WAKEFIELD EXCITATION AND ELECTRON ACCELERATION AT DETUNING BUNCH REPETITION FREQUENCY AND FREQUENCY OF EIGEN PRINCIPAL MODE OF WAKEFIELD

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Theoretical results of process of the wake field excitation in the dielectric waveguide by sequence of relativistic bunches in the presence of detuning between the bunch repetition frequency and the frequency of the fundamental wave waveguide. In this case the wake waves radiated by individual bunches, there is the phase shift, and, beginning with some number of the bunch, the phase shift will be. The polarity of the field is changed and the subsequent bunches gain energy. Energy spectra of accelerated (decelerated) particles are obtained.

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INTRODUCTION

Linear accelerators are an effective device for obtaining sequences of relativistic electron bunches of high quality. Such sequences bunches can be used for excitation of intense wake fields in dielectric waveguide. Wake fields in turn can be used to accelerate electrons (positrons). We note that various aspects of the wake fields excitation by relativistic electron bunches are considered in [1-8]. Development of accelerator based on wakefields excited by a long sequence of bunches is encountered with the physical and technical difficulties associated with the injection of electron bunches in the accelerating phase of the wake field. To solve the problem of injection of accelerated particles the scheme is proposed, in which a separate accelerator-injector is not required. This scheme is based on input of the detuning between the bunch repetition frequency and frequency of the excited eigenmode of the dielectric waveguide. In this case for the wakefield, excited by individual bunches, a phase shift will originate. Beginning from certain number of bunches, which is determined by the frequency detuning the phase shift will be accumulated to π and, therefore, the polarity of wakefield is reversed. Subsequent bunches will gain energy. Thus, in this acceleration scheme, using the same sequence of bunches, one part of bunches moving in the head of the sequence will lose its energy to excite wakefield and another part of bunches beginning from a mentioned number, will be accelerated.

In this paper the results of theoretical investigations of wake field excitation by a sequence of electron bunches and accelerate bunches of the same sequence in the dielectric waveguide in the presence of frequency detuning are presented.

1. STATEMENT OF THE PROBLEM

Let's consider an infinite dielectric waveguide, which is a cylindrical metal tube of radius b, filled with homogeneous dielectric of permittivity ε, having axial vacuum channel of radius a. In the vacuum channel along of the longitudinal axis z a periodic sequence of N axisymmetric relativistic electron bunches with arbitrary spatial period L propagates. The longitudinal and transverse density profile of bunches are arbitrary.

We will find the wakefields in a dielectric waveguide excited by a finite sequence of relativistic electron bunches. In the ultrarelativistic case, when relativistic factor of the electron bunches $\gamma_0 >> 1$, the motion of electrons in the longitudinal and transverse directions is strongly "frozen." Therefore, at the first step of the investigation the approximation of the "given motion" of bunches.

For the taken geometry of the dielectric waveguide and electron bunches the wakefields will be excited as E-type electromagnetic waves. The wakefield excitation in a dielectric waveguide by a sequence of relativistic electron bunches describes by inhomogeneous Maxwell equations. We will solve the problem as follows. Firstly we determine the wakefield of an infinitely thin ring electron bunch with a longitudinal current density

$$dj_{ext} = -edN_0 \frac{\delta(r-r_0)}{2\pi r_0} \delta(t - z/V_0 - z_0/V_0), \quad (1)$$

where r_0 – is the radius of the ring, t – is the current time, z_0 – is the initial coordinate of the ring bunch, V_0 – is the longitudinal velocity, z – is the longitudinal coordinate, e is the electron charge, $\delta(x)$ – is the delta function, dN_0 – is the number of particles in the electron ring associated with the current density of the bunch at the dielectric waveguide

$$dN_0 = -\frac{1}{eV_0} j_0(z_0, r_0) 2\pi r_0 dr_0 dz_0, \quad (2)$$

$j_0(z_0, r_0)$ is given initial current density electron bunch. The resulting electromagnetic wake field of the bunch of arbitrary axisymmetric form may be obtained by summation (integration over initial longitudinal coordinates z_0 and initial radii r_0) wake fields of the elementary ring bunches.

2. WAKE FIELD EXCITATION BY SEQUENCE OF RELATIVISTIC BUNCHES

Wakefield of the periodic sequence of thin relativistic ring electron bunches with period $T = L/V_0$ is determined by direct summation of individual bunch wakefields. The expression for the longitudinal component of the electric wakefield of a sequence of thin relativistic ring bunches (1) has the form

$$E_{zN} = \frac{4edN_0}{aV_0\gamma_0} \sum_{n=1}^{\infty} \frac{F_0(k_{\perp n}a, k_{\perp n}b)}{D'_n} \frac{I_0(\kappa_n r_0)}{I_0(\kappa_n a)} I_0(\kappa_n r) \times \times \sum_{m=0}^{N} \cos(\omega_n(\tau - mT))\theta(\tau - mT), \quad (3)$$

$D'_n = dD(\omega)/d\omega$ at $\omega = \omega_n$, ω_n is the frequencies of dielectric waveguide eigenmodes,

$$D(\omega) = I_1(\kappa a)F_0(\kappa_\perp a, \kappa_\perp b) - \frac{\kappa\varepsilon}{\kappa_\perp} I_0(\kappa a)F_1(\kappa_\perp a, \kappa_\perp b)$$

is dispersion equation, functions F_0 and F_1 are $F_0(\kappa_{\perp n}a, \kappa_{\perp n}b) = J_0(\kappa_{\perp n}a)N_0(\kappa_{\perp n}b) - J_0(\kappa_{\perp n}b)N_0(\kappa_{\perp n}a)$, $F_1(\kappa_{\perp n}a, \kappa_{\perp n}b) = J_1(\kappa_{\perp n}a)N_0(\kappa_{\perp n}b) - J_0(\kappa_{\perp n}b)N_1(\kappa_{\perp n}a)$, $\kappa_{\perp n}^2 = \frac{\omega_n^2}{V_0^2}(\beta_0^2\varepsilon - 1)$, $\beta_0 = \frac{V_0}{c}$, $\kappa_n^2 = \frac{\omega_n^2}{V_0^2\gamma_0^2}$, $\varepsilon -$ dielectric constant, $\tau = t - z/V_0$.

Let's consider the case when the bunch repetition frequency $\omega_r = 2\pi/T$ is close to the fundamental mode frequency ω_1 of the dielectric waveguide, i.e. $|\alpha| << 1$, $\alpha = (\omega_1 - \omega_r)/\omega_r$. Then, for large amount of bunches $N >> 1$ (in our case of the nonequidistant radial modes) the excitation of the only fundamental mode $n = 1$ will be prevail. In such singlemode approximation the expression for the wakefield (3) takes the form

$$E_{zm} = E_0 I_0(\kappa r)\,\mathrm{Re}\,\Lambda_m e^{-i\omega\tau}, \quad (4)$$

where $\Lambda_m = \frac{1 - \exp[i(m+1)\Delta]}{1 - \exp(i\Delta)}$, $|\Lambda_m| = \left|\frac{\sin[(m+1)\Delta/2]}{\sin(\Delta/2)}\right|$, $\Delta = 2\pi\alpha$, E_0 is the wakefield amplitude of separate ring bunch, $\kappa \equiv \kappa_1, \omega \equiv \omega_1$. Factor Λ_m describes the interference of the wakefield excited by separate bunches and depends on the frequency detuning. In turn, the function $|\Lambda_m|$ takes into account the change of amplitude with the number of bunch in the sequence. In the absence of detuning $\Delta = 0$ we have $|\Lambda_m| = m + 1$. Wakefield amplitude increases linearly with the number of bunch. Linear growth is due to the coherent summation of wakefields of separate bunches. Dependence $|\Lambda_m|$ on the number of bunch at small detuning is presented in Fig. 1. At the beginning, linear growth of amplitude with increasing number of the bunch takes place. Then the amplitude growth diminishes and for bunch with number $m_1 + 1 = \pi/|\Delta|$ amplitude reaches the maximum value $|\Lambda_m| = 1/\sin(|\Delta|/2) \approx 2/|\Delta|$. Further the amplitude begins to decrease and for bunch with number $m_2 + 1 = 2\pi/|\Delta|$ becames zero, and th picture is periodically repeated. Bunches with numbers up to $m = \pi/|\Delta|$ increases wakefield due to summation and, therefore, lose its energy. On the other hand the region of slowdown of amplitude wakefield for bunches with number $2\pi/|\Delta| > m > \pi/|\Delta|$ contrary take away energy from the wakefield spend its energy on corresponding bunches acceleration.

Now we consider the change of the bunch energy with the bunch number m. It is described by the equation

$$\frac{d\gamma_m}{d\zeta} = -\varepsilon\frac{\sin[\Delta(m+1/2)]}{2\sin(\Delta/2)}, \quad m = 0,1,2,...N, \quad (5)$$

where $\zeta = z/L$, $\varepsilon = eE_0L/m_ec^2$, γ_m is the relativistic factor. At the exact synchronism condition of the bunches with the fundamental wave (i.e. detuning is absent $\Delta = 0$) we have instead of (5) the equation $d\gamma_m/d\zeta = -\varepsilon(m+1/2)$. The first bunch ($m = 0$) experiences the deceleration field, which is equal $-\varepsilon/2$, the second one $-3\varepsilon/2$, the third one $-5\varepsilon/2$, etc. In the presence of frequency detuning in result of phase shift the polarity of the wakefield will be changed. The process of deceleration will be replaced by process of bunches acceleration.

The graph of function $F_m = -\sin[\Delta(m+1/2)]$, which determines the force sign in the right side of the equation (5), is shown in Fig. 1. Bunches with numbers $\pi/|\Delta| > m > 1$ ($|\Delta| << 1$) are decelerated, i.e. $d\gamma_s/d\zeta < 0$. Accordingly, the wakefield amplitude is increased. In turn, bunches with numbers $2\pi/|\Delta| > m > \pi/|\Delta|$ gain the energy. The amplitude of the wakefield decreases. Bunch with number $m_{\gamma\max} = 3\pi/2|\Delta|$ has maximum of acceleration rate $d\gamma_{\max}/d\zeta = 1/|\Delta| >> 1$.

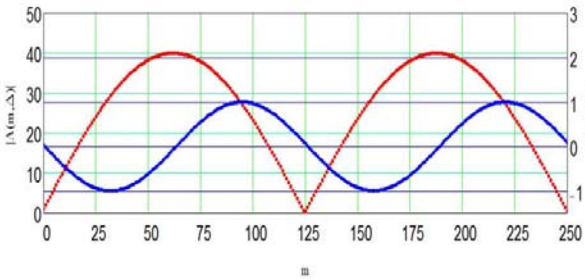

Fig. 1. $|\Lambda_m|$ (red line) and F_m (blue line) as functions of bunch number. Frequency detuning α=0.05

Fig. 2 shows the dependence of the longitudinal component of the electric field on the axis upon τ for sequence of 10 infinitely thin ring bunches for the radius of bunches $\frac{r_b}{a} = 0,5$, $a = 1,44$ cm, radius of the waveguide $b = 4,325$ cm, the permittivity $\varepsilon = 2,1$, bunch energy $E_{bunch} = 4,5$ MeV in the absence of frequency detuning α=0 and in its presence α=0.1. For given parameters of the waveguide and the energy of the electron bunches the frequency of the excited fundamental radial mode is $f_1 = 2,804$ GHz. In the resonant case (without detuning) Fig. 2 shows a coherent (linear) growth of wakefield with bunch number with its jumps that corresponds to adding excited field of the each next bunch. If detuning α = 0.1 the growth of wakefield after 5-th bunch is changed by its decrease up to zero on 10-th bunch.

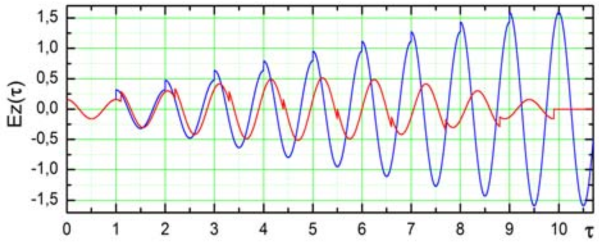

Fig. 2. Dependence of the longitudinal component E_z of wakefield excited by a sequence of infinitely thin ring bunches upon τ at absence of frequency detuning α=0 (blue line) and its presence α=0.1 *(red line)*

Wakefield of the bunch of arbitrary form and, therefore, wakefield of the sequence of such bunches can be found by summation of wakefields of elementary thin ring bunches (3). For this in formula (2) we must set the current density of a sequence of bunches at waveguide entrance. Let's consider wakefield of bunches sequence with rectangular longitudinal and transverse profiles. The expression for the longitudinal component of the electric field excited by a sequence of bunches has the view

$$E_{zN} = \frac{4Q_b}{a^2}\sum_{n=1}^{\infty}\frac{2I_1(\kappa_n r_b)}{\kappa_n r_b}\frac{\kappa_n a}{\omega_n D_n'}\frac{F_0(k_{\perp n}a, k_{\perp n}b)}{I_0(\kappa_n a)} I_0(\kappa_n r) Z_{Nn}(\tau),$$

$$Z_{Nn}(\tau) = \sum_{m=0}^{N} Z_{mn}(\tau)\theta(\tau - mT), \qquad (6)$$

$$Z_{mn}(\tau) = \frac{1}{\omega_n t_b}\begin{cases}\sin\omega_n(\tau - mT); \ mT + t_b > \tau > mT \\ \sin\omega_n(\tau - mT) - \sin\omega_n(\tau - mT - t_b); \\ \qquad (m+1)T > \tau > mT + t_b,\end{cases}$$

t_b is the duration of the rectangular bunch, Q_b is its charge, r_b is the radius of the bunch, $\theta(\tau - mT)$ is the unit Heaviside function.

Let's consider the wakefield between the bunches. In this case for first mode, performing the summation in accordance with the formula (6), we find

$$Z_{N1}(\tau) = \frac{2}{\omega_1 t_b}\sin(\omega_1 t_b/2)\,\mathrm{Re}[\exp(i\omega_1(\tau - t_b/2))\Lambda_N]. \quad (7)$$

Factor $\dfrac{\sin(\omega_1 t_b/2)}{\omega_1 t_b/2}$ describes the interference of wakefield excited by head and back fronts of each separate rectangular bunches, and the factor Λ_N takes into account the interference of wakefields excited by a sequence of bunches.

Fig. 3 presents the dependence of the longitudinal component of the electric field upon τ for sequence of 10 bunches with rectangular longitudinal and transverse profiles with duration $\tau_b = \omega_1 t_b = \pi/3$ in the absence of frequency detuning α=0 and in its presence α=0.1. The other parameters are the same as in Fig. 2, and correspond to the experimental facility of the accelerator "Almaz-2". It is seen that the behavior of the field differs little from the case of infinitely short bunches (see Fig. 2). Jumps of the field becomes smoother and maximum amplitude of the beating is slightly less. So bunches of duration $\tau_b = \omega_1 t_b = \pi/3$ can be considered as infinitely short ones. Region of growth of wakefield amplitude corresponds to the deceleration of electron bunches (bunches give up energy to wakefield). When wakefield amplitude decreases the acceleration of the electron bunches takes place

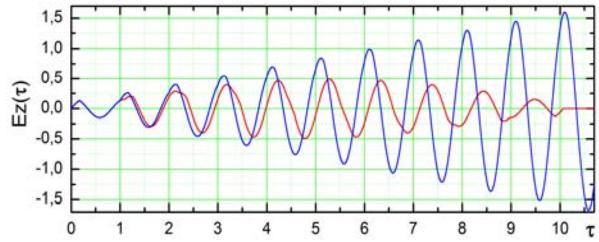

Fig. 3. Dependence of the longitudinal component E_z of wakefield excited by a sequence of rectangular bunches of duration $\tau_b = \omega_1 t_b = \pi/3$ upon τ at absence of frequency detuning α=0 (blue line) and its presence α=0.1 *(red line)*

3. ENERGY SPECTRA OF BUNCH PARTICLES

The energy spectrum of particles in the single-mode approximation (the fundamental mode) for the case of thin circular bunches (red line) and the length of the rectangular bunches (blue line) is shown in the graphics of Fig. 4. Interaction length has been set at six wavelengths of the fundamental mode a wake field. Detuning is α=0.002. The values of other parameters listed above. Changing of the frequency of the exciting electromagnetic field, depending on the energy of the bunch is not taken into account.

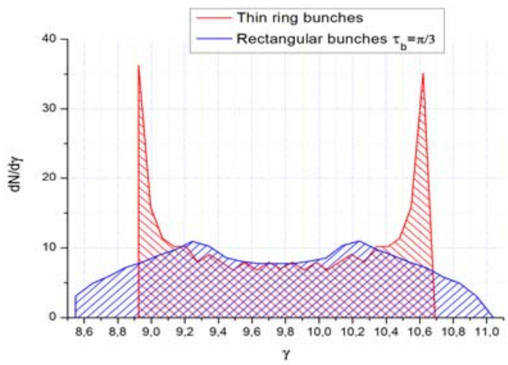

Fig. 4. Energy spectra particles sequences of bunches for infinitely thin ring bunches (red line) and rectangular (blue line) at detuning α=0.002, *N*=500

For infinitely thin bunches energy spectrum has two narrow peaks corresponding to acceleration and deceleration of bunches. For the chosen parameters the maximum increase of the relativistic factor is $\Delta\gamma_0 = 0.9$ or 410 keV on the initial value 4.5 MeV. For bunches of finite width is a significant broadening of the energy spectrum of particles. Instead of narrow peaks have weakly expressed maxima. The maximum of the particle acceleration has been the increment 612 keV. The reason for the broadening of the energy spectrum is the gradient of accelerating (decelerating) electric field inside the bunches, which leads to the gradient energy and, consequently, to a broadening of the energy spectrum.

Fig. 5 illustrates the above, which shows the change of the relativistic factor of the first particle bunch (lilac line) and last (blue line). Red curve corresponds to an infinitely thin bunch. It is seen that for all bunches, except bunches near the front and rear edges of the chain, where the electric field is zero. There is a significant difference in the increase in the energy of the first and the last particles in each bunch.

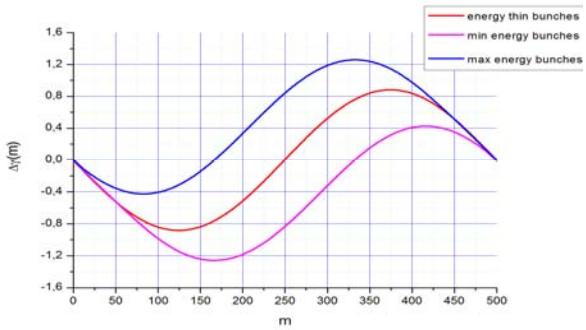

Fig. 5. Dependences of energy first particle (magenta line), last particle (blue line) of rectangular bunches upon bunch number. Red curve corresponds case of infinitely thin ring bunches. Detuning is α=0.002, *N*=500

Thus, in this paper we have shown that the introduction of small detuning between the frequency of the fundamental eigen mode excited in dielectric waveguide and bunch repetition frequency allows to excite wakefield efficiently and on the other hand, to solve the problem of injection of relativistic electron bunches in the accelerating phase of the wakefield. In this acceleration scheme the bunches of the head of the sequence, excite the wakefield and thus they are decelerated. For bunches with large numbers, in result of the phase shift, polarity of the wakefield is reversed, and these bunches are occurred in the acceleration process.

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ВОЗБУЖДЕНИЕ КИЛЬВАТЕРНОГО ПОЛЯ И УСКОРЕНИЕ ЭЛЕКТРОНОВ ПРИ НАЛИЧИИ РАССТРОЙКИ ЧАСТОТЫ СЛЕДОВАНИЯ СГУСТКОВ И ЧАСТОТЫ ОСНОВНОЙ СОБСТВЕННОЙ КИЛЬВАТЕРНОЙ ВОЛНЫ

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Изложены результаты теоретических исследований процесса возбуждения кильватерного поля в диэлектрическом волноводе последовательностью релятивистских сгустков при наличии расстройки между частотой следования сгустков и частотой основной волны волновода. В этом случае для кильватерных волн, излучаемых отдельными сгустками, возникает набег фазы и, начиная с некоторого номера сгустка, набег фазы составит π . Изменяется полярность поля и последующие сгустки набирают энергию. Получены энергетические спектры ускоренных (замедленных) частиц.

ЗБУДЖЕННЯ КІЛЬВАТЕРНОГО ПОЛЯ І ПРИСКОРЕННЯ ЕЛЕКТРОНІВ ЗА НАЯВНОСТІ РОЗСТРОЙКИ ЧАСТОТИ СЛІДУВАННЯ ЗГУСТКІВ І ЧАСТОТИ ОСНОВНОЇ ВЛАСНОЇ КІЛЬВАТЕРНОЇ ХВИЛІ

В.А. Балакірєв, І.М. Оніщенко, О.П. Толстолужський

Викладено результати теоретичних досліджень процесу збудження кільватерного поля в діелектричному хвилеводі послідовністю релятивістських згустків при наявності розстройки між частотою проходження згустків і частотою основної хвилі хвилеводу. У цьому випадку для кільватерних хвиль, випромінюваних окремими згустками, виникає набіг фази і, починаючи з деякого номера згустку, набіг фази складе π . Змінюється полярність поля і наступні згустки набирають енергію. Отримані енергетичні спектри прискорених (уповільнених) частинок.