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Fracture Investigation of Wood under Mixed Mode I/II Loading Based on Maximum Shear Stress Criterion

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Анализ разрушения деревянных образцов по смешанной моде I/II с помощью критерия максимальных касательных напряжений

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Рассматривается применение нового критерия разрушения для прогнозирования направления инициирования и роста трещин в деревянных образцах при смешанной моде нагружения I/II. Данный критерий учитывает распределение максимальных касательных напряжений в окрестности вершины трещины. Его главное преимущество по сравнению с другими известными критериями – более точный учет влияния зоны повреждения на разрушение дерева. Изменение податливости поврежденного материала, связанное с ростом микротрещин, учитывается с помощью специального коэффициента повреждаемости. Предложенный критерий применяется в случае разрушения образцов с произвольной ориентацией трещин относительно ортотропных осей материала. Установлено, что такой подход хорошо описывает механизм разрушения деревянных образцов. Приведено сравнение расчетных результатов с известными экспериментальными.

Ключевые слова: критерий разрушения, смешанная мода нагружения, дерево, касательное напряжение, зона повреждения.

Introduction. Wood is a natural orthotropic material which is broadly used in civil and construction fields. Different factors like easy accessibility, the high ratio of mass to strength, remarkable durability and performance, absorbing and dissipating of vibrations under some conditions of use and also good insulating properties of dry wood against heat, sound and electricity of wood have been rendered them as a good candidate for structural elements in construction [1]. But the most important parameter which plays a key role in growing demand for the application of wood is its availability in many species, sizes and shapes. Moreover, the simple manufacturing process of wood from renewable sources is another factor encouraging industry to employ them.

Efficient and safe design of timber structures necessitates the proper understanding behavior of wood fracture which is a complicated issue attributed to the inherent variability, heterogeneity and anisotropy of wood.

Wood elements are either most likely to experience damages during service, or contain geometric discontinuity. Subjected to the mixed mode I and II of loading, intrinsic damages in wood are identified as cracks with various sizes [2]. The mechanism of wood fracture can be hardly distinguished with linear elastic fracture mechanics (LEFM) principles due to the damage zone around the crack tip and development of micro-cracks in the crack tip vicinity resulted in wood specimens' fracture [3].

Almost all proposed criteria for fracture investigations of wood specimens under mixed mode I/II loading are developed for the specimens with initial notches along the wood fibers. The complicated mechanism of wood fracture has hinder different researchers from studying this phenomenon theoretically and limited most studies to satisfy by curve fitting on experimental data [4, 5].

Recently, Jernkvist [6] tried to develop a criterion for wood structures under mixed mode I/II loading by extending the well-known isotropic fracture criteria, namely maximum strain energy release rate [7] and minimum strain energy density criteria [8]. The obtained results are conservative and are not in agreement with experimental data [9]. This inconsistency is originated from neglecting the wasted energy caused by microcrack formations and their growth in fracture process zone.

Therefore, the theoretical investigations of wood fracture are suffering from robust and general criterion especially for mixed mode fracture. The main objective of this article is to develop a general criterion to study mixed mode fracture of wooden specimens relying on modification of maximum shear stress criterion while not only the influence of fracture process zone in crack tip region is taken into account but also the robust modeling accounts for arbitrary oriented cracks.

1. **Framework of Modeling**. From the material point of view, wood can be described as a continuum, homogenous and orthotropic material at macro scale [1]. Material symmetry axes are defined as the longitudinal direction (L) of fibers, the radial direction (R) of rays and the tangential direction (T) to the growth rings. Consequently, there are six principal systems of crack propagation introduced by Smith et al. [2] as TL, RL, LR, TR, RT, and LT. The first letter denotes the normal direction to the crack plane; while the second letter indicates the direction of crack propagation [1]. In one hand, due to the particular design of timber structures, the RL and TL systems represent the most frequent planes of crack propagation. From the other hand, mode II and mixed mode I and II are most experienced fracture modes for wooden structural members than the others. Therefore, development of a reliably accurate criterion for mixed mode I/II fracture in the RL and TL crack propagation systems for wood is considered in this research. Figure 1 shows the coordinate system which is used for analysis wherein x-axis corresponds to the wood longitudinal (L) direction and y is R direction.

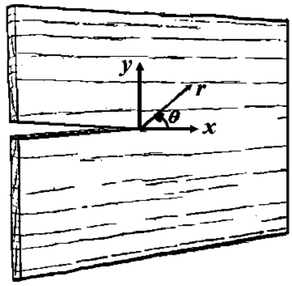


Fig. 1. Crack tip coordinate system used in the analysis of cracked wooden specimens.

Table 1

Elastic Properties of Scots Pine and Norway Spruce Applied in the Analysis

Species	E_R, GPa	E_T, GPa	E_L, GPa	G_{RL}, GPa	ν_{LR}	ν_{LT}	ν_{TR}	K_{Ic}, $\text{MPa}\sqrt{\text{m}}$	K_{IIc}, $\text{MPa}\sqrt{\text{m}}$
Norway spruce (RL) [9]	0.81	0.64	11.84	0.63	0.38	0.56	0.34	0.58	1.52
Scots pine (RL) [10]	1.10	0.57	16.30	1.74	0.47	0.45	0.31	0.49	1.32

Norway spruce and Scots pine are two well known wood species that have been used in this study. The elastic properties of these materials are summarized in Table 1.

2. **Prediction of Crack Propagation Direction**. Motivated by the physical observation in isotropic materials that failure occurs along the plane of maximum shear stress and paying attention to this fact that the strength of wood can be highly directional, it is assumed that the direction of crack extension is controlled by the ratio of shear stress to strength at the given plane. It is postulated that the crack grows along the plane on which this ratio is a maximum. Thus, the model is a shear stress criterion which includes the anisotropic strength of the material.

Mathematically, the criterion states that given the shear stress $\tau(r_0, \varphi)$ at some small distance r_0 from the crack tip and the tensile strength $T(\varphi)$ on the plane φ, the crack will grow along the plane on which the ratio $R(r_0, \varphi)$ is a maximum, where

$$R(r_0, \varphi) = \frac{\tau(r_0, \varphi)}{T(\varphi)}. \tag{1}$$

For wood, the tensile strength $T(\varphi)$ is strongly dependent on the orientation of the plane φ with respect to the fiber orientation. Strength properties in directions ranging from parallel to perpendicular to the fibers can be approximated using Hankinson type formula [11]:

$$T(\varphi) = X_T \sin^2 \beta + Y_T \cos^2 \beta, \tag{2}$$

where Y_T is strength perpendicular to grain and X_T is strength parallel to grain. The angle β is defined as the angle from the plane of possible crack extension to the material principal direction as depicted in Fig. 2.

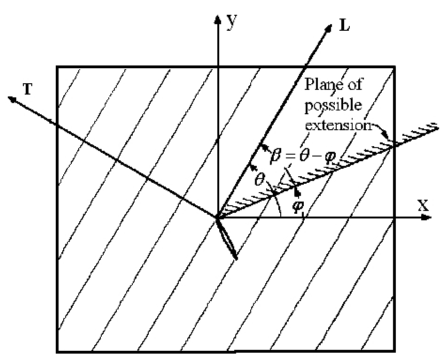


Fig. 2. Plane of possible extension and associated angles.

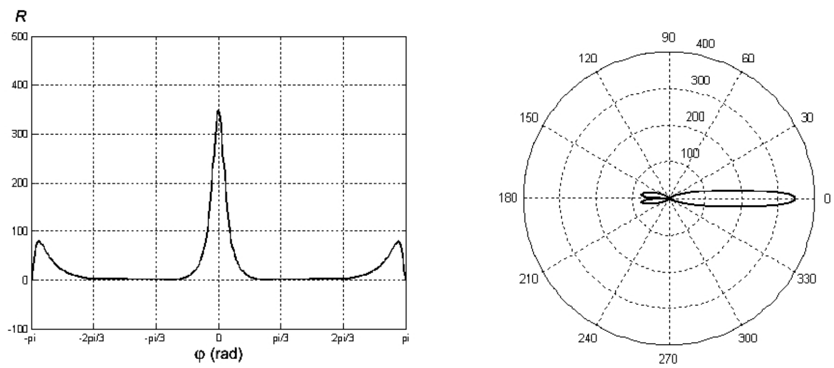
Since the mixed mode I/II loading is considered, maximum shear stress criterion in $x = y$ plane will be given by

$$\tau_{\max} = \sqrt{[(\sigma_{11} - \sigma_{22})/2]^2 + \sigma_{12}^2}. \qquad (3)$$

Value of $\tau(r_0, \varphi)$ in front of crack tip can be written by using the following singular stress state at the crack tip vicinity:

$$\sigma_{ij} = \frac{K_{\rm I} f_{ij}(\theta)}{\sqrt{2\pi r}} + \frac{K_{\rm II} g_{ij}(\theta)}{\sqrt{2\pi r}} \qquad (i, j = 1, 2). \qquad (4)$$

The angular functions f_{ij} and g_{ij} are dependent on the plane strain constitutive matrix [12]. Now the $R(r_0, \varphi)$ can be easily found by substituting Eqs. (2) and (4) into Eq. (1) and crack growth direction is coinciding with the angle of maximum $R(r_0, \varphi)$. The distribution of $R(r_0, \varphi)$ for fiber inclination $\theta = 0$ is plotted in Figs. 3 and 4 for Norway spruce in two sample cases of $K_{\rm I} = K_{\rm Ic}$ and $K_{\rm I} = 0.4$ MPa$\sqrt{\rm m}$, $K_{\rm II} = 1.0$ MPa$\sqrt{\rm m}$.



a b

Fig. 3. Distribution of *R*-function in crack tip vicinity in Cartesian (a) and polar (b) coordinates for $K_{\rm I} = K_{\rm Ic}$ and $\theta = 0$.

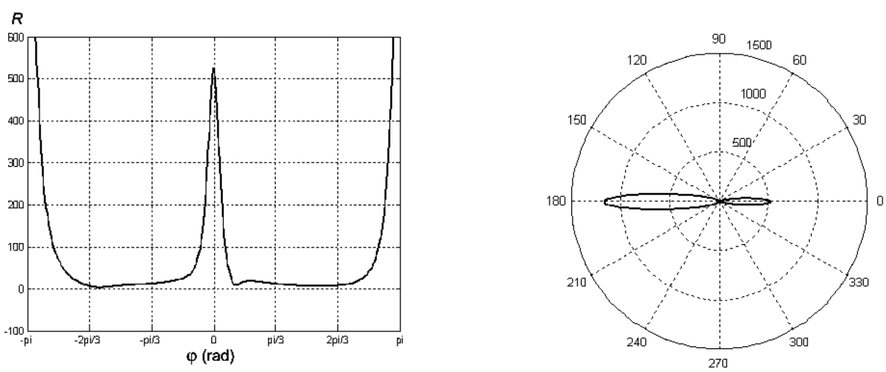


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Fig. 4. Distribution of *R*-function in crack tip vicinity in Cartesian (a) and polar (b) coordinates for $K_{\rm I} = 0.4$ MPa$\sqrt{\rm m}$, $K_{\rm II} = 1.0$ MPa$\sqrt{\rm m}$, and $\theta = 0$.

The results of modified shear stress criterion and experimental observations [13] are summarized in Table 2. It can be observed that predicted crack propagation angle with modified shear stress criterion completely coincides with experimental data. The results show that the propagation of crack with any inclination with respect to wood fibers will be along the wood fibers. Crack kinking behavior is independent from mode mixity and is justifiable with respect to highly orthotropic behavior of wood.

Table 2

Crack Propagation Angle for Wood Specimens with Different Crack Inclination

θ, rad	K_I, MPa$\sqrt{m}$	K_{II}, MPa$\sqrt{m}$	Crack propagation angle (rad)	
			Modified maximum shear stress	Experimental data [13]
0	K_{Ic}	0.0001	0	0
	0.0001	K_{IIc}	0	0
	0.4	1.0	0	0
$\pi/6$	K_{Ic}	0.0001	$\pi/6$	$\pi/6$
	0.0001	K_{IIc}	$\pi/6$	$\pi/6$
	0.4	1.0	$\pi/6$	$\pi/6$
$\pi/3$	K_{Ic}	0.0001	$\pi/3$	$\pi/3$
	0.0001	K_{IIc}	$\pi/3$	$\pi/3$
	0.4	1.0	$\pi/3$	$\pi/3$
$\pi/2$	K_{Ic}	0.0001	$\pi/2$	$\pi/2$
	0.0001	K_{IIc}	$\pi/2$	$\pi/2$
	0.4	1.0	$\pi/2$	$\pi/2$

3. **Fracture Criterion for Cracks along the Wood Fiber**. In this section, onset of crack growth is assumed as soon as τ_{max} at some finite distance $r=\delta$ from the crack tip reaches a critical value. In accordance with mentioned assumption, the crack propagation direction is supposed to be in fiber direction. Using Eqs. (3) and (4), in front of crack tip can be written as

$$\tau_{max} = \sqrt{\left[\frac{K_I}{\sqrt{2\pi r}}\left(\frac{f_{11}(0)-f_{22}(0)}{2}\right)\right]^2 + \left(\frac{K_{II}}{\sqrt{2\pi r}}\right)^2}. \qquad (5)$$

And requiring it to be valid also for pure mode I loading, the following relation is found:

$$\tau_{cr} = \frac{K_{Ic}}{\sqrt{2\pi r}}\left(\frac{f_{11}(0)-f_{22}(0)}{2}\right) \qquad (6)$$

by definition of

$$\beta = \left(\frac{f_{11}(0) - f_{22}(0)}{2} \right)^2 \tag{7}$$

Eq. (6) takes the following simple form:

$$\frac{1}{\sqrt{\beta}} \sqrt{K_{\rm I}^2 \beta + K_{\rm II}^2} - K_{{\rm I}c} = 0. \tag{8}$$

Assuming the fact that, mixed mode limiting fracture curve obtained from the modified maximum shear stress fracture criterion should pass through the point describing the critical value of the mode II stress intensity factors, we find the following relation between mode I and mode II fracture toughness:

$$K_{{\rm II}c} = K_{{\rm I}c} \sqrt{\beta}. \tag{9}$$

Substituting β from Eq. (9) into Eq. (8), yields

$$K_{\rm I}^2 + \rho K_{\rm II}^2 = K_{{\rm I}c}^2 \tag{10}$$

in which

$$\rho = (K_{{\rm I}c} / K_{{\rm II}c})^2, \tag{11}$$

ρ is introduced as a “toughness damage factor,” demonstrating the variation compliance of damaged material in crack tip vicinity due to growth of microcracks. Equation (10) shows a simple mixed mode I/II fracture criterion in terms of stress intensity factors $K_{\rm I}$ and $K_{\rm II}$.

Several mixed mode fracture criterion such as modified minimum strain energy density, maximum strain energy release rate and maximum principal stress were proposed for mixed mode fracture analysis of wood [9]. In order to show the superiority of the proposed method, fracture limit curves related to these criteria is plotted in Figs. 5 and 6 for Norway spruce and Scots pine wood in comparison with modified maximum shear stress criterion.

A comparison between experimental data related to Norway spruce and Scots pine and theoretical models reveals that developed approach is more compatible with the nature of fracture phenomena in wood. It can be understood that the fracture limit curve related to modified maximum shear stress criterion could predict the experimental data very well.

4. **Fracture Criterion for Different Crack Orientation**. The most general case of crack configuration is studied in this section. In this case crack axis and the main orthotropy axes do not coincide with each other. It is known that crack is susceptible to grow along a particular physical plane with the highest risk of the fracture. Therefore, crack growth at an arbitrary notch angle with respect to wood fibers deviates by angle ω from the original notch orientation as illustrated in Fig. 7a. Therefore, it is assumed that arbitrary crack in wood after in-plane loading will depart via a sharp kink and propagates along the fibers when the fracture toughness at the tip of the kink is exceeded. This phenomenon is shown in Fig. 7b.

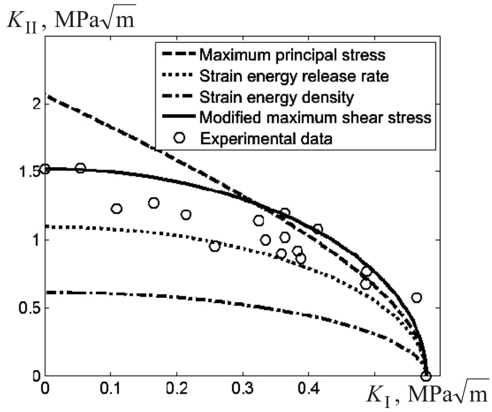


Fig. 5

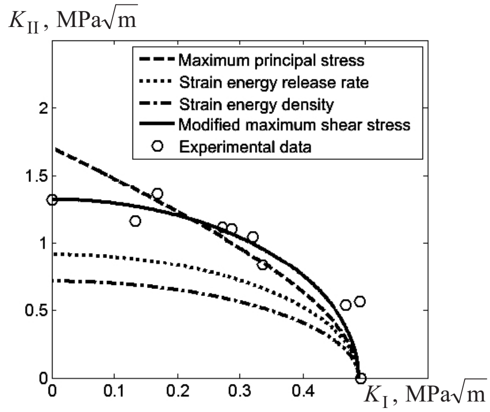
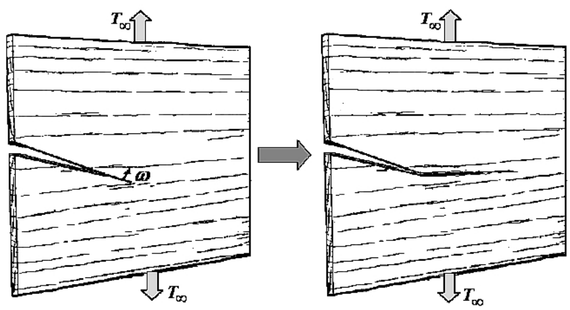


Fig. 6

Fig. 5. Proposed criterion for wood fracture investigation in RL direction in comparison with Norway spruce experimental data [9].
Fig. 6. Proposed criterion for wood fracture investigation in RL direction in comparison with Scots pine experimental data [10].



a b

Fig. 7. Wood specimen with arbitrary orientation initial notch (a); kinking in an arbitrary crack orientation (b).

Since the kink is parallel to the wood fibers, it represents a plausible microcrack or intercellular flow in the material. So, it is permissible to use the developed fracture criterion in the preceding section [i.e., Eq. (10)] to obtain onset of fracture for this arbitrary oriented crack position. Substituting the local stress intensities at the crack tip of the kink (i.e., $K_{\rm I}^{kink}$, $K_{\rm II}^{kink}$) into Eq. (10), we have

$$(K_{\rm I}^{kink})^2 + \rho (K_{\rm II}^{kink})^2 = K_{{\rm I}c}. \qquad (12)$$

Linearly elastic material is considered and the stress intensity factors at the tip of the kink, $K_{\rm I}^{kink}$ and $K_{\rm II}^{kink}$ can be expressed by linear combinations of the stress intensity factors for the main arbitrary oriented cracks:

$$\begin{aligned} K_{\rm I}^{kink} &= a_{11} K_{\rm I} + a_{12} K_{\rm II}, \\ K_{\rm II}^{kink} &= a_{21} K_{\rm I} + a_{22} K_{\rm II}, \end{aligned} \qquad (13)$$

where a_{ij} are functions of kink angle ω, and material compliance properties C'_{ij}, and they can be calculated using an integral equation method [14, 15].

For the sake of model validation, experimental data for Norway spruce [9] in comparison with fracture limit curves are shown in Fig. 8 when initial notches are along (RL, $\omega = 0$) and across (LR, $\omega = -90°$) the wood fibers.

Fracture limit curves for Scots pine wood in comparison with mixed mode experimental results [16] for specimens with the crack inclination angles $\omega = 0$, 45, 67.5, and 90° are also presented in Fig. 9.

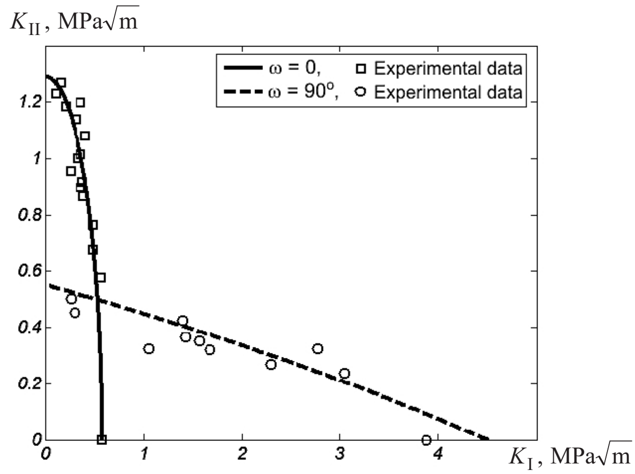


Fig. 8. Presented criterion for different crack orientation in comparison with experimental data [9] for Norway spruce wood.

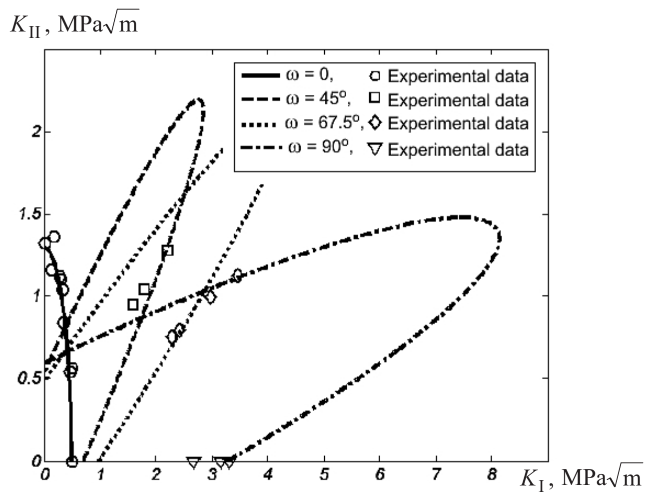


Fig. 9. Presented criterion for different crack orientation in comparison with experimental data [15] for Scots pine wood.

The presented results and comparison in Figs. (8) and (9) imply the efficiency of developed model for fracture investigations in wood. It is worth mentioning that the applicability of the derived fracture criterion for cracks with arbitrary directions with respect to wood fibers is restricted to those geometric configurations wherein non-singular stress parallel with the crack plane is close to zero.

Conclusions. In this article, a new mixed mode fracture criterion for investigation of crack growth condition and crack propagation direction in wood species was extracted based on maximum shear stress criterion. This criterion is capable of considering fracture process zone effects on crack growth initiation. The criterion is very simple and contains mode I and II fracture toughness as the only material dependent fracture parameters. This criterion provides a better and more efficient mathematical tool which could be applied to fracture problems including specimens with arbitrarily oriented notches with respect to wood fibers. Different case studies for a typical set of wood specimens demonstrate the effectiveness of the proposed idea and could properly approximate the experiment results. However, completeness and simplicity, as well as the noticeable results reliability are the main features of the proposed approach justify using of this criterion in mixed mode fracture analysis of wood.

Резюме

Розглядається використання нового критерію руйнування для прогнозування напрямку ініціювання і росту тріщин у дерев'яних зразках за змішаної моди навантаження I/II. Даний критерій враховує розподіл максимальних дотичних напружень в околі вістря тріщини. Його головна перевага порівняно з іншими відомими критеріями – більш точне врахування впливу зони пошкодження на руйнування дерева. Зміна піддітливості пошкодженого матеріалу, пов'язана з ростом мікротріщин, враховується за допомогою спеціального коефіцієнта пошкодженості. Запропонований критерій використовується у випадку руйнування зразків із довільною орієнтацією тріщини відносно ортотропних осей матеріалу. Установлено, що такий підхід добре описує механізм руйнування дерев'яних зразків. Наведено порівняння розрахункових результатів із відомими експериментальними.

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