POLARIZATION OF SYNCHROTRON RADIATION FROM RELATIVISTIC ELECTRONS MOVING WITHIN TOROIDAL MAGNETIC FIELDS

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Using the formulas previously obtained by the author, the polarization of synchrotron radiation is computed for toroidal magnetic field configurations. The radiated power and direction of polarization of the synchrotron radiation spot of runaway electrons for medium-sized tokamaks are estimated. It is shown that the polarization measurements give additional diagnostics for the radiating relativistic electrons.

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INTRODUCTION

Synchrotron radiation from ultrarelativistic electrons is the main mechanism of non-thermal emission from cosmic sources, the curvature radiation of charged bunches used to explain the pulsar emission.

In [1] the formulas have been derived to describe the synchrotron radiation of ultrarelativistic electrons moving at arbitrary pitch angles along a helical trajectory that winds about a curved magnetic field line.

They examined an electron trajectory that results from two motions: particles exhibit a circular motion around the guiding center, which is moving with a constant speed along the circular magnetic field lines. The writers called their radiation mechanism as synchrocurvature to underline that the curvature of magnetic force lines was taken into account.

In [2, 3] the radiation formulae of a relativistic charge moving along bent spiral path have been generalized. The trajectory of a charged particle has been refined, the centrifugal drift and the variations of the curvature radius at different points of the trajectory was taken into account. The new expressions for the spectral angular distribution and polarization components of synchrotron radiation were obtained.

The synchrotron radiation spectrum of runaway electrons in tokamak was obtained in [4].

In [1 - 3], the Poynting vector is calculated to find the emission spectrum, in [4] the method of radiation field work is used. In [5] we compare these both methods. In [6, 7] the undulator radiation of relativistic electrons is considered.

So, we have analogous topologies for relativistic electron motions in astrophysical magnetic fields and for toroidal magnetic fields in tokamaks.

The synchrotron radiation is the powerful tool to diagnose the relativistic runaway electron distribution. This diagnostic provides a direct image of the runaway beam inside the plasma. This radiation was measured in infra-red and visual wavelength ranges [8 - 10].

In present paper, the polarization of synchrotron radiation of electrons in a toroidal magnetic field configuration is calculated using the spectrum formula from paper [3].

The topology of toroidal magnetic fields and drift trajectories, the electron energies take values that are typical for medium-size tokamaks.

The paper is organized as follows. The magnetic field and drift trajectory are discussed in Section 2. In Section 3, the spectrum and polarization properties of radiation emitted by relativistic elecrrons are considered. The polarization in synchrotron radiation spot is calculated and discussed, a conclusion about possibility to use polarization diagnostic is given in Section 4.

1. TOROIDAL MAGNETIC FIELDS AND ELECTRON TRAJECTORIES

Cartesian coordinates (x, y, z) are taken as shown in Fig. 1. The torus is described with coordinates (r, ϑ, φ): coordinates (r, ϑ) with center at major radius R and the toroidal angle φ, $\mathbf{e}_r, \mathbf{e}_\vartheta, \mathbf{e}_\varphi$ are the corresponding orts, $\mathbf{e}_\varphi = [\mathbf{e}_r, \mathbf{e}_\vartheta]$. Magnetic surfaces and drift surfaces have a toroidal topology. The major radius of nested magnetic surfaces is $R = R_0$, the major radius of drift surfaces is $R = R_0 + \delta$. So, there are two coordinate systems (r, ϑ, φ) and $(r_f, \vartheta_f, \varphi)$ corresponding to drift and magnetic torus, respectively.

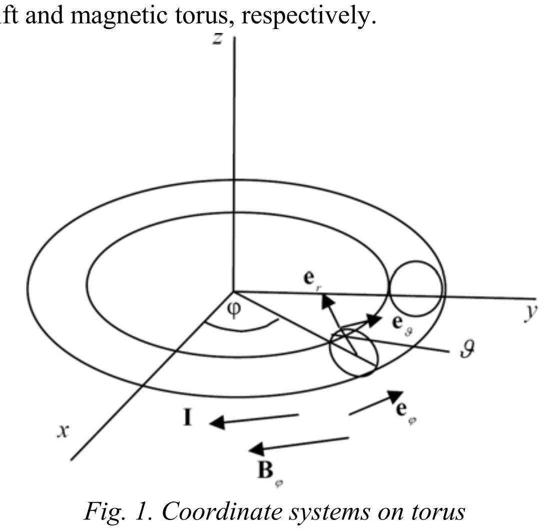

Fig. 1. Coordinate systems on torus

Suppose, the toroidal magnetic field $\mathbf{B}_\varphi$ and plasma current $\mathbf{I}$ are clockwise, radiating electrons are moving counterclockwise.

Magnetic fields take the form [11, 12]

$$\mathbf{B}(r, \vartheta) = \frac{-B_0}{1 - r\cos\vartheta / R_0}\left[\frac{r}{q(r)R_0}\mathbf{e}_\vartheta + \mathbf{e}_\varphi\right], \qquad (1)$$

here $r = r_f, \vartheta = \vartheta_f$ are the radius and poloidal angle of magnetic surface, $q(r)$ the safety factor. The subscript f is omitted in eq. (1).

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Suppose that the safety factor of the magnetic field lines $q(r_f)$ is equal that of the particle drift orbits $q_D(r)$ when equal radii of magnetic field line surface r_f and drift surface r are considered [8].

The guiding center is moving along drift trajectory with speed $V_\parallel$. The electron pitch angle is $\alpha \approx V_\perp/V_\parallel \ll 1$, where $V_\perp = \omega_B r_B$, $\omega_B = eB/(\gamma mc)$, $e = |e|, m, \gamma$ are the electron charge, mass, and Lorentz-factor, $B = B(r,\vartheta)$ the magnetic field at the point with drift coordinates $r(r_f,\vartheta_f), \vartheta(r_f,\vartheta_f)$, r_B is the Larmor radius.

The electron velocity vector is given by

$$\mathbf{v} = v_\parallel \boldsymbol{\tau}_D + v_\perp(-\sin\Theta \mathbf{v}_D - \cos\Theta \mathbf{b}_D), \qquad (2)$$

here $\boldsymbol{\tau}_D, \mathbf{v}_D, \mathbf{b}_D$ are the tangent, normal, and binormal to the drift path, the angle Θ is measured from normal $\mathbf{v}_D$ to the direction of vector $-\mathbf{b}_D$.

The detector is located at the point P with Cartesian coordinates $(-D, R_0, 0)$. The line of sight is directed from the detection point P to the radiation point P_e, denote unit vector in the direction PP_e as $\mathbf{n}$. The high energetic electrons emit their synchrotron radiation practically along their velocity vector (2).

The coordinates of radiation point are founded out after equating components $-n_y, -n_z$ to the directional cosines of velocity vector.

For any r, ϑ the third coordinate of radiation point takes a value $\varphi = \pi/2 + \Delta\varphi$, where $\Delta\varphi$ is the first-order correction with respect to $r/R \ll 1$, $\alpha \ll 1$.

The electron position in Larmor circle, angle Θ, is given by

$$\cos\Theta = \frac{r}{\alpha q_D(r) R \cos\vartheta_0} \cos(\vartheta - \vartheta_0), \qquad (3)$$

where $\cos\vartheta_0 = D\big/\sqrt{D^2 + (q_D R)^2}$,

$q_D(r) = 3q_0 \dfrac{r^2/a^2}{1-(1-r^2/a^2)^3}$ is the safe factor of drift path [12], a the small tokamak radius. Larmor radius r_B is not taken into account because of its smallness.

The range of angles ϑ depends on pitch-angles α. As can see from eq. (3), there are constraints on acceptable angles ϑ when $\alpha < r/(q_D R_D)$.

2. SYNCHROTRON RADIATION

The spectral power density of the synchrotron radiation emitted by relativistic electrons moving within magnetic field lines with curvature radius R is expressed by [3]

$$\frac{dP_i}{d\omega} = \frac{P_C}{\omega_C} f_i(y_C, q_a) \qquad (4)$$

$$f_i(y_C,q_a) = \frac{9\sqrt{3}}{16\pi} y_C \left\{ \int\limits_{\frac{y_C}{|1-q_a|}}^{\infty} dx F_i(x) + \frac{1}{\pi} \int\limits_{\frac{y_C}{1+q_a}}^{\frac{y_C}{|1-q_a|}} dx F_i(x) \left(\frac{\pi}{2} + \arcsin\frac{1+q_a^2 - y_C^2/x^2}{2q_a} \right) \right\}$$

here $P_C = \dfrac{2}{3}\dfrac{e^2}{c}\gamma^4\beta_\parallel^2\Omega^2$ is the total power emitted by a charged particle moving with velocity $V_\parallel$ along a circular orbit of radius R, $y_C = \omega/\omega_C$, $\omega_C = (3/2)\gamma^3\Omega\beta_\parallel/\beta \approx (3/2)\gamma^3\Omega$ the characteristic radiation frequency, $q_a = \dfrac{\omega_B^2 r_B}{\Omega^2 R_D}$, $\Omega = \dfrac{V_\parallel}{R_D(r,\vartheta)}$, $i = p, s$ denote the cases of p and s -polarization,

$$F_\pi = K_{5/3} + dK_{2/3}/dx = (1/2)(K_{5/3} - K_{1/3}),$$
$$F_\sigma = K_{5/3} - dK_{2/3}/dx = (1/2)(3K_{5/3} + K_{1/3}),$$

K_ν is the Macdonald function.

Formulas (4) contain the parameter q_a that determines the radiation mode. The parameter q_a is defined as the quotient of the acceleration of the particle motion in the small Larmor circle to the centrifugal acceleration due to movement on the larger circumference of radius equal to the radius of curvature of the magnetic field line. Formula (4) has classical synchrotron or curvature radiation limits when $q_a \gg 1$ or $q_a \ll 1$, respectively [3].

The parameter q_a can be written as

$$q_a = \frac{e}{mc^2} B(r_f,\vartheta_f) R_D(r,\vartheta) \frac{\alpha}{\gamma}, \qquad (5)$$

where $R_D = R_0 + \delta + \left(-1 + \dfrac{1}{q_D^2}\right) r\cos\vartheta$ is the curvature radius of drift trajectory in a first-order approximation. The parameter q_a is denoted as η in the paper [4].

To calculate the magnetic field at the point $P_e = \left(r, \vartheta, \dfrac{\pi}{2}\right)$ the displacement in equatorial plane of the drift torus with respect to magnetic torus is taken into account, then $r_f = \sqrt{r^2 + \delta^2 - 2\delta r\cos\vartheta}$ and $\cos\vartheta_f = \dfrac{r\cos\vartheta - \delta}{r_f}$. Then, the magnetic field value is expressed by

$$B = B_0\left(1 + \frac{r\cos\vartheta - \delta}{R_0}\right).$$

2.1. POLARIZATION VECTOR

As known, the direction of the larger axis of polarization ellipse for synchrotron emission of relativistic electrons moving on a circular orbit coincides with the direction of particle's acceleration [13]. Let χ be an angle between the axis $0z$ and the electron acceleration $\mathbf{a}$. Taking into account the smallness of angle between the line of sight and axis $0x$, we find that $\sin\chi = \dfrac{a_y}{a}$, $\cos\chi = \dfrac{a_z}{a}$.

Then

$$\sin\chi = \frac{-1 + q_a\cos\Theta - \left[b_1 + b\dfrac{q_a}{q_D}\sin\vartheta\sin\Theta\right]}{\sqrt{1 + 2b_1 + 2q_a[-1+b_1]\cos\Theta + q_a^2}}, \qquad (6)$$

and

$$\cos\chi = \frac{q_a \sin\Theta + \dfrac{b}{q_D}\left(-1 + q_a\cos\Theta\right)\sin\vartheta}{\sqrt{1 + 2b_1 + 2q_a\left[-1 + b_1\right]\cos\Theta + q_a^2}}, \qquad (7)$$

where $b = \dfrac{r}{q_D(r)R_D(r,\vartheta)}$, $b_1 = b\cdot\left(q_D - \dfrac{1}{q_D}\right)\cdot\cos\vartheta$.

To describe the polarization properties we use the Stokes parameters I, Q, U, V. According [13, eq. (5.28)]

$$I \propto \frac{dP_\sigma}{d\omega} + \frac{dP_\pi}{d\omega},$$

$$Q \propto \left(\frac{dP_\sigma}{d\omega} - \frac{dP_\pi}{d\omega}\right)\cos 2\tilde{\chi},$$

$$U \propto \left(\frac{dP_\sigma}{d\omega} - \frac{dP_\pi}{d\omega}\right)\sin 2\tilde{\chi},$$

where $\tilde{\chi} \in (0,\pi)$ is the angle between some arbitrary fixed direction, axis $0z$ in our case, and the major axis of the ellipse of polarization. The homogeneity of the electron distribution function implies that $V = 0$.

It follows from (3) that each ϑ corresponds to the two values of Θ. We add up Stokes parameters for these two emitting points, $Q = Q_1 + Q_2$, $U = U_1 + U_2$. Expressions (6), (7) are substituted for $\sin\tilde{\chi}$, $\cos\tilde{\chi}$ in trigonometric formulas for doubled angles.

Then the degree of polarization is defined as [13]

$$\pi(\omega) = \frac{\sqrt{Q^2 + U^2}}{I}, \qquad (8)$$

and angle $\tilde{\chi}$

$$\mathrm{tg}\, 2\tilde{\chi} = \frac{U}{Q}.$$

By definition, the angle $\tilde{\chi}$ describes the direction in the picture plane in which the intensity of the polarized components has maximum.

Using the formulae (3)-(8) we will calculate the distributions of the total intensity I and polarization directions $\tilde{\chi}$ in the synchrotron radiation spot from a homogenous electron beam with radius r_b.

3. DISCUSSION

The monoenergetic distribution function and a given pitch angle are supposed [8, 9]. The distribution is also homogenous in space.

The distribution of total intensity in the area of synchrotron spot is shown in Fig. 2. The radiation point coordinates (y,z) are plotted in horizontal plane, the vertical axis shows the radiation power (in arbitrary units) at a given frequency ω (in this case, the wavelength $\lambda = 5\ \mu$m). The parameters for calculations are taken as in the middle-sized tokamaks [8 - 10].

The power emitted at a given spot point depends strongly on the values of α and γ. In Fig. 2 we see that the radiation is stronger in the area of larger magnetic fields (closer to axis z) than in area with the smaller magnetic field (further from axis z). When γ increases in number this correlation is inversed.

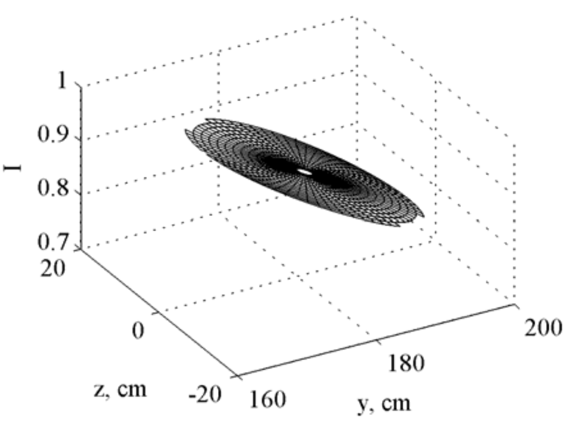

Fig. 2. Total radiated power at the frequency $\omega = 4.8\cdot10^{14}$ Hz with parameters: $B_0=3\cdot10^4$ G; $\gamma=50$; $\alpha = 0.1$; $R_0 = 175$ cm; $\delta = 6$ cm; $D = 200$ cm; $q_0 = 1$; $r_b = 15$ cm; $a = 40$ cm

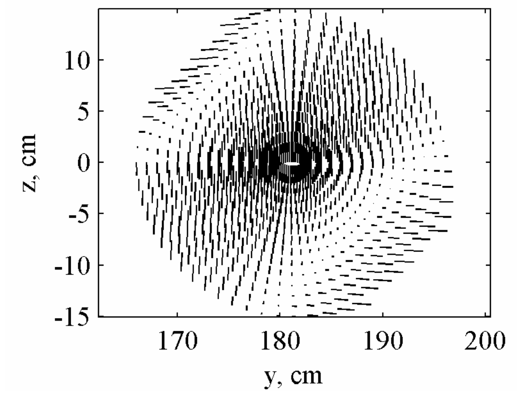

Fig. 3. Polarization of the synchrotron spot (poloidal projection). The parameters as in Fig. 2

This dependence may be the cause of the observed heterogeneity in the synchrotron radiation spots [9, Fig. 12].

At different values of the given (experimental) parameters, the parameter q_a often takes value near unity that evidenced in favor of the using Eqs. (4).

The directions of polarization in the synchrotron radiation spot are shown in Fig. 3. The length of lines is proportional to the degree of polarization that varies from 0 to 80%. This range of values depends on α and γ.

In the case of small α the polarization directions are determined by accelerations of guiding center, i. e. by the normal vector to the drift trajectory, along axis y. For large α the polarization direction is generated by centrifugal accelerations within Larmor circle. This is a direction along axis z.

In Fig. 3 we see these polarization directions as well as areas of zero polarization. It should be noted the change of polarization direction on 90°.

Assume that a similar cause may be responsible for changing the direction of polarization in the emission of pulsar.

Thus the synchrotron radiation is polarized and the distribution pattern of polarization directions in the spot and the degree of polarization can be used for diagnostics of electron beams.

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ПОЛЯРИЗАЦИЯ СИНХРОТРОННОГО ИЗЛУЧЕНИЯ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОНОВ, ДВИЖУЩИХСЯ В ТОРОИДАЛЬНОМ МАГНИТНОМ ПОЛЕ

Я.М. Соболев

Используя формулы, ранее полученные автором, проведены расчеты поляризации синхротронного излучения (СИ) в тороидальной конфигурации магнитного поля. Оценена излучаемая мощность и направления поляризации в пятне синхротронного излучения убегающих электронов с параметрами, характерными для токамаков средних размеров. Показано, что измерение поляризации СИ может быть дополнительным средством для диагностики излучающих релятивистских электронов.

ПОЛЯРИЗАЦІЯ СИНХРОТРОННОГО ВИПРОМІНЮВАННЯ ЕЛЕКТРОНІВ, ЯКІ РУХАЮТЬСЯ У ТОРОІДАЛЬНОМУ МАГНІТНОМУ ПОЛІ

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Застосовуючи формули, які були отримані раніше автором, виконано розрахунки поляризації і спектру синхротронного випромінювання (СВ) в тороідальній конфігурації магнітного поля. Дані оцінки потужності випромінювання і напрям поляризації у плямі синхротронного випромінювання тікаючих електронів для токамаків середніх розмірів. Показано, що вимірювання поляризації СВ може слугувати додатковим засобом для діагностики випромінюючих релятивістських електронів.