theory sources do not show the existence of a size effect of different cases of geometric imperfections. On the other hand, the threat posed by combinations of dents and gouges was reported [3–5] in which the local stress concentration resulting from dent and gouge combinations was considered.

Holst et al. [6] studied the strains developed by fabrication misfit of perfect and imperfect shells to attain equivalent residual stresses. Showkati and Golzan [7] conducted six tests on frusta specimens (truncated conical shells) under uniform external pressure considering ordinary geometric imperfections which are usually caused by fabrication process. They showed that buckling loads obtained from experiments are lower than the results obtained from finite element analysis (FEA) and theoretical predictions due to the existence of initial imperfections. Shen and Chen [8] investigated the buckling and post-buckling behavior of perfect and imperfect shells under axial loading and peripheral pressure. They showed the effects of geometry, loading and initial imperfections on the buckling response of such structures. An attempt was made by Schneider et al. [9] to develop a new non-destructive buckling test technique for imperfect shells based on the concept in which natural frequency in the buckling mode shape goes to zero when the critical buckling load is reached. Showkati [10] studied experimentally and numerically the buckling behavior of imperfect thin-walled cylindrical shells under uniform external pressure considering different boundary conditions. Yamaki [11] also fully investigated the nonlinear behavior of cylindrical shells under external pressure and effects of geometrical imperfections. He showed that geometric irregularities are the most important factor on the response of shell buckling behavior.

The motivation behind this paper was to execute the role of longitudinal weld induced depression caused at the immediate vicinity zone of longitudinal weld in conical shells during the fabrication process (Fig. 1). It is known that the value of the geometric depression depends on the methods used for welding. In the present study each model has numerically simulated with different depth of longitudinal imperfections to evaluate the effect of such geometric parameter on the buckling load of such structures. It was obtained that, as the depth of the longitudinal imperfection increases, the bucking load of the conical shell structures decreases particularly for higher values of imperfection. The aims of the present study are briefly summarized as follows:

(i) numerical simulations of locally imperfect slender conical shells;

(ii) comparison between the corresponding experimental results and the present results;

(iii) evaluation of the results of the numerical study and the results of theoretical predictions and recommendations;

(iv) evaluation of the effect of local imperfections on the buckling load of conical shell structures.

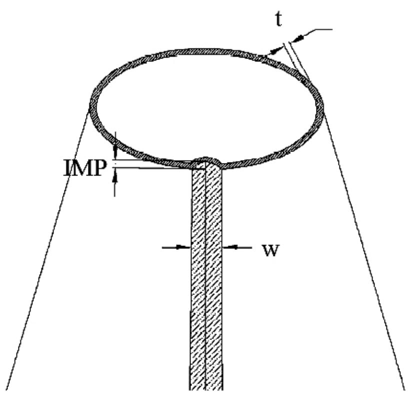

Fig. 1. Geometric specifications for imperfect region.

1. **Previous Experimental Study**. Golzan and Showkati [7] evaluated the buckling behavior of six nearly geometrically perfect conical shells under external pressure, considering unavoidable manufacturing imperfections. In this paper, a comparison has been made between the test results of the experimental study and the corresponding locally imperfect models of the present research.

It is worth bearing in mind that truncated conical shells subjected to external uniform pressure were discussed closely in the previous research concerning the experimental responses and numerical results in contrast. The buckling aptitude of such shells was obtained to be contingent upon the basic geometric ratio of "slant-length to radius" (L/R). In this paper, the relevant finite element (FE) models of three frusta specimens with longitudinal imperfections have been taken into account having a comparison with the later published results. The geometric specifications of the models are tabulated in Table 1. Schematic illustration of the frusta models is also shown in Fig. 2.

Table 1

Geometric Specifications of the Models

Model label	r, mm	R, mm	t, mm	h, mm	SL/R
SC1	100	300	0.6	223.6	1.0
SC2	100	300	0.6	403.2	1.5
SC3	100	300	0.6	565.7	2.0

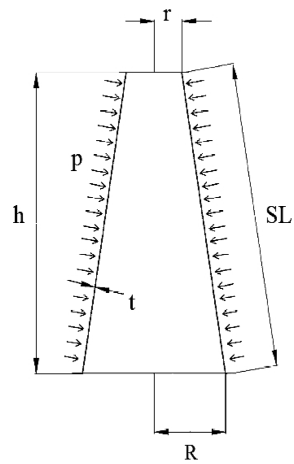

Fig. 2. Geometric parameters of the conical shells.

2. **Finite Element Analysis**. FE results presented in this paper were derived using the general-purpose finite element program ABAQUS designed specifically for advanced structural analyses. The program has been shown to give accurate predictions by widely-used comparisons between theoretical and experimental results and has been applied extensively to study buckling, post-buckling and collapse behavior of shells of revolution.

2.1. ***Type of Analysis***. Generally speaking, buckling studies of shell structures basically require two types of analysis. First, eigenvalue analysis is usually used to obtain estimates of the buckling loads and modes. Such study also provides guidance in mesh design because mesh convergence studies are required to ensure that the eigenvalue estimates of the buckling load have converged. This requires that the mesh should be adequate to model the buckling modes, which are usually more complex than the pre-buckling deformation mode. The second phase of the study is load-displacement analysis, usually using the “riks” method to handle possible instabilities. This analysis would typically study imperfection sensitivity of shell structures to investigate the effect of the geometric perturbation on the responses [12].

It is worth to note that the derived upshots are referred to as linear and nonlinear analyses respectively in this study. In the bifurcation buckling analysis which is based on linear theory, geometrically perfect shells under external pressure are considered. On the other hand, local imperfections are taken into account in nonlinear buckling analysis which determines the response based on the locally imperfect shell geometry.

The models of nonlinear analysis which have been studied herein are intended for the elasto-plastic analysis of shells of revolution. Note that comprehensive modeling of the structures requires determination of the entire equilibrium path until collapse occurs. In the analysis of these structures, material and geometrical nonlinearities were undertaken. In the present study, the "arc-length-type" method which is the most efficient method for this purpose and it is now predominantly used in structural nonlinear analyses was used [13]. In the arc-length method, a constraint equation controls the load increment in order to force the iteration path to follow either a plane normal to the tangent at the starting point of the iteration, or a sphere with its center at the starting point.

2.2. ***FEA Modeling of Thin-Walled Conical Shells***.

2.2.1. *Element Type*. The element S4R was used for modeling such shell elements. Element S4R has four nodes including five independent degrees of freedom. Note that these are the three orthogonal translations and the dimensions changing of two independent components of a unit vector normal to the surface of the shell, which is considered as the rotations. It is to be said that the normal vector's third component is derived from the condition in which the normal vector length is assumed to be equal to unity. The independent degrees of freedom are all interpolated linearly. Externally, three rotational and three translational degrees of freedom per node are available to the user. This element is well-suited for modeling shell structures. Large deflection, stress stiffening and nonlinear analysis are implied by its capabilities.

2.2.2. *FE Model Specifications*. The geometric nonlinear analysis has been taken into account in which uniform external pressure was modeled as a follower force. Boundary conditions were modeled in such a way that the radial constraint was provided at two edges of the models. Note, however, that the models were free to move in axial direction. The stress–strain input to the model matched those used in [7] and is given in Table 2.

Table 2

Material Properties

Young modulus (GPa)	Yield stress (MPa)	Failure stress (MPa)	Poisson's ratio
210	277	373	0.3

2.2.3. *Imperfections*. The imperfections were modeled directly as a table of node numbers and coordinate perturbations in the global coordinate system. The depth of local imperfections is different in the models in which for each model the imperfections with the depth of 0.5, 1, 1.5, 2, 2.5, and 3 mm were modeled and they are called the imperfect cases of IMP.1 to IMP.6, respectively, in this study, bearing in mind that local imperfections in the current models are formed longitudinally along the slant length of each model (Fig. 3).

3. **Results and Discussion**. It should be remembered that, since the present models are slender shells with high R/t ratio, they buckle prior to attaining their plastic capacity. Numerical results confirm this fact, in which the stresses in the pre-buckling phase are always less than the yield stress. Figure 4 shows the values

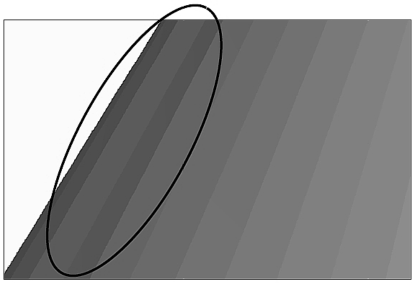

Fig. 3. Imperfect region layout of a typical numerical model.

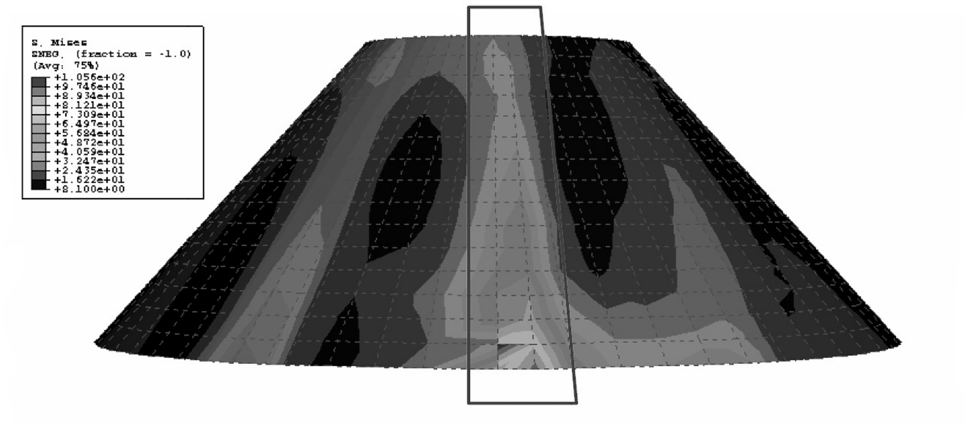

Fig. 4. Pre-buckling stress magnitude (MPa) and distribution for SC1, IMP.1 at the zone of imperfection.

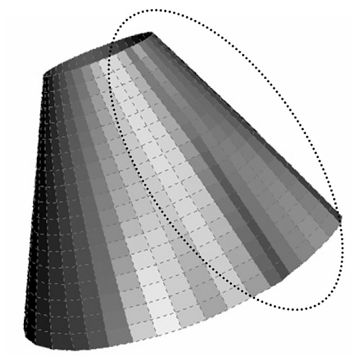

Fig. 5. Deformed mode of SC2 at the zone of imperfection.

of the stresses and stress distribution at the pre-buckling stage of SC1 at the region of local imperfection in which the maximum stresses happen around the locally imperfect zone of the present models. Note, however, that along with formation of large deformations (Fig. 5), the material becomes plastic at depression and projection areas.

Table 3 represents the comparison between the buckling load of different models and test results. It also shows the effect of depression type of imperfections on the reduction of the buckling load (Fig. 6). It is noteworthy that for the case

IMP.1 of the model SC1, which have the slightest imperfection value in comparison to the other cases, one can observe 6% difference with the corresponding experimental model, whereas for IMP.6 this difference value increases to 37.4%. Moreover, for the model SC2 the differences between the case IMP.1 and IMP.6 with the experimental buckling load are 1.5 and 28.2%, respectively. Accordingly, it is clear that for slight imperfections (mostly IMP.1) the value of the numerical results for all the models are even more than the buckling load values obtained from the experiments; so that, for lower values of imperfection the effect of the local imperfection does not seem significant and the results approach the experimental conical shells with ordinary manufacturing imperfections. However, for deeper imperfections the numerical results are quite lower than the experimental results, while for the cases of IMP.4 to 6 the decreasing effect of the longitudinal imperfection is noticeable.

T a b l e 3

Buckling Load of Conical Shells

Model label	Experimental buckling load of [7] (kPa)	P_{cr}, kPa, for IMP.1	P_{cr}, kPa, for IMP.2	P_{cr}, kPa, for IMP.3	P_{cr}, kPa, for IMP.4	P_{cr}, kPa, for IMP.5	P_{cr}, kPa, for IMP.6
SC1	25	26.5	23.8	22.1	20.7	19.5	18.2
SC2	20	19.7	18.5	17.5	16.7	16.1	15.6
SC3	14	18.3	17.2	16.3	15.7	15.1	14.8

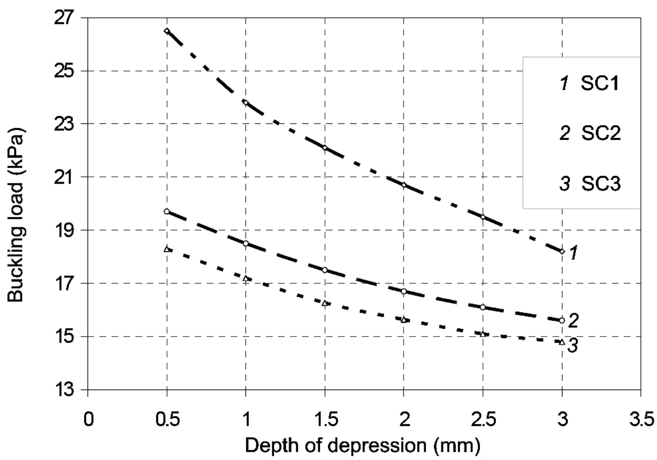

Fig. 6. Buckling load reduction versus the depth of imperfection.

3.1. ***Comparison with Theoretical Predictions and Recommendations***. In fact, there are limited sources reporting on critical pressure prediction of the externally pressurized truncated conical shells. Thus, available predictions and recommendations are presented herein to have comparison with the present numerical models (Table 4 and Fig. 7).

3.1.1. *Simplified Theoretical Equations*. Jawad [14] has pointed out that conical shells subjected to external pressure may be analyzed as cylindrical shells

T a b l e 4

Comparison with Theoretical Predictions and Standards

Model label	P_{cr}, kPa, obtained from Eq. (1)	P_{cr}, kPa, obtained from Eq. (2)	P_{cr}, kPa, obtained from ECCS recommendation	P_{cr}, kPa, obtained from DTMB formula	P_{mc}, kPa, obtained from BSI, PD 5500
SC1	35.40	54.8	39.9	41.9	35.2
SC2	32.30	43.6	35.0	36.5	31.7
SC3	24.68	33.5	26.1	29.4	28.0

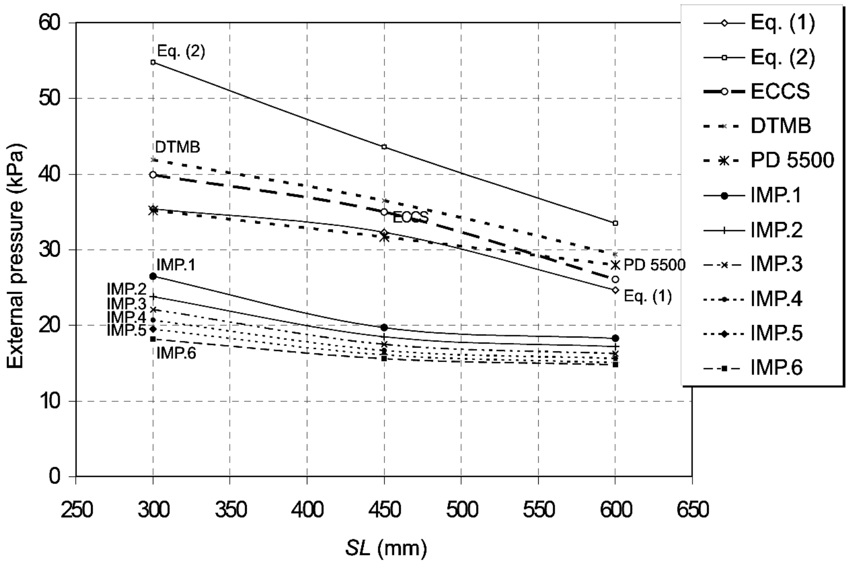

Fig. 7. Critical external pressure obtained from different solutions.

by applying the effective thickness and length in the calculations. Thus, Eq. (1) was proposed to determine the critical buckling pressure of metallic conical shells:

$$P_{cr} = \frac{0.92 E (t_e / R)^{5/2}}{L_e / R}. \tag{1}$$

Considering this equation, the discrepancies of 33.6, 64, and 35% between the predictions and the numerical results of the case IMP.1 are obtained for the models SC1 to SC3, respectively. It is noted that these values for the case IMP.6 are 94.5, 107.1, and 66.8%, respectively, which can also show the significant influence of the longitudinal geometric imperfections.

Equation (2) also represents the results of linear buckling analysis of intermediate-length conical shells subjected to uniform external pressure [15]:

$$P_{cr} = \frac{0.92 E t}{SL \cos\alpha} \left(\frac{t}{r_m} \right)^{3/2} (\cos\alpha)^{3/2}. \tag{2}$$

The boundary conditions are assumed in such a way that the shell ends cannot displace in the direction normal to the shell mid-surface. Indeed, the biggest difference is derived comparing the results of the present study and the critical pressure of Eq. (2) in comparison to the other theoretical predictions outlined in this study. This difference may be attributed to various factors: firstly, as noted above, the boundary conditions assumed in this equation are different from those of the present models. Note that this difference can noticeably govern the buckling pressure of the shells. Secondly, in Eq. (2) the critical buckling pressure was derived using linear analysis, whereas in the present numerical study both geometric and material non-linearity were applied to the models. Thus, one may note that all these differences along with the effect of geometric imperfections can significantly affect the buckling pressure of the shells and make the differences between the different solutions.

3.1.2. *Standards and Recommendations*. To the knowledge of authors, there are also limited codes and recommendations evaluating the buckling pressure of externally pressurized conical shells. Here, three available recommendations are used to determine the critical pressure and to compare the results with the locally imperfect numerical models.

At first, European recommendation for buckling of steel shells (ECCS) [16] is undertaken to appraise the results. In this recommendation, a simplified equation is used to determine the buckling pressure:

$$P_{cr} = E(t/r_e)\beta_{\min}. \tag{3}$$

It is noted here that the difference between the numerical imperfect models and Eq. (3) for SC1 to SC3 and for the cases of minor and major imperfection are 50.6, 77.7, and 42.6% (for IMP.1) and 119.2, 124.4, and 76.4% (for IMP.6). These differences also show the influence of initial geometric imperfection on the buckling behavior of conical shell structures.

A further predicting formula reported by Ross [17] and known as David Taylor model basin (DTMB) equation, which is assumed to be the basic equation of the design chart proposed by Ross to estimate the inelastic buckling pressure of conical shells is

$$P_{cr} = \frac{2.6E(t/d)^{5/2}}{l/d - 0.45(t/d)^{1/2}}. \tag{4}$$

It is mentioned that DTMB equation shows the discrepancies of 58.1%, 85.3% and 60.7% with the imperfect cones of the numerical models of SC1 to SC3 for the case of IMP.1, while for IMP.6 these values changes into 130.2%, 134% and 98.6%.

Ultimately, the results of the present models are compared with the British Standards Institution (BSI) rules (PD 5500) which are used to obtain the elastic instability pressure for the conical sections [18]:

$$P_{mc} = \frac{Et\varepsilon\cos^3\alpha}{R_{mean}}. \tag{5}$$

It is worthy to note here that with a comparative view one can verify this fact that although the results obtained from Eq. (5) overestimate the buckling pressure of the present numerical models too, this equation generally gives closer results, in comparison to the other theoretical predictions.

In all, the main conclusion to be drawn is that the results of Eqs. (1) and (5), in comparison with the other predictions discussed in this study, may appear to give closer results for slender truncated cones under uniform external pressure. So that, it is recommended to use the aforementioned equations to predict the buckling pressure of slender shells' within the range of this study. On the other hand, the lower the value of the imperfection the closer the numerical results to the theoretical predictions.

It is also noteworthy that the longest models (SC3) show the smallest differences with theoretical predictions and recommendations. In our opinion, due to the fact that the longer specimens are less sensitive to the boundary effects, and also owing to the shells different boundary conditions which have been assumed in different solution; it can make lower discrepancies between theoretical recommendations and test results rather for the longer specimens than the shorter ones.

Conclusions. Although a large body of literature grew up on the effects of geometric imperfections of the shell structures, quite few are the studies devoted to the related problems of local longitudinal imperfections of the shells under external pressure. For instance, the effect of longitudinal depression-shaped imperfections caused by welding process has remained untouched. To this end, in this study some effort has been put toward defining the effect of such initial geometric imperfections on thin-walled conical shells which are classified as highly imperfection sensitive structures. Some salient points arising from the present study are worth considering as follows:

1. Under external pressure, the shorter the slant length, the greater the efficiency of the longitudinal imperfection on the buckling pressure.

2. For lower values of imperfection (e.g., IMP.1) the effect of local imperfection does not seem significant and the results approach the experimental conical shells with ordinary manufacturing imperfections.

3. Due to high R/t ratio of the present models, the conical shells buckle prior attaining their plastic capacity. Indeed, the stresses in the pre-buckling phase are always less than the yield stress. Note, however, that along with formation of large deformations, the material becomes plastic and the stresses exceed the yield stress especially at the zone of imperfection.

4. The results of Eqs. (1) and (5), in comparison with the other predictions discussed in this study, give closer results for slender truncated cones under uniform external pressure.

5. In the present models, the lower the value of the imperfection the closer the numerical results to the theoretical predictions.

6. Generally, the longest model (SC3) shows the smallest difference with theoretical predictions and recommendations.

Ultimately, it is noteworthy that, although the results presented herein are reliable due to the reasonable correlation between the present results and test results and the theoretical methods, overall, it is not clear whether the results will yield a similar accuracy for all kinds of shell structures. Thus, the current work is merely a point of departure, from which additional studies may be made on the various shell structures with different cases of imperfections.

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Р е з ю м е

У багатьох конструкціях використовуються конічні оболонки з різного роду геометричними недосконалостями. Однак відомо дуже мало робіт, де досліджується вплив геометричних недосконалостей на критичне значення зовнішнього тиску, за якого відбувається втрата стійкості конічних оболонок. У даній роботі досліджується порушення геометричної форми конічної оболонки у вигляді западини у зоні поздовжнього зварного шва, описуються різні геометричні моделі, у тому числі скінченноелементні та обговорюється вплив геометричної недосконалості форми оболонки на величину критичного навантаження, за якого має місце втрата стійкості таких конструкцій. Отримані результати порівнюються з раніше наведеними експериментальними даними, деякими аналітичними результатами та зіставляються з рекомендаціями і стандартами.

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